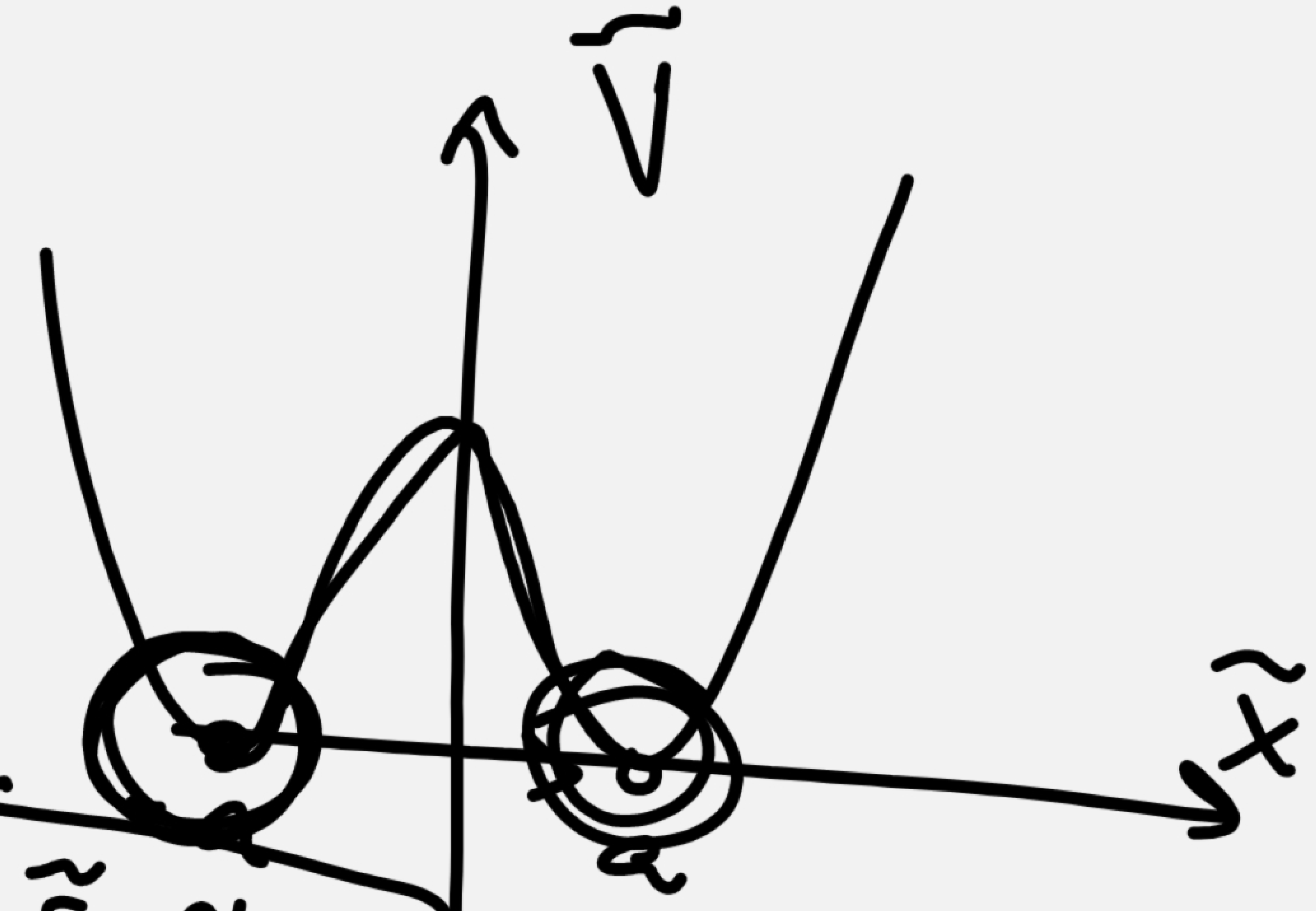
Туннелирование в КТП.

Лекция 2.

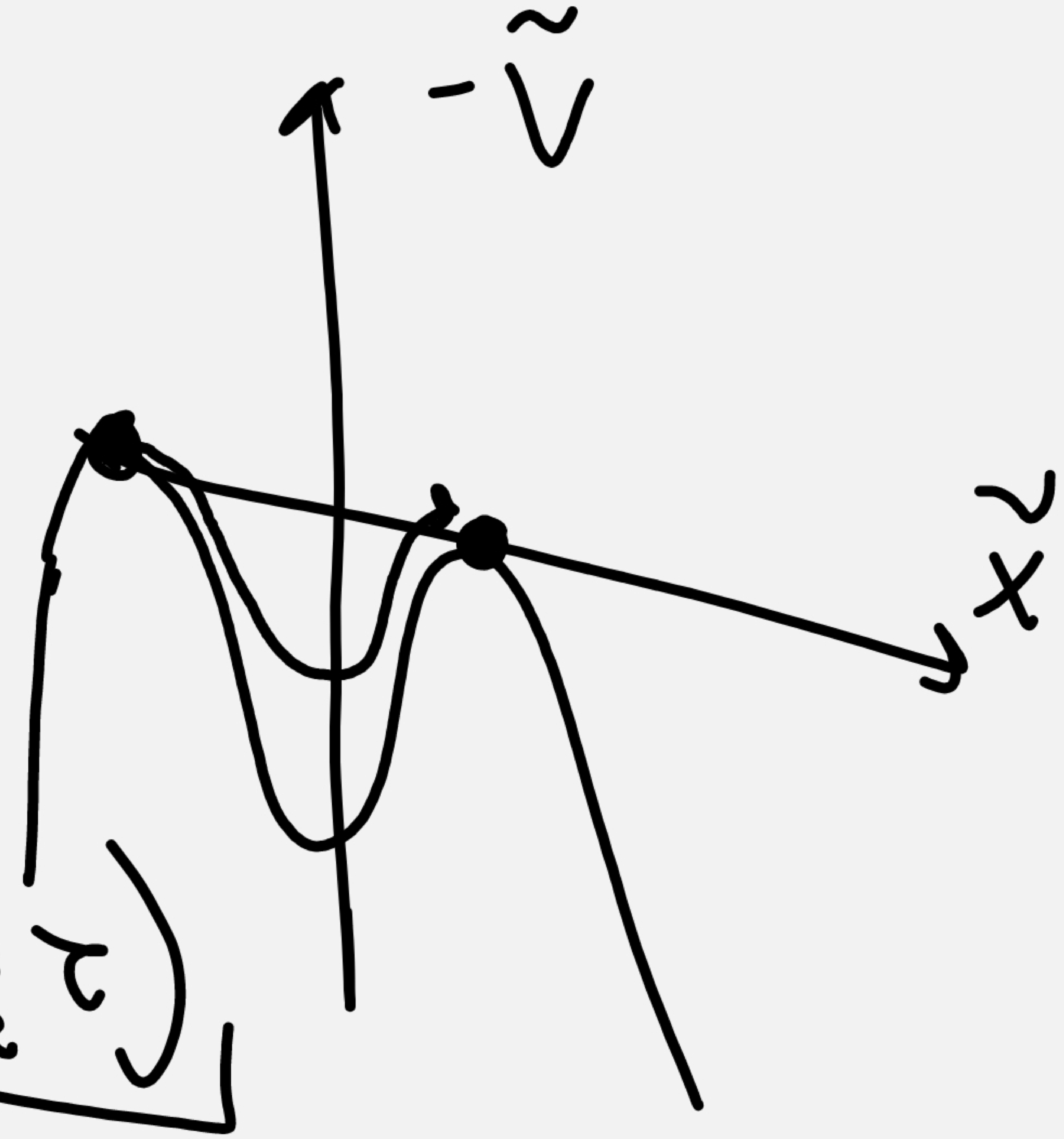
$$V = \frac{1}{g^2}\,\frac{1}{4}\left(\tilde{x}^2 - a^2\right)^2$$



$$\langle +a|\, e^{-H\tau_{fi}}\, |-a\rangle = \int \mathcal{D}\tilde{x}(\tau)\, e^{-\frac{\tilde{S}[\tilde{x}]}{g^2}}$$

$$\tilde{S} = \int d\tau \left(\frac{1}{2}\left(\frac{d\tilde{x}}{d\tau}\right)^2 - V \right)$$

$$\frac{d^2}{d\tau^2}\tilde{x} = -\frac{d}{dx}\left(-\tilde{V}\right) \longrightarrow \tilde{x}_I = a\,\mathrm{th}\left(\frac{a}{\sqrt{2}}\tau\right)$$



$$\tilde{x}_I = a\,\mathrm{th}\left(\frac{a}{\sqrt{2}}\tau\right)$$

$$x = x_I + \delta x$$

$$\langle a|\, e^{-H\tau_{fi}}\, |-a\rangle = N\, \frac{e^{-\frac{\tilde{S}[\tilde{x}_I]}{g^2}}}{\sqrt{\det\left(-\frac{d^2}{d\tau^2} + \tilde{V}''(x_I)\right)}} \qquad \tau_{fi} \to \infty$$

$$\left.\frac{d\psi}{d\tau}\right|_{\tau_i} = 1, \qquad \psi(\tau_i) = 0, \qquad \hat{L}\psi = 0$$

$$\hat{L} = -\frac{d^2}{d\tau^2} + \hat{V}''(\tilde{x})$$

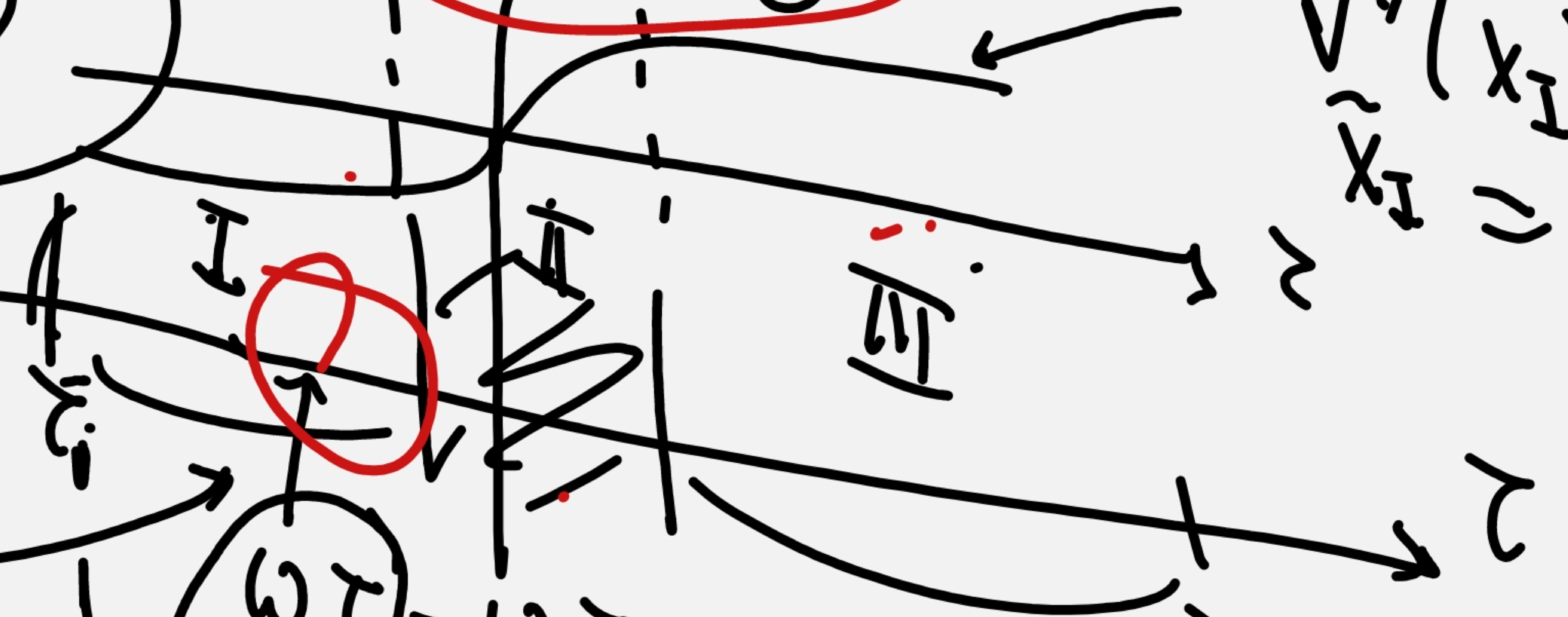
$$\hat{L}\Big|_{\text{большие } |\tau|} \approx -\frac{d^2}{d\tau^2} + \omega^2$$

$$2a^2 = \omega^2$$

при $|\tau| \gg \frac{1}{a}$

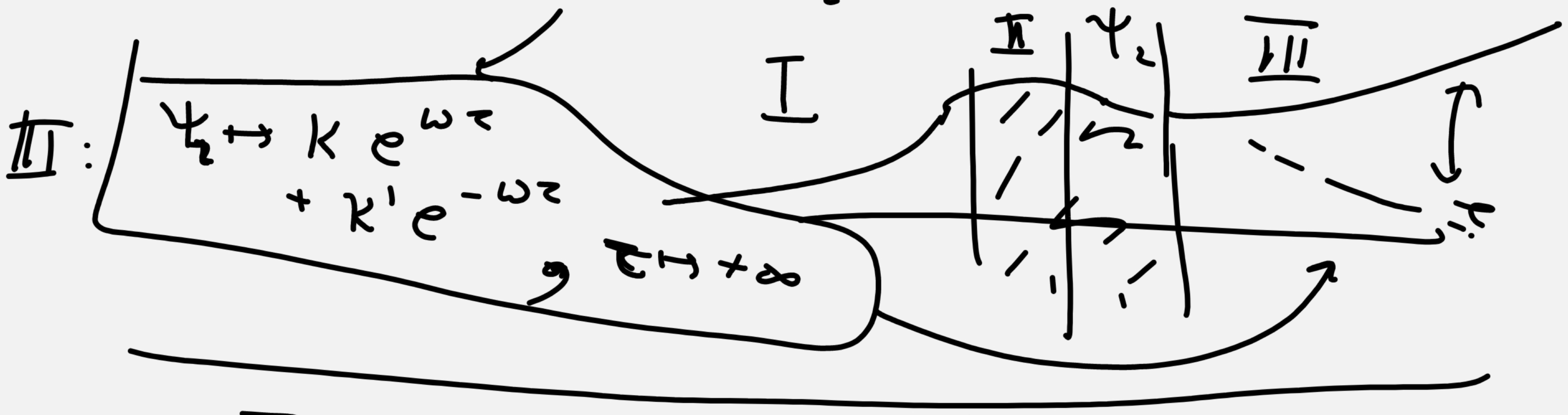
$$\tilde{V}''(x_I) \approx 2a^2$$

$$\tilde{x}_I \approx \mathrm{sgn}(\tau)\cdot a$$



$$\psi = \frac{1}{\omega}\,\mathrm{sh}\left(\omega(\tau - \tau_i)\right) \to \frac{1}{2\omega} e^{\omega\tau - \omega\tau_i} = \frac{1}{2\omega} e^{-\omega\tau_i}\, \psi_2$$

$$\psi_2 : \begin{cases} \hat{L}\psi_2 = 0 \\ \psi_2 \to e^{\omega\tau}, \text{ при больших отр. } \tau. \end{cases}$$



III $\quad \psi = \frac{1}{2\omega} e^{-\omega\tau_i} \psi_2 \xrightarrow{\text{больше } \tau} \frac{1}{2\omega} e^{-\omega\tau_i} \cdot K e^{\omega\tau} + \dots$

$$\det\left(-\frac{d^2}{d\tau^2} + V''(x_I)\right) = N\psi(\tau_f) = \underbrace{\frac{1}{2\omega} e^{\omega(\tau_f - \tau_i)}}_{\text{осцилл.}} K$$

$$\langle a | e^{-H\tau_{fi}} | -a \rangle = \frac{\alpha N^{-1/2}}{\sqrt{K}} e^{-\frac{\omega}{2}(\tau_{fi}) - \sqrt{2\omega}} e^{-\frac{S[X_I]}{g^2}}$$

Осциллятор.

$$= \sqrt{\frac{\omega}{\pi}}\, e^{-\frac{\omega}{2}\tau_{fi}} \frac{1}{\sqrt{K}} e^{-\frac{S[X_I]}{g^2}}$$

$\tilde{x}_I$

$$\tilde{S}[\tilde{x}_I] = \int d\tau \left(\frac{1}{2}\left(\frac{d\tilde{x}_I}{d\tau}\right)^2 + V(\tilde{x}_I) \right)$$

Ур-ние Ньютона в пот. $(-V)$

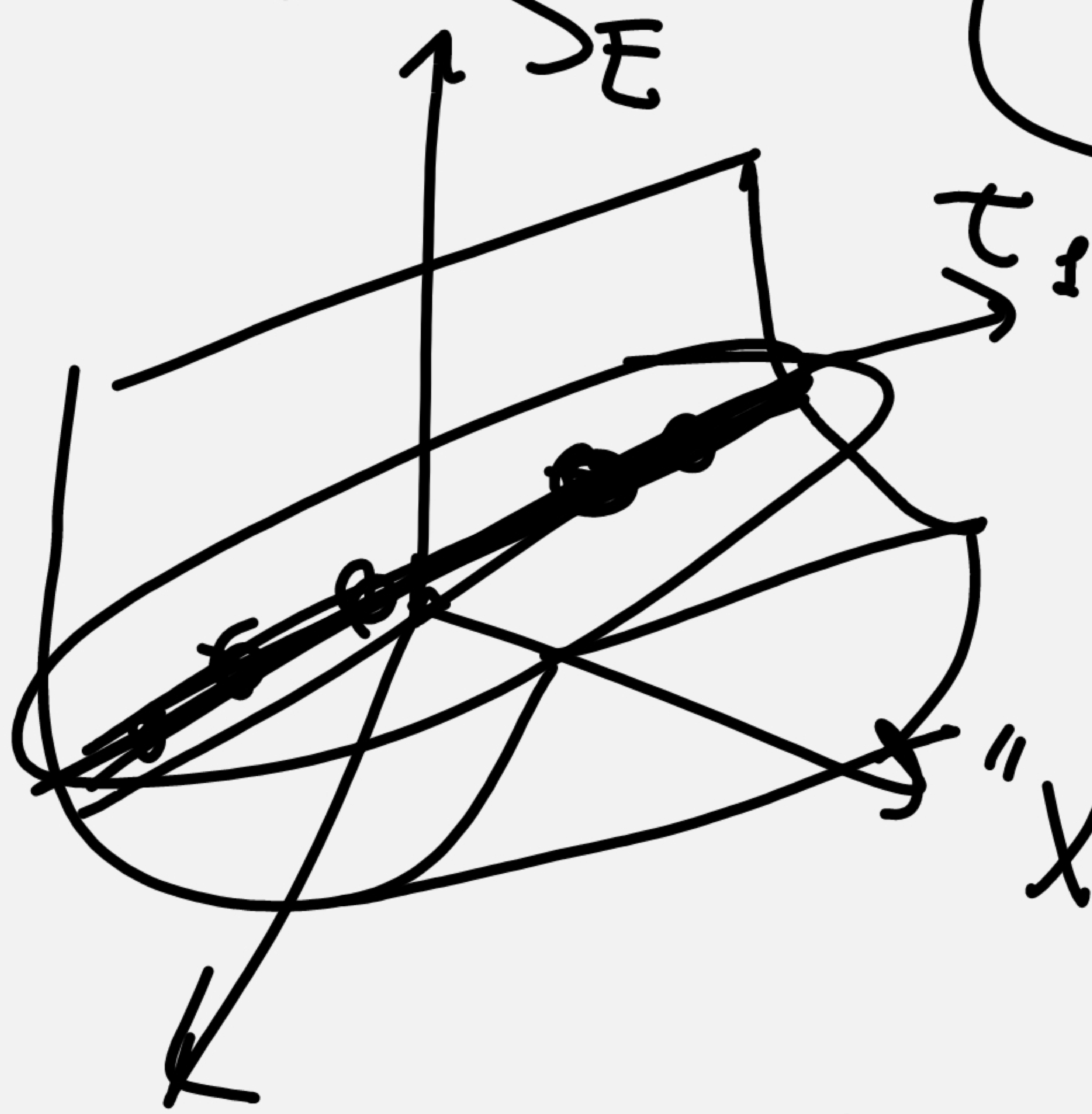
$$\tilde{E} = \frac{1}{2}\left(\frac{d\tilde{x}_I}{d\tau}\right)^2 - V_I = 0 \Rightarrow \frac{d\tilde{x}_I}{d\tau} = \sqrt{2V_I}$$

$$\tilde{S}[\tilde{x}_I] = \int d\tilde{x}_I \frac{d\tilde{x}_I}{d\tau} = \int_{-a}^{a} d\tilde{x}_I \sqrt{2V}$$

$$X_I = a\,\mathrm{th}\left(\frac{a}{\sqrt{2}}(\tau-\tau_1)\right)$$

cagno.

$$\det\left(-\frac{d^2}{d\tau^2}+V''_{I}\right)=0$$



$$\frac{d^2X_I}{d\tau^2} = V'(X_I).$$

$$A = \int \mathcal{D}x(\tau)\, e^{-\frac{\tilde{S}_E[\tilde{x}]}{g^2}}$$

$$\frac{d^2}{d\tau^2}\left(\frac{dX_I}{d\tau}\right) = V''\frac{dX_I}{d\tau}.$$

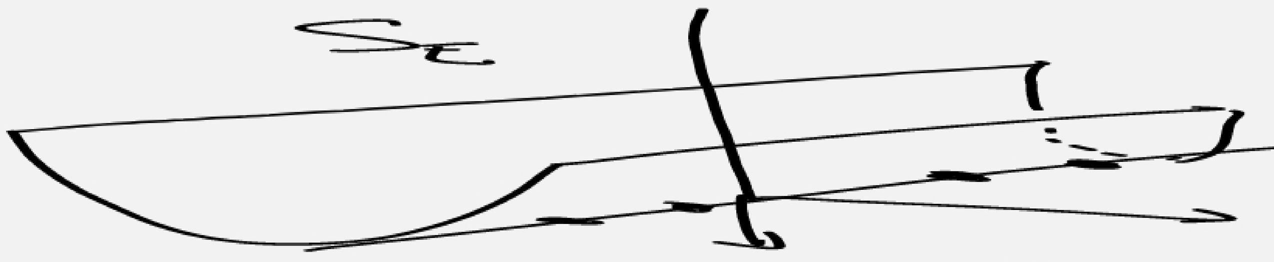
$$L\,\frac{dX_I}{d\tau} = 0 \implies \det L = 0 \iff K = 0$$

$$A = \langle -a|e^{-H\tau_{fi}}|a\rangle = \int_{\tau_i}^{\tau_f} d\tau_1 \frac{e^{-\frac{\omega}{2}\tau_{fi}}}{\sqrt{K}}\, e^{-\frac{S[\tilde{X}_I]}{g^2}}$$

Лекция 3.

$\bar{x}_I = a\,\mathrm{th}\left(\frac{a}{\sqrt{2}}(\tau-\tau_1)\right).$

$\bar{x}(\tau_i) = -a$
$\bar{x}(\tau_1) = 0$
$\bar{x}(\tau_f) = a.$



$1 = \int_{\tau_i}^{\tau_f} d\tau_1\, \delta(x(\tau_1))\, \dot{\bar{x}}(\tau_1)$

$= \int d\tau_1\, \dot{x}(\tau_1)\, \delta(x(\tau_1)) = 1$

$\langle +a | e^{-H\tau_{fi}} | a \rangle = \int d\tau_1 \int \mathcal{D}x(\tau)\, \dot{x}(\tau_1)\, \delta(\bar{x}(\tau_1))\, e^{-\frac{S[x]}{g^2}}$

$= \int d\tau_1 \left(\int \mathcal{D}x(\tau) \Big|_{\tau_i}^{\tau_1} e^{-\frac{S[x]}{g^2}} \right) \times \quad \leftarrow \bar{x}_I = a\,\mathrm{th}\left(\frac{a}{\sqrt{2}}(\tau-\tau_1)\right)$

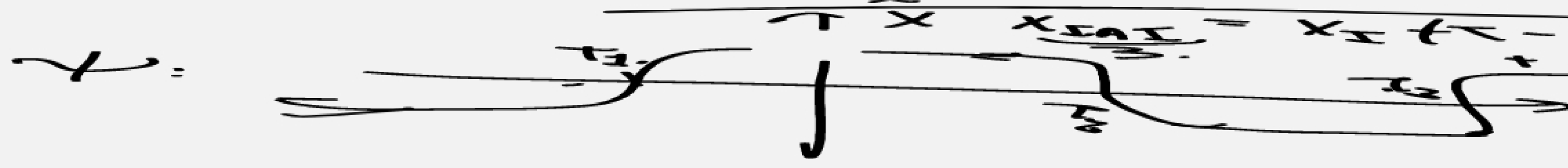
$\times \left(\int \mathcal{D}x(\tau) \Big|_{\tau_1}^{\tau_f} e^{-\frac{S[x]}{g^2}} \right) \quad \dot{x}(\tau_1) =$

$= \int d\tau_1\, e^{-\frac{S(\bar{x}_I)}{g^2}} \frac{1}{\sqrt{k'}}\, e^{-\frac{\omega}{2}(\tau_f-\tau_i)} \sqrt{\frac{\omega}{\pi}}$

$\frac{1}{\sqrt{k'}}\, e^{-\frac{\omega}{2}(\tau_f - \tau_1 - \tau_i + \tau_1)}. \quad \sqrt{\frac{\omega}{\pi}} = \dot{x}(\tau_1)\, \det\left(-\partial_\tau^2 + V''(x_I)\right)^{-1/2} \quad \det L \Big|_0^{\tau_f-\tau_i} \quad \frac{\tau_f - \tau_i}{2}$

$\dot{x}_I(\tau_1) = \frac{a^2}{\sqrt{2}} \qquad a\sqrt{2} = \omega$

$\langle a | e^{-H\tau_{fi}} | -a \rangle = \int_{\tau_{fi}} d\tau_1 \sqrt{\frac{\omega}{\pi}}\; \frac{e^{-\frac{\omega}{2}(\tau_f-\tau_i)}}{\sqrt{k'}}\; e^{-\frac{S(\bar{x}_I)}{g^2}}$



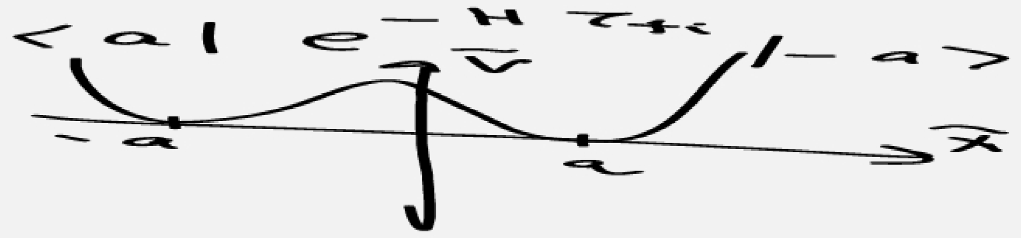
$x_{I\bar{A}I} = x_I(\tau-\tau_1) + x_A(\tau-\tau_2) + x_I(\tau-\tau_3)$

$x_I = a\,\mathrm{th}\left(\frac{a}{\sqrt{2}}(\tau-\tau_1)\right).$

$x_A(\tau-\tau_2) = -a\,\mathrm{th}\left(\frac{a}{\sqrt{2}}(\tau-\tau_2)\right)$

$\omega\tau_{fi}\, e^{-\frac{S_I}{g^2}} \sim 1$

$\langle a | e^{-H\tau_{fi}} | -a \rangle = \sqrt{\frac{\omega}{\pi}} \sum_{\substack{n=2k+1 \\ k=0}} e^{-\frac{nS_I}{g^2}}\, \frac{e^{-\frac{\omega}{2}(\tau_f-\tau_i)}}{(\sqrt{k'})^n}\, \frac{\tau_{fi}^n}{n!}$



$\frac{\tau_{fi}^n}{n!} \leftarrow \frac{1}{n!} \int d\tau_1 \dots d\tau_n$

$= \sqrt{\frac{\omega}{\pi}} \left(\sum_{\substack{k=0 \\ n=2k+1}} \frac{1}{n!} \left(\frac{\tau_{fi}\, e^{-\frac{S_I}{g^2}}}{\sqrt{k'}} \right)^n \right) e^{-\frac{\omega}{2}\tau_{fi}}$

$\psi(\tau)\big|_{\tau \sim \tau_i} \sim \frac{1}{\omega}\,\mathrm{sh}(\omega(\tau-\tau_i))$

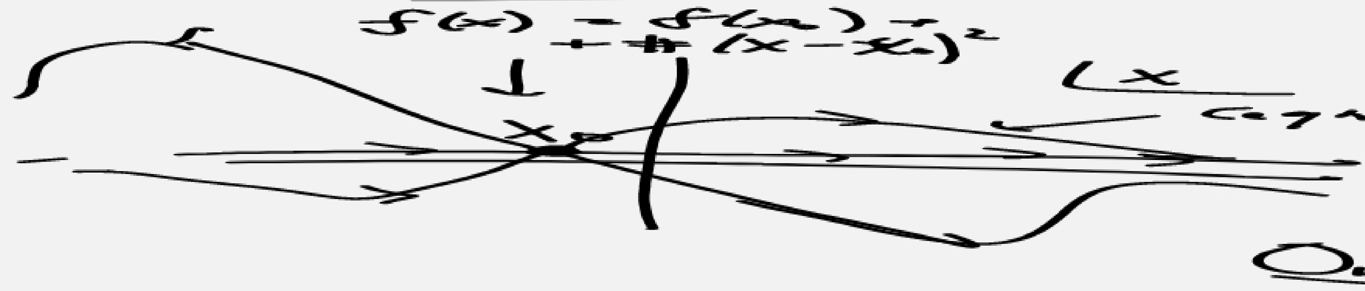
$= \sqrt{\frac{\omega}{\pi}}\; \mathrm{sh}\left(\frac{\tau_{fi}\, e^{-\frac{S_I}{g^2}}}{\sqrt{k'}} \right) e^{-\frac{\omega}{2}\tau_{fi}}$

$\Delta E\, \tau_{fi}$

$= \frac{1}{2}\sqrt{\frac{\omega}{\pi}} \left(e^{-\frac{\omega}{2}\tau_{fi} + \Delta E\, \tau_{fi}} - e^{-\frac{\omega}{2}\tau_{fi} - \Delta E\, \tau_{fi}} \right)$

$e^{-E_+ \tau_{fi}} \qquad e^{-E_- \tau_{fi}}$

$E_\pm = \frac{\omega}{2} \mp \Delta E \qquad \Delta E = \frac{1}{\sqrt{k'}}\, e^{-\frac{S_I}{g^2}}$



$Z = \int_{-\infty}^{+\infty} dx\, e^{-\frac{f(x)}{g^2}} \Big|_{g^2 \to 0} = e^{-\frac{f(x_0)}{g^2}} \left(A_0 + g^2 A_1 + g^4 A_2 + \dots \right)$

$\swarrow x$ седл. контур. $\qquad e^{-\frac{f}{g^2}}$

Опр: Седл. контур: $\mathrm{Im}\, f = \mathrm{Im}\, f(x_0)$

Опр Контур наиск. спуска = (седл. контур, где $f(x_0)$ — лок. мин.

Опр — " — наиб. подъёма — где $f(x_0)$ — лок. макс.

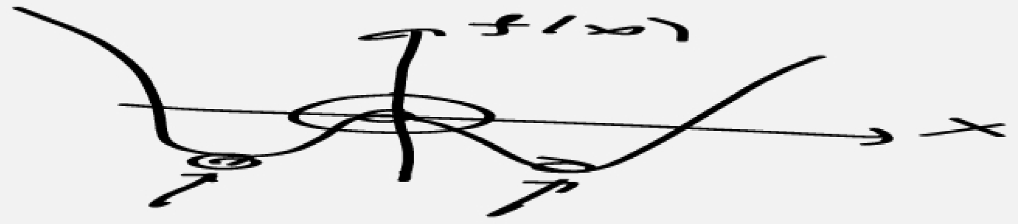
Опр: Контур Лефшеца = контур наиск. спуска. (Lefshetz thimble).

$\underline{\mathrm{th}}$ Конт. Лефшеца $\Rightarrow$ сумм. по Борелю к точному ответу. < 1 седл. точкой

$f = x^2 + \lambda x^4$
$x = 0$ — седл.
$\lambda > 0.$

$\int_{x\in\mathbb{R}} dx\, e^{-\frac{f(x)}{g^2}} = $ PSG

$f(x) = -\frac{1}{2}x^2 + \lambda x^4 \quad \lambda > 0.$



$\int_{x\in\mathbb{R}} dx\, e^{-\frac{f(x)}{g^2}}$

$$I_1 = \frac{\lambda}{2}\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2+m^2} \longrightarrow \frac{\lambda}{2}\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2+m^2} = I(d)$$

$$I(d) = \frac{\lambda}{(2\pi)^d}\int d\Omega_d \int_0^\infty dk\,\frac{k^{d-1}}{k^2+m^2}$$

$$\int d\Omega_d = \frac{2\pi^{d/2}}{\Gamma(d/2)}$$

$$\frac{\pi^{d/2}}{\Gamma(d/2)(2\pi)^d} = \quad d=4 \quad \frac{\pi^2}{16\pi^{4}2}$$

$$\frac{1}{m^2}\int_0^\infty dk\,\frac{k^{d-1}}{1+\frac{k^2}{m^2}} = \left\langle \frac{k^2}{m^2}=t,\ k=m\sqrt{t},\ dk=\frac{1}{2}\frac{m}{\sqrt{t}}dt \right\rangle =$$

$$= \frac{1}{2m^2}\int_0^\infty dt\,\frac{m}{\sqrt{t}}\frac{m^{d-1}\,t^{\frac{d-1}{2}}}{(1+t)} = \frac{1}{2m^{2-d}}\boxed{\int_0^\infty dt\,\frac{t^{d/2-1}}{1+t}} =$$

$$\boxed{\int_0^\infty dt\,\frac{t^{d/2-1}}{1+t}} = B(d/2, 1-d/2)$$

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

$$= \frac{1}{2m^{2-d}}\frac{\Gamma(d/2)\Gamma(1-d/2)}{\Gamma(1)} \xrightarrow{d\to 4} \Gamma(-1)$$

$$\triangleleft \quad d = 4-2\varepsilon$$

$$\frac{1}{2m^{2-d}}\cdot\Gamma(1-2+\varepsilon) = \frac{1}{2m^{2-d}}\ \frac{\Gamma(2-d/2)}{1-d/2} = -\Gamma(\varepsilon)$$

$$\Gamma(2-d/2) = (1-d/2)\Gamma(1-d/2)$$

$$\Gamma(z) = \boxed{\frac{1}{z}-\gamma} + \frac{1}{2}\left(\gamma^2+\frac{\pi^2}{6}\right)z$$

$$\Gamma(\varepsilon) = \frac{1}{\varepsilon}-\gamma$$

$$-\mu^{2\varepsilon}\frac{\lambda}{2}\frac{1}{2m^{2+2\varepsilon}}\frac{1}{(4\pi)^{2-\varepsilon}}\left(\frac{1}{\varepsilon}-\gamma+O(\varepsilon)\right)$$

$$\frac{\lambda}{4}\frac{m^2}{16\pi^2}\left(\frac{\mu^2\,4\pi}{m^2}\right)^{\varepsilon} = \frac{\lambda}{4}\frac{m^2}{16\pi^2}\left(1+\varepsilon\ln\frac{m^2}{\mu^2 4\pi}\right)\cdot\left(\frac{1}{\varepsilon}-\gamma\right) =$$

$$= -\frac{\lambda}{32}\frac{m^2}{\pi^2}\left(\frac{1}{\varepsilon}-\gamma+\ln\frac{m^2}{4\pi\mu^2}\right)$$

$$\beta(\lambda) = \frac{3\lambda^2}{16\pi^2} + c\lambda^4 \qquad a = \frac{3}{16\pi^2}$$

$$\mu \frac{d\lambda}{d\mu} = \frac{3\lambda^2}{16\pi^2} + c\lambda^4$$

$$\int_{\lambda_0}^{\lambda} \frac{d\lambda}{a\lambda^2 + c\lambda^4} = \int_{\mu_0}^{\mu} \frac{d\mu}{\mu} \qquad \frac{1}{\lambda} = t;\ dt = \frac{-d\lambda}{\lambda^2}$$

$$-\int_{\frac{1}{\lambda_0}}^{\frac{1}{\lambda}} \frac{dt}{a + \frac{c}{t^2}} = \ln\mu \Big|_{\mu_0}^{\mu}$$

$$\int \frac{dt\, t^2}{at^2 + c} = \ln\mu - \ln\mu_0$$

$$-\frac{1}{a}\int \frac{dt\left(\frac{a}{c}t^2 + 1 - 1\right)}{\frac{a}{c}t^2 + 1} = -\frac{1}{a}\int_{\frac{1}{\lambda_0}}^{\frac{1}{\lambda}} dt + \frac{1}{a}\int_{\frac{1}{\lambda_0}}^{\frac{1}{\lambda}} \frac{dt}{\frac{a}{c}t^2 + 1} =$$

$$= -\frac{1}{a}\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) + \frac{1}{a}\frac{1}{\sqrt{\frac{a}{c}}}\, \text{arctg}\left(\sqrt{\frac{c}{a}}\, t\right)\Big|_{\frac{1}{\lambda_0}}^{\frac{1}{\lambda}}$$

$$= -\frac{1}{a}\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) + \sqrt{\frac{c}{a^3}}\left(\text{arctg}\left(\sqrt{\frac{c}{a}}\frac{1}{\lambda}\right) - \text{arctg}\left(\sqrt{\frac{c}{a}}\frac{1}{\lambda_0}\right)\right)$$

$$= \left(\frac{1}{\lambda_0} - \frac{1}{\lambda}\right) + \sqrt{\frac{c\,16\pi^2}{3}}\left(\text{arctg}\sqrt{\frac{16\pi^2}{3c}}\frac{1}{\lambda} - \text{arctg}\sqrt{\frac{16\pi^2}{3c}}\frac{1}{\lambda_0}\right) = \ln\mu - \ln\mu_0$$

$$\text{arctg}(1/x) = \frac{\pi}{2} - \text{arctg}\, x$$

$$= \frac{1}{\lambda_0} - \frac{1}{\lambda} + c\left(\frac{1}{\lambda_0} - \frac{1}{\lambda}\right) \quad - \text{arctg}\sqrt{\frac{3c}{16\pi^2}}\frac{1}{\lambda} + \text{arctg}\sqrt{\frac{3c}{16\pi^2}}\frac{1}{\lambda_0}$$

$$-\sqrt{3c}\,\frac{1}{\lambda} + \sqrt{3c}\,\frac{1}{\lambda_0}$$

$$-\frac{1}{2}\, c\, \frac{1}{\lambda} + \frac{1}{2}\, c\, \frac{1}{\lambda_0}$$

$$\frac{1}{\lambda_0} - \frac{1}{\lambda} = \frac{\ln\frac{\mu}{\mu_0}}{1 + c}$$

$$\lambda = \frac{1}{\frac{1}{\lambda_0} - \frac{1}{1+c}\ln\frac{\mu}{\mu_0}}$$

$$\frac{a\ln\mu}{a^n(a\ln\mu)}\ ;\ \frac{a^2\ln\mu}{a^3\ln\mu}$$

$$\lambda = \frac{\lambda_0}{1 - \frac{\lambda_0}{1+c}\ln\frac{\mu}{\mu_0}}$$

$$\frac{1}{a}\left(\sqrt{\frac{b}{a}}\,\text{arcctg}\left(\sqrt{\frac{b}{a}}\,\lambda(\mu)\right) + \frac{1}{\lambda(\mu)}\right) = -a\ln\frac{\mu}{\mu_0} + \frac{1}{\lambda(\mu_0)} + \frac{1}{a}\sqrt{\frac{b}{a}}\cdot \text{arcctg}\sqrt{\frac{b}{a}}\lambda_0$$

$$\frac{\lambda(\mu)}{1 + \sqrt{\frac{b}{a}}\,\lambda(\mu)\,\text{arcctg}\left(\sqrt{\frac{b}{a}}\lambda(\mu)\right)} = \frac{\lambda(\mu_0)}{1 - a\lambda(\mu_0)\ln\frac{\mu}{\mu_0} + \sqrt{\frac{b}{a}}\,\text{arcctg}\left(\sqrt{\frac{b}{a}}\lambda(\mu_0)\right)}$$