PRIMORDIAL BLACK HOLES

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Content :

* Creation of BH in the early Universe
* Detection of BH & conctraints
* Capture of PBH by stars
* Capture at star birth
* Perspectives for observations/ constraints

Lecture 1.

⊛ Dark Matter (DM) — main motivation for PBH

- DM is needed to explain many observational facts of different nature:
  - ▸ rotation curves
  - ▸ gas temperature in clusters of galaxies
  - ▸ CMB anisotropy data
  - ▸ structure formation
  - ▸ Bullet cluster

⇛ all this can be explained by DM

<u>BUT</u>: only indirect evidence
+ only of gravitational origin
[no direct detection so far]

⇛ no clue what is the DM mass.

⊛ PBH are an attractive candidate for DM

- basically invisible
- only interacting gravitationally, including with ordinary matter
- no new stable particle is required if constitute all of the DM

⊛ BH – reminder $[c = \hbar = 1]$

$M_{pe} = 10^{19}$ GeV
$M_{\odot} = 10^{57}$ GeV

- formed in collapse of massive stars

P1.1. Esimate J_{max}

- only characterized by mass m and momentum J

- horizon size r_g vs. mass m:

$$\boxed{r_g = 2Gm = \frac{2m}{M_{pe}^2}}$$

Note: linear scaling with m

For $m = M_{\odot} = 10^{57}$ GeV $= 10^{33}$ g

$$r_g \simeq 3 \text{ km}$$

- temperature

$$\boxed{T = \frac{1}{4\pi} \frac{1}{r_g}} \propto \frac{1}{m}$$

- Hawking radiation (black body with temp. T)

$$\frac{dm}{dt} = -4\pi r_g^2 \cdot \sigma_B T^4 \propto T^2 \propto \frac{1}{m^2}$$

P1.2 Evaporation time

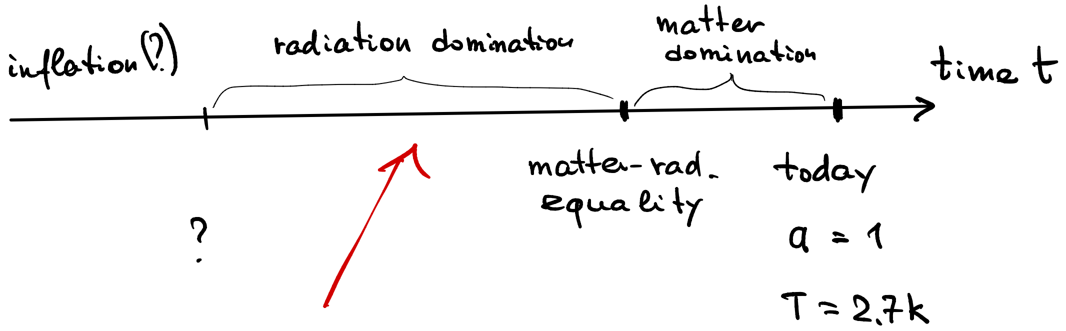
Note: negative heat capacity!

⊛ BH formation in the early Universe

* flat FRWL metric $ds^2 = dt^2 - a^2(t)\, d\vec{x}^2$

a = scale factor ; $a=1$ today

* Standard cosmology :



This is our range of interest

$$\boxed{a \propto \sqrt{t}}$$

* Key quantity – expansion rate H

$$H \equiv \frac{\dot a}{a} \qquad \text{(Hubble parameter)}$$

From Friedmann equation

$$\boxed{H^2 = \frac{8\pi}{3} G \rho}$$

energy density

Newton's constant

$$G = \frac{1}{M_{pe}^2} \; ; \; M_{pe} = 10^{19}\,\text{GeV}$$

At radiation stage

$$\rho \propto T^4 \quad \Rightarrow \quad \boxed{H \propto \frac{T^2}{M_{pe}}}$$

- There is a characteristic length scale – <u>horizon</u>

$$R_H \sim \frac{1}{H} \sim \frac{M_P}{T^2}$$

$\left[\text{at the radiation stage } R_H = \frac{1}{2H}\right]$

$$R_H \propto a^2 \propto t$$

- At the radiation stage the horizon size is the maximum size of a causally-connected region $\Rightarrow$ maximum mass of a BH!

- Total mass inside horizon:

$$M_H \sim \rho R_H^3 \sim T^4 \cdot \left(\frac{M_P}{T^2}\right)^3 = \frac{M_P^3}{T^2}$$

$\Rightarrow$ maximum mass of a BH formed at temperature T is

$$\boxed{m \lesssim M_H = 0.02 \frac{M_{Pl}^3}{T^2}}$$

T	$M_H \sim m$
100 GeV	$3\times10^{-6}\, M_\odot = 6\cdot10^{27}$ g
100 MeV	$3\, M_\odot = 6\cdot10^{33}$ g
1 MeV	$3\cdot10^{4}\, M_\odot = 6\cdot10^{37}$ g

$M_\odot = 2\cdot10^{33}$ g

* Is it easy to produce enough PBH to constitute <u>all</u> of the DM? – yes!

Indeed,

$$\rho_{PBH} \propto \frac{1}{a^3}$$

$$\rho_{rad} \propto \frac{1}{a^4}$$

$\Rightarrow$ As a increases, the relative abundance of PBH <u>grows</u>

P1.3

$$\frac{\rho_{PBH}}{\rho_{rad}} \propto a$$

$\Rightarrow$ ρ_{PBH} must be subdominant at the moment when they are created

⊛ How to make a BH in the early Universe?

To form a BH one must compactify a mass into a region smaller than its Schwarzschild radius [Hoop conjecture, Thorn 1972]

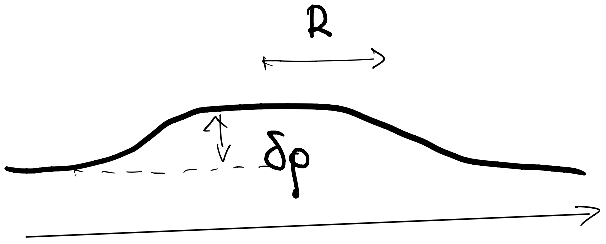
* Many mechanisms at various cosmological epochs have been proposed:

- from primordial density perturbations
- bubble collisions at phase transitions
- from collapse of cosmic strings
- at reheating
- at preheating
- ...

most popular now

* BH from primordial density perturbations

Assume we have $\delta\rho$ at scale R.



$\Rightarrow$ mass of the bump is

$$m \sim \delta\rho \cdot R^3$$

$$r_s = \frac{\delta\rho \cdot R^3}{M_p^2} \qquad (\text{its grav. radius})$$

$\Rightarrow$ to form a BH require $r_s \gtrsim R$ (Hoop conjecture)

$$\Rightarrow \boxed{\frac{\delta\rho \cdot R^2}{M_p^2} \gtrsim 1}$$

$\Rightarrow$ smaller $\delta\rho$ correspond to larger R

But the largest R is $R = R_H$.

Then we have

$$H^2 = \frac{\rho}{M_p^2}$$

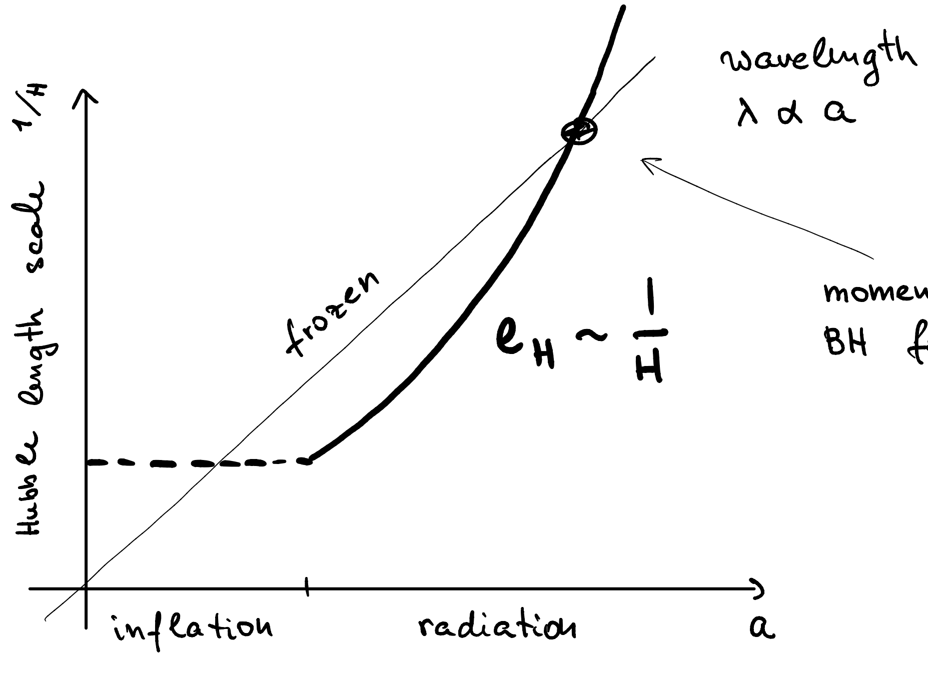
$$R_H^2 = \frac{1}{H^2} = \frac{M_p^2}{\rho}$$

$$1 \sim \frac{\delta\rho\, R_H^2}{M_p^2} = \frac{\delta\rho}{M_p^2}\,\frac{M_p^2}{\rho} = \frac{\delta\rho}{\rho}$$

$\Rightarrow$ the smallest $\delta\rho$ stile sufficient to form a BH satisfies:

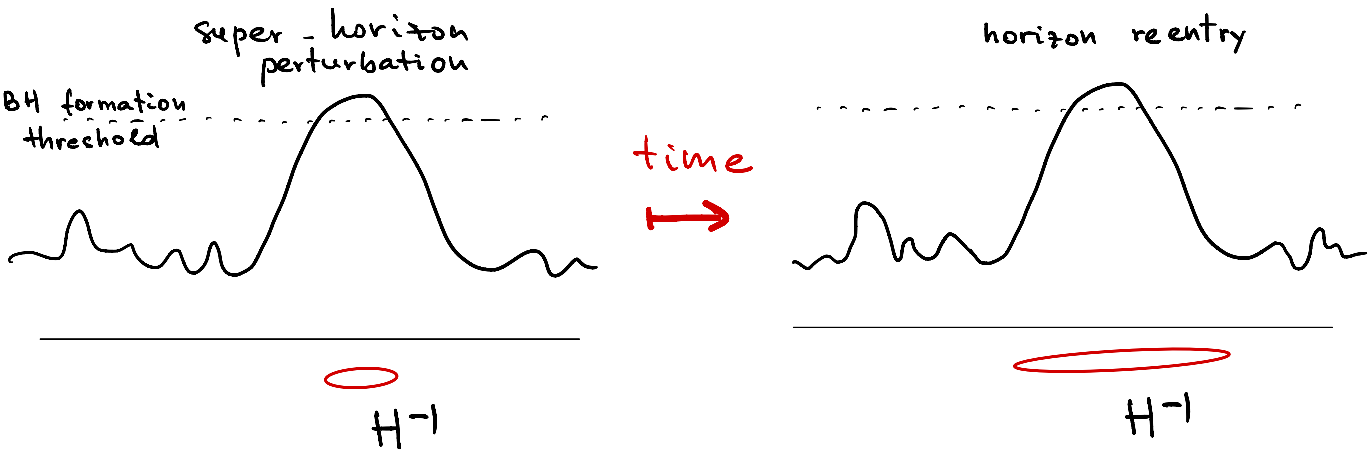
$$\boxed{\frac{\delta\rho}{\rho} \sim 1} \text{ at horizon scales}$$

⊛ Collapse of primordial perturbations



- inflation $a \propto e^{Ht}$ → perturbations exit horizon and after that stay frozen

- RD stage → perturbations re-enter the horizon. If they overshoot the threshold $\delta\rho/\rho \sim 1$ they form a BH after re-entry. Otherwise - oscillate.



⊛ Because we want to convert into BH only a small fraction of Hubble volumes at the time of production, we do not need $\delta\rho/\rho \sim 1$ on average. Then only rare fluctuations will reach $\delta\rho/\rho \sim 1$ and form BH. In fact, one should have

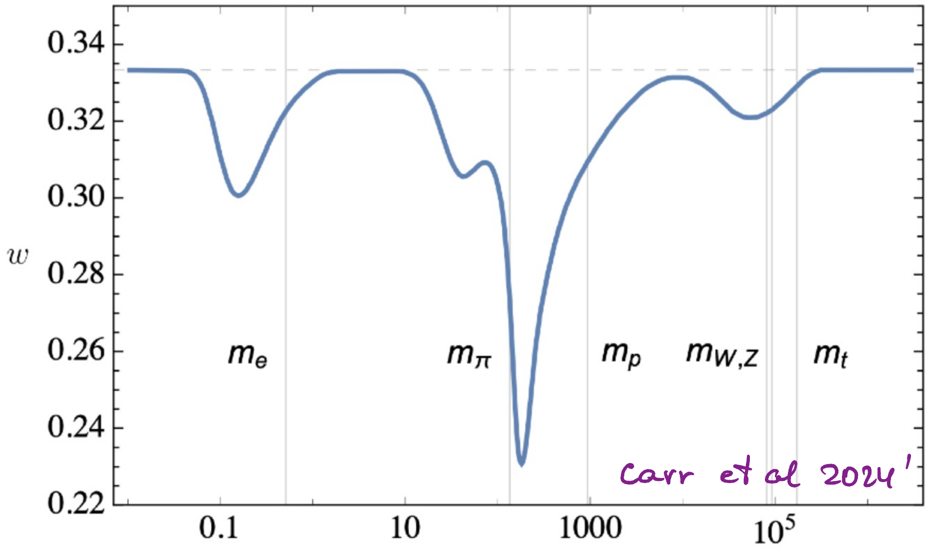
$$\delta\rho/\rho \sim 10^{-2}$$

⊛ The exact threshold of BH formation depends on the equation of state,

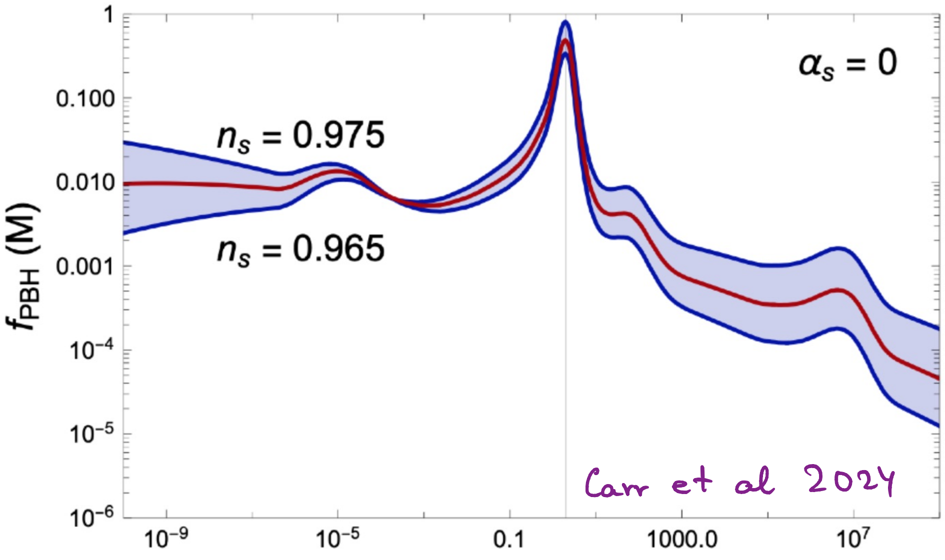
$$p = w\rho$$

Pressure opposes collapse ⇒ the smaller w, the lower the threshold.

Softening at the QCD phase transition at $T \sim 100$ MeV ⇒ enhanced production of $\sim M_\odot$ PBHs.



Carr et al 2024'



Carr et al 2024

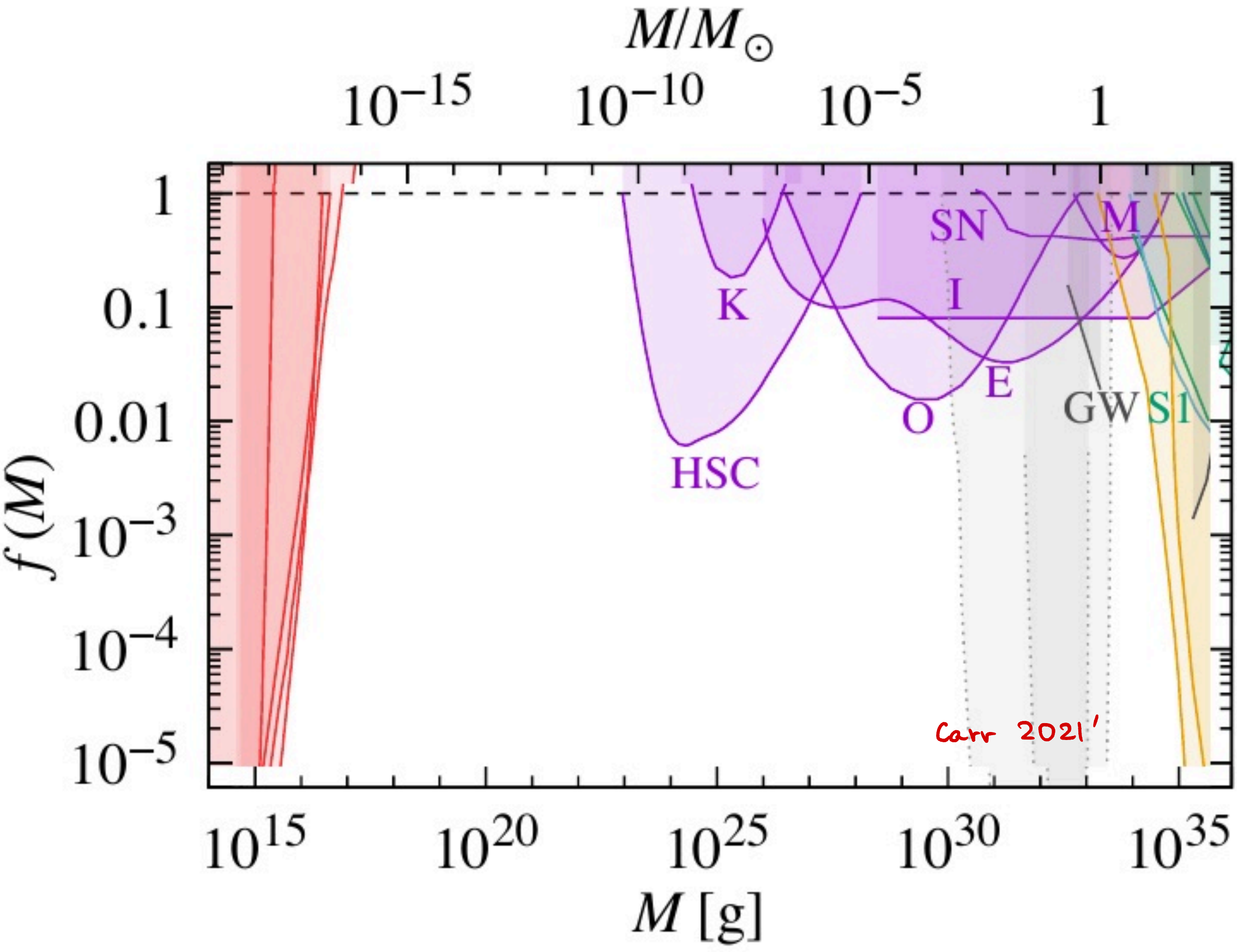
CONCLUSION:

No real constraints from theory on the mass spectrum of PBH and their abundance

Lecture 2

CONSTRAINTS

- Hawking radiation
- lensing
- LIGO/VIRGO
- CMB



Carr 2021'

Hawking evaporation

$$\frac{dm}{dt} = -\frac{1}{15360\,\pi\, G^2 m^2}$$

$$-15360\,\pi \frac{1}{M_p^4} m^2\, dm = dt$$

$$5120\,\pi \frac{m^3}{M_p^4} = \tau \quad \leftarrow \text{lifetime}$$

take $m = 10^{20}\,g = 5.3\times 10^{43}$ GeV

$$\tau = 5120\pi \left(\frac{m}{M_p}\right)^3 \frac{1}{M_p} =$$

$$= 1.3\cdot 10^{59}\ \frac{1}{\text{GeV}} =$$

$$= 3\times 10^{27}\ \text{yr}$$

Evaporation time scale:

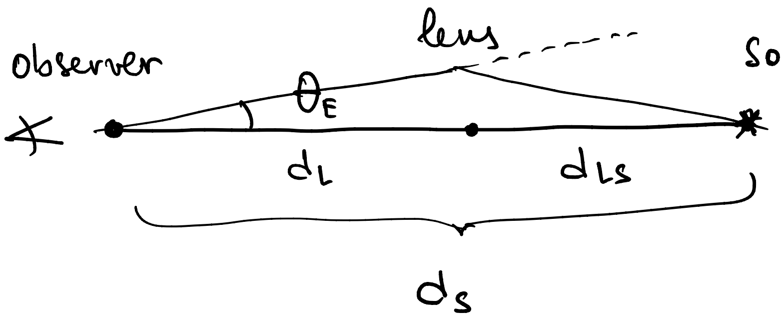
$$\boxed{\tau \sim 3\cdot 10^{9}\ \text{yr} \left(\frac{m}{10^{14}\,g}\right)^3}$$

$\Rightarrow$

- $*$ PBH lighter than $\sim 10^{15}$ g completely evaporate by now and cannot be a DM
- $*$ Moreover, the lifetime must be larger than $\sim 10^{27}$ s $\sim 10^{20}$ yr, otherwise the produced radiation overshoots the experimental limits.

P2.1 DM lifetime

Lensing



Source: stars in LMC, SMC, galactic bulge
(OGLE, MACHO, EROS)

* Einstein ring angular radius

$$\theta_E = \sqrt{\frac{2R_g \cdot d_{LS}}{d_s d_L}}$$

take $d_s, d_L, d_{LS} \sim$ kpc
$R_g = 3$ km (sun)
$\theta_E \sim 10^{-8}$
$\sim 3 \cdot 10^{-3}$ arcsec

Ideal Earth-based observations – 0.4 arcsec

* Time duration

$$t_E = \frac{R_E}{v} = \frac{1}{v}\sqrt{\frac{2R_g \cdot d_L d_{LS}}{d_s}}$$

$$= 44 \text{ days} \left(\frac{m}{M_\odot}\right)^{1/2} \left(\frac{d_L d_{LS}}{d_s \cdot 4\text{kpc}}\right)^{1/2} \left(\frac{220 \text{ km/s}}{v}\right)$$

* Amplification factor A

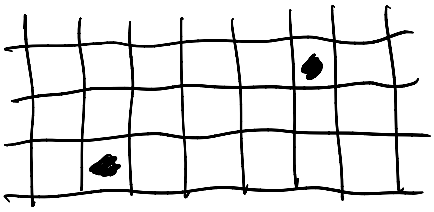
define $u \equiv \frac{\Delta\theta}{\theta_E}$ ← minimum approach

P2.2.
Amplification

$$A = \frac{u^2+2}{u\sqrt{u^2+1}}$$

LIGO/VIRGO [$m \sim$ few to 10's of $M_\odot$]

- PBH formation is a random process: at the time of formation collapse happens in only a few Hubble volumes



Important: PBH have zero velocities.

P2.3
Binary formation

- As the Universe expands, the Hubble volume grows and some of the PBH find themselves close enough to form a bound pair. They start to orbit each other and finally coalesce.

- Once the formation time (equivalently, the PBH mass) is fixed, the only free parameter in this process is the fraction of PBH in the DM:

$$\left(\begin{array}{c}\text{coalescence rate}\\ \text{today}\end{array}\right) = \text{function of } f = \frac{\Omega_{PBH}}{\Omega_{DM}}$$

↑ this is measured by LIGO/VIRGO

⇓

$$\boxed{\text{constraint on } f}$$

CMB distortions $[\gtrsim 10 M_\odot]$

- PBH are formed ~~at~~ in the radiation era at temperatures $\gg$ eV

  $\Rightarrow$ they must have been present at and after recombination, at the time ~~of~~ of CMB formation.

- Accretion of gas on these PBH after recombination produces radiation that leads to ionisation before the star formation, which is excluded / constrained from CMB observations.

> Note: accretion is a complicated process $\Rightarrow$ potentially large uncertainties.

P2.4 Collisions with Earth & Sun

TO SUMMARIZE:

- With larger or smaller confidence in all mass ranges except from $\sim 10^{17}$ g to $\sim 10^{23}$ g PBH are excluded as <u>the only</u> component of DM

This mass range goes under the name of "asteroid-mass window"

HOW CAN PBH OF THESE MASSES BE CONSTRAINED?