Lectures 3-5 ASTEROID-MASS WINDOW

Capture in stars → accretion → destruction.

ACCRETION

* In principle, a common astrophysical phenomenon (gravity being an attractive force).
  Still, a very controversial subject.
  Multiple models.

  In our context – a number of simplifications:
  - PBH are very small → its vicinity is, to a good approximation, a homogeneous medium
  - for the same reason (small size) angular momentum is not important at least at first stages.

In these conditions there is a simple solution known as Bondi accretion – a spherically symmetric incoming adiabatic flow.

* characterized by variables only depending on r: $\rho(r)$, $T(r)$, $p(r)$, $v(r)$.
* adiabatic means there is no energy exchange between layers of different r (no diffusion).

Bondi solution for accretion

* The flow is governed by two equations:
  - matter conservation

(1) $$4\pi r^2 \rho v = \dot{m}$$ ← this is the total accretion rate, for now a free parameter

  - Newton's law (non-relativistic case) known in this context as Euler equation:

(2) $$v \frac{dv}{dr} = -\frac{1}{\rho}\frac{dp}{dr} - \frac{Gm}{r^2}$$

We have 2 equations and 3 unknowns: ρ, p and v.

⇒ need one more equation

* Adiabatic approximation (no radiation, diffusion, etc)

(3) ↳ $$p = K\rho^{\Gamma}$$

K, Γ = constants

Γ = adiabatic index

P3.1 Rescaling

P3.2 Bondi radius

sound speed

$$c_s^2 = \frac{dp}{d\rho} = \Gamma \frac{p}{\rho}$$

* Using (3) one may integrate eq.(2):

(2') $$\frac{1}{2}v^2 + \frac{1}{\Gamma-1}c_s^2 - \frac{Gm}{r} = \frac{1}{\Gamma-1}c_\infty^2$$

↑ asymptotic value of sound speed

⇒ 3 algebraic eqs for 3 unknowns.

* From these equations one can show (see problems) that there is <u>one</u> particular value of $\dot{m}$ for which the flow velocity monotonically grows from 0 to the free-fall velocity $(2Gm/r)^{1/2}$ as r decreases from ∞ to 0.

This value is

$$\dot{m} = 4\pi\lambda \cdot \frac{G^2 m^2}{c_\infty^3} \cdot \rho_\infty$$

λ = const = $O(1)$

depends on Γ

* The flow becomes supersonic at the value of r called Bondi radius,

$$r_B = \frac{5-3\Gamma}{4} \frac{Gm}{c_\infty^2}$$

* Assume Bondi accretion. Then

$$\dot{m} = 4\pi\lambda \frac{G^2 m^2}{c_\infty^3} \rho_\infty$$

P3.3 accretion time

$$t_{acc} = \frac{c_\infty^3}{4\pi G^2 \rho_\infty m} \simeq 5\times 10^6 \text{ yr} \left(\frac{c_\infty}{500\,\text{km/s}}\right)^3 \left(\frac{150\,\text{g/cm}^3}{\rho}\right)\left(\frac{10^{20}\,\text{g}}{m}\right)$$

short time compared to the age of the Universe

* Lifetime is determined by the <u>initial</u> stages

* At final stages Bondi regime may break, What happens then – nobody knows for sure. This is not important for destruction time, but is important for other observational signatures apart from destruction.

* Validity of Bondi regime:

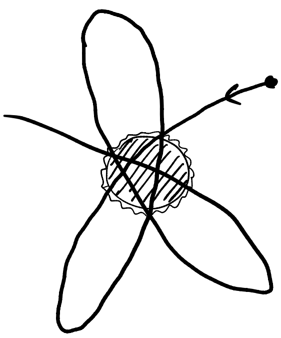
  - photon mean free path $\ll r_B$
  - angular momentum
  - relative motion of BH and medium

CAPTURE

* From time to time PBHs from DM halo collide with stars

* PBH passing through a star does not stop, but it gets slowed down (see below).

* As a result, it may become gravitationally bound to the star. If this happens, the PBH continues to cross the star, every time loosing energy until it is completely inside the star.



After that it accretes the star as discussed above.

Consider this capture mechanism in more detail. We will see that it is not efficient, but we will learn a few useful things.

* In order to get captured by a star, a PBH has to loose more energy than it has at infinity.

$\Rightarrow$ consider energy losses

ENERGY LOSSES

There are two ways how a PBH looses energy when crossing a star:

1. Accretion of star matter. In the star frame the PBH total momentum does not change, while the mass increases $\Rightarrow$ velocity decreases. This is equivalent to the drag force

$$F_{acc} = -\dot{m}\, v$$

what is $\dot{m}$?

PBH velocity is mildly supersonic (750 km/s vs 500 km/s)

$\Rightarrow$ collection with free particles should be a reasonable approximation. Then

$$\dot{m} = \rho_* \sigma_{BH} \cdot v$$

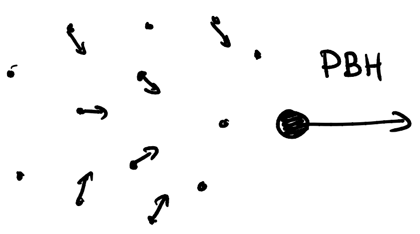
$$\sigma_{BH} = 16\pi \frac{G^2 m^2}{v^2}$$

LL vol 2

сечение захвата черной дырой

$$\Rightarrow \boxed{F_{acc} = -16\pi \rho_* G^2 m^2}$$

2. Dynamical friction (Chandrasekhar '49)



- perpendicular momentum averages out
- longitudinal momenta add up

$\hookrightarrow$ BH looses momentum

$\Rightarrow$ force.

Simple calculation gives:

$$\boxed{F_{df} = -4\pi \rho_* G^2 m^2 \frac{\ln\Lambda}{v^2}}$$

P3.4
dynamical
friction

$\ln\Lambda$ = Coulomb logarithm

$\simeq \ln\left(R_*/R_g\right) \simeq 30$

$\Rightarrow$ F_{df} is parametrically larger than F_{acc} at small velocities $v \ll 1$

$\Rightarrow$ neglect F_{acc}

* Average energy loss (assuming uniform distribution in the impact plane):

$$E_{loss} = \frac{1}{\pi R_*^2} \int ds\, dl\, F_{dyn} = \frac{4 M_* G^2 m^2}{R_*^2 v^2} \ln\Lambda$$

Here v is the velocity with which the PBH passes through the star. To a good approximation this is the escape velocity

$$v^2 \simeq v_{esc}^2 = \frac{2 G M_*}{R_*}$$

Thus we have

$$\boxed{\frac{E_{loss}}{m} = \underbrace{\frac{R_g}{R_*}}_{\sim 10^{-5}} \cdot \underbrace{\frac{m}{M_*}}_{10^{-13}} \cdot \ln \Lambda} \ll 1$$

for $m = 10^{20}\,g$

* To understand how small is the energy loss let us rewrite it in terms of the asymptotic kinetic energy of the PBH by equating

$$\frac{1}{2} m v_\infty^2 \sim E_{loss}$$

From here we find

$$v_\infty \sim \left(\frac{E_{loss}}{m}\right)^{1/2} \sim 10^{-8} = 0.003 \ \text{km/s}$$

(compare to typical Galactic velocity ~ 200 km/s)

$\Rightarrow$ Two conclusions:

- only extremely slow PBH can become gravitationally bound after one crossing
- those who are captured have a very large initial orbit

$$r_{in} = \frac{G M_* m}{E_{loss}} = \frac{M_*}{2m \ln \Lambda} R_*$$

CAPTURE RATE

* Assume Maxwellian distribution in v:

$$dn = \frac{\rho_{DM}}{m}\left(\frac{3}{2\pi\sigma^2}\right)^{3/2}\exp\left(-\frac{3v^2}{2\sigma^2}\right)d^3v$$

↑ velocity dispersion ~ 270 km/s in MW

⇒ capture rate (formal)

$$\frac{dn}{dt} \simeq 6\sqrt{6\pi}\,\frac{\rho_{DM}}{\sigma^3}\,\frac{G M_* R_*}{m^2}\,E_{loss}$$

$$= 24\sqrt{6\pi}\,\frac{\rho_{DM}}{\sigma^3}\,G^2 M_* \ln\Lambda$$

$$\frac{dn}{dt} \propto \frac{\rho_{DM}}{\sigma^3}$$

$$= 0.5\ \mathrm{Gyr}^{-1}\left(\frac{7\ \mathrm{km/s}}{\sigma}\right)^3\left(\frac{\rho_{DM}}{100\ \mathrm{GeV/cm^3}}\right)$$

* very small rate, can only capture a few over the age of the Universe

!! But the real rate is even much smaller!

* For a successful capture two more conditions have to be satisfied:

    (1) "cooling" time ($\simeq$ time to shrink the orbit to the star radius R_*) is smaller than the age of the Universe

    (2) trajectories are not perturbed by nearby stars

Consider what do these conditions imply.

(1) COOLING

* Between star crossings the PBH moves on a Keplerian orbit. The time it takes is

$$\Delta t = \pi \frac{a^{3/2}}{\sqrt{R_g}} \qquad [a = \text{apastron}]$$

* After the crossing the energy changes by $-E_{loss}$, so that the apastron changes by

$$\Delta a = -\frac{a^2}{G M_* m} \cdot E_{loss}$$

$$\Rightarrow \frac{da}{dt} \approx \frac{\Delta a}{\Delta t} = -\frac{2 E_{loss}}{\pi m \sqrt{R_g}} \sqrt{a}$$

Solving this equation we find

$$t_{cool}(a_0) = \frac{\pi\sqrt{R_g}\cdot m}{E_{loss}}\sqrt{a_0} = \frac{\pi M_* R_*}{m\sqrt{R_g}\ln\Lambda}\sqrt{a_0}$$

$10^{13}\cdot 10^{6}\cdot 10^{8}$ km $=$

$= 10^{27}$ km $\simeq 3\cdot 10^{21}$ s

$\sim 10^{14}$ yr $\times\frac{1}{\ln\Lambda}\Rightarrow 10^{13}$ yr

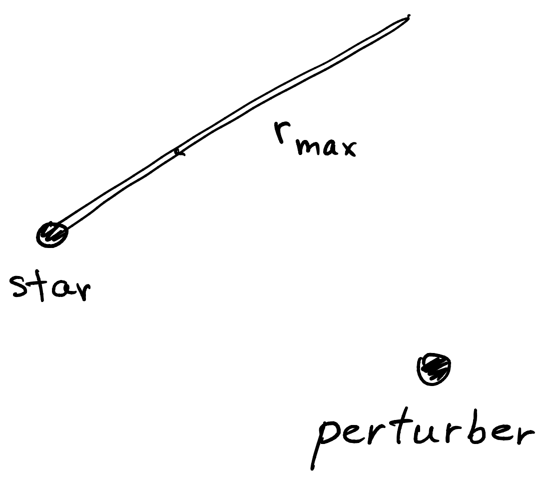
Take:

$$m = 10^{20}\,g$$
$$M_* = M_\odot = 10^{33}\,g$$
$$R_* = 7\cdot 10^{5}\,km$$
$$R_g = 3\,km$$
$$a_0 = \frac{M_*}{2m\ln\Lambda}R_* = 4\,kpc\left(\frac{10^{20}g}{m}\right)$$
$$\simeq 10^{17}\,km$$

$$\Rightarrow \boxed{t_{cool} \sim 2\times 10^{13}\,yr\left(\frac{10^{20}g}{m}\right)^{3/2}}$$

$\hookrightarrow$ too large, unless $m \gtrsim 10^{23}\,g$!

(2) PERTURBATIONS



* Gravitational perturbations from nearby stars should not make PBH to miss the star

* We want periastron to be inside the star, that is

$$r_{min} < R_*$$

* For large nearly radial orbits we have

$$r_{min} = \frac{J^2}{m^2 R_g} \qquad J = \text{angular momentum}$$

$\Rightarrow$ $r_{min} < R_*$ translates into $J < J_{max}$ with

$$\frac{J_{max}}{m} = \sqrt{R_* R_g}$$

* Perturbations change J. Suppose the perturber grav. potential is $U(\bar{x})$ acting in the plane (xy), and the trajectory is $x(t)$, i.e. along x. Then the change of J over one fall from r_{max} to $\sim R_*$ is

$$\frac{\Delta J}{m} = \int_0^{T/4} x^2(t)\, U_{xy}\, dt \qquad \left[U_{xy} = \partial_x \partial_y U \right]$$

Assume $U_{xy} = \text{const}$, then

$$\frac{\Delta J}{m} = \frac{5\pi}{16} \cdot \frac{r_{max}^{7/2}}{R_g^{1/2}} \cdot U_{xy}$$

We require that

$$\frac{\Delta J}{m} \lesssim \frac{J_{max}}{m} = \sqrt{R_* R_g}$$

* Assume $U(x)$ is the gravitational potential of a star of the same mass M_* at a distance $d \ll r_{max}$. Then we get the condition

$$r_{max} < d \left(\frac{R_*}{\alpha d} \right)^{1/7}$$

parameter $O(1)$ depends on relative orientation

determined by the star density

* In a most promising dwarf galaxy Triangulum II

$$d \sim 6\,\text{pc}$$

$$\Downarrow$$

r.h.s: $$d \left(\frac{R_*}{\alpha d} \right)^{1/7} \sim 0.4\,\text{pc}$$

On the other hand, after 1 star crossing the PBH orbit has

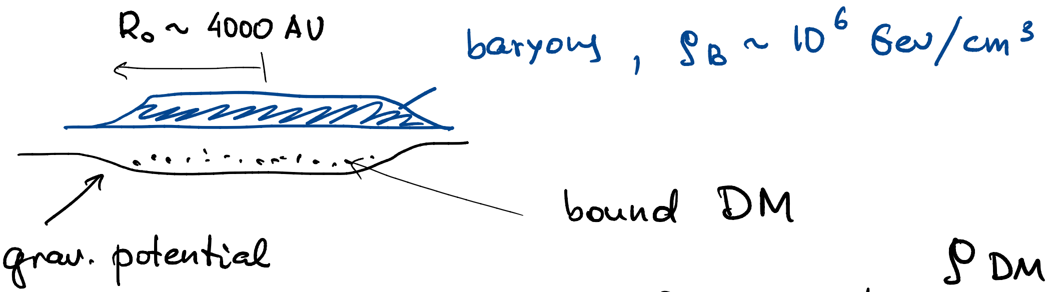
l.h.s: $$r_{max} \sim 4\,\text{kpc} \left(\frac{10^{20}\,\text{g}}{m} \right)^{3/2}$$

$\Rightarrow$ does not work!

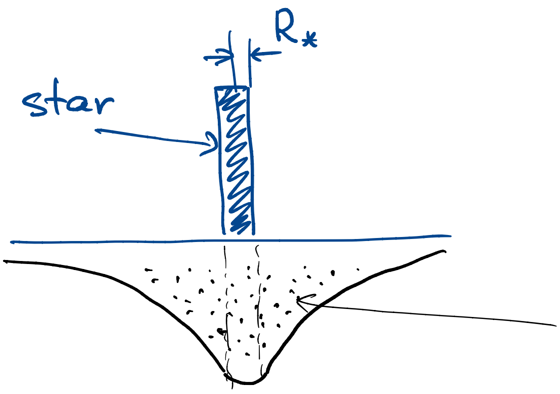
CAPTURE AT BIRTH

* There is an alternative mechanism where PBH are captured in the process of the star formation.

Stars are formed from giant molecular clouds. Some PBH are gravitationally bound to these clouds.



$$\rho_{\text{bound}} \propto \frac{\rho_{DM}}{\sigma^3}$$



DM develops a spiked profile: some trajectories are contained in the star

Some pass through the star and may get captured later

* One may calculate analytically the number of captured PBH assuming phase-space density conservation during adiabatic contraction of the gas cloud.

Before contraction,

$$Q = \frac{\rho_{DM}}{m} \left(\frac{3}{2\pi \sigma^2} \right)^{3/2} \quad \propto \quad \frac{\rho_{DM}}{\sigma^3}$$

Assuming Q is preserved, after contraction

$$N = \int Q \, d^3r \, d^3v$$

integrate over trajectories which are
(i) bound
(ii) have periastron $\leq R_*$
in the Newtonian potential of a mass M_*

P... # of captured PBH

The integral diverges at large orbit sizes. It reads

$$N = \frac{\rho_{DM}}{m \sigma^3} \left(3\pi R_g \right)^{3/2} R_* \sqrt{2 r_{max}}$$

* We now have to apply the cooling time condition and the constraint from perturbers. It turns out that for masses $m \sim 10^{20}\,g$ the cooling condition is <u>stronger</u>. If we recall that

$$t_{cool} = \frac{\pi M_* R_*}{m \sqrt{R_g} \ln\Lambda} \sqrt{r_{max}}$$

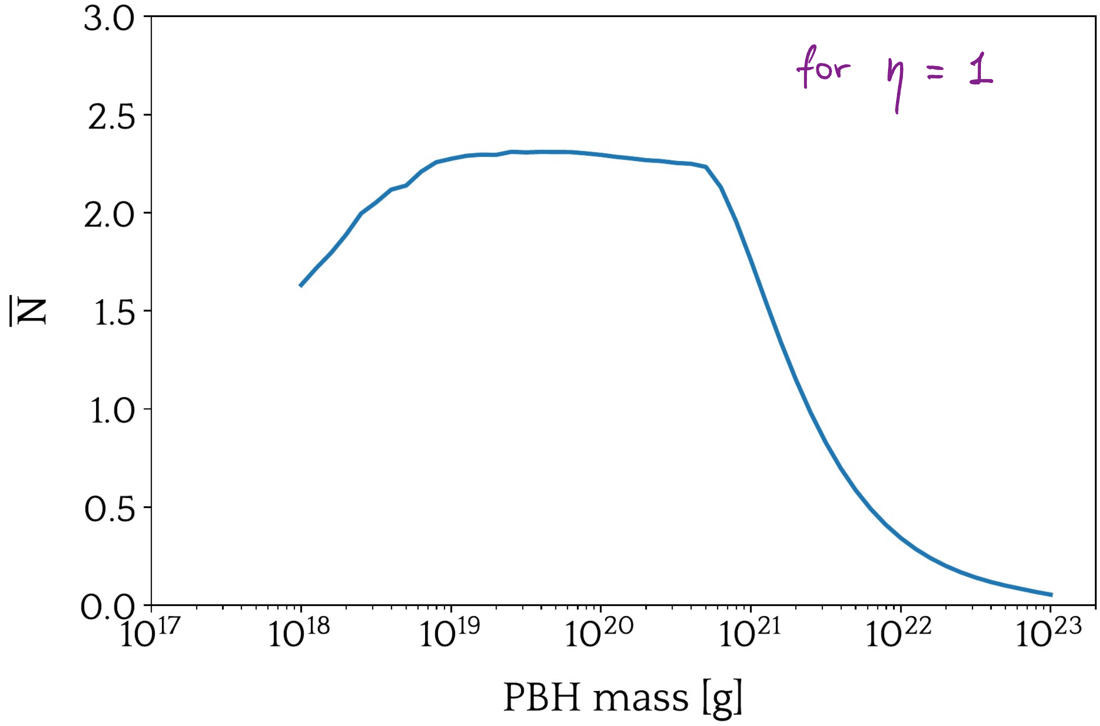
we may rewrite the captured number as follows:

$$N = 3\sqrt{6\pi}\, R_g^2 \frac{\ln\Lambda}{M_*} \cdot \frac{\rho_{DM}}{\sigma^3} t_{cool}$$

$$\hookrightarrow \boxed{N = 2.3 \underbrace{\left(\frac{\rho_{DM}}{100\ \mathrm{GeV/cm^3}}\right)\left(\frac{7\ \mathrm{km/s}}{\sigma}\right)^3}_{\text{dwarf galaxy "merit factor" } \eta} \frac{t_{cool}}{10\ \mathrm{Gyr}}}$$

typical captured number

* The numerical calculation with all the effects included gives the following result:



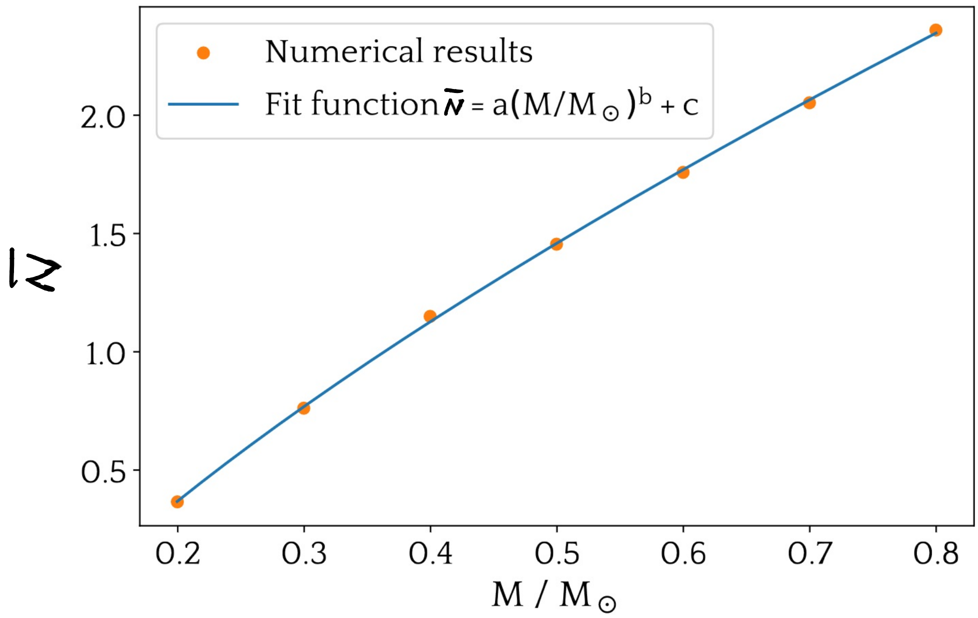
$\bar{N} \gtrsim 1 \Rightarrow$ possibility of constraints or detection!

* Most promising objects – ultra-faint dwarf galaxies

	$R_{1/2}$ [pc]	σ [km/s]	$\rho_{\rm DM}$ [GeV/cm^3]	n_* [10^{-3}pc^{-3}]	η
Triangulum II	16	< 5.9	161	9.2	0.95
Tucana III	37	< 2.1	3.7	0.67	0.51
Draco II	19	< 10.2	343	2.6	0.39
Segue 1	24	6.4	85	2.1	0.39
Grus I	28	5.0	38	9.6	0.37

* What is the best strategy to detect/exclude disappearance of stars in UFDs?

A promising idea is to use the fact that the probability of capture depends on the star mass [see our analytic formula]. Numerically, this dependence is as follows:

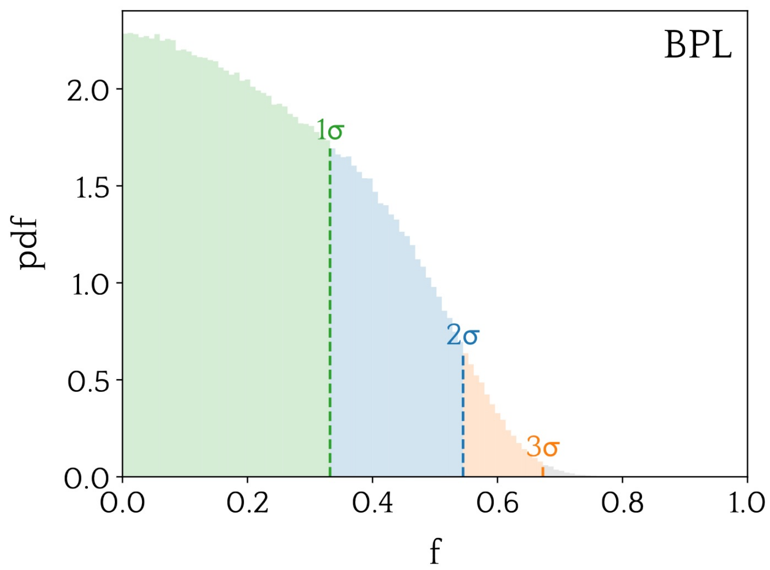


$\Rightarrow$ PBH preferentially destroy *heavier* stars

$\Rightarrow$ PBH modify the dwarf galaxy star *mass function*.

* The following numerical experiment shows that this is a promising method:
  - generate a synthetic star population with no PBH effects
  - define the likelihood that depends on the standard mass function parameters + the fraction of PBH f in the total amount of DM
  - run Bayesian analysis on the synthetic sample to recover the mass function parameters and the fraction f. Marginalize over all params. except f

    $\Rightarrow$ p.d.f. for f



If it was real data, this plot would exclude $f > 0.7$ at 3σ confidence

$\Rightarrow$ $\boxed{\text{READY TO GO TO REAL DATA}}$!