Туннелирование в КТП-1

$$V(x) = \frac{\omega^2 x^2}{2}$$

$$\langle x_f | e^{-\hat{H}(\tau_f - \tau_i)} | x_i \rangle = N \int \mathcal{D}x(\tau)\, e^{-S_E[x]}$$

$t = e^{-i\tau}$ $\qquad \begin{cases} x(\tau_f) = x_f \\ x(\tau_i) = x_i \end{cases}$

$$\langle x_f | e^{-\hat{H}\tau_{fi}} | x_i \rangle = \sum e^{-E_n \tau_{fi}} \langle x_f | E_n \rangle \langle E_n | x_i \rangle$$

$1 = \sum |E_n\rangle\langle E_n|$ $\longrightarrow e^{-E_0 \tau_{fi}} \langle x_f | 0 \rangle \langle 0 | x_i \rangle + \dots$

$t = -i\tau$

$$S_M = +i S_E \qquad S_E = \int d\tau \left(\left(\frac{dx}{d\tau}\right)^2 \frac{1}{2} + \frac{\omega^2 x^2}{2} \right).$$

$$\int d\tilde{x}\, e^{-\frac{f(\tilde{x})}{g^2}} \Big|_{g \to 0} = \int d\tilde{x}\, e^{-\frac{f(\tilde{x}_{cl})}{g^2} - \frac{f''(\tilde{x}_{cl})}{2g^2}(\tilde{x} - \tilde{x}_{cl})^2} =$$

$f'(\tilde{x}_{cl}) = 0 \longrightarrow$ седловые точки

$$= \frac{\sqrt{2\pi}\, g}{\sqrt{f''(\tilde{x}_{cl})}}\, e^{-\frac{f(\tilde{x}_{cl})}{g^2}} \qquad \begin{pmatrix} a_1 & & \\ & \ddots & \\ & & a_N \end{pmatrix}$$

$$\int d^N \tilde{x}\, e^{-\frac{f(\tilde{x})}{g^2}} = \int d^N \tilde{x}\, e^{-\frac{f(\tilde{x})}{g^2} - \frac{\delta\tilde{x}_i\, \partial_i \partial_j f|_{\tilde{x}_{cl}}\, \delta\tilde{x}_j}{2g^2}} =$$

$\frac{\partial f}{\partial x}\Big|_{x_{cl}} = 0 \longleftarrow$ седла $\qquad \tilde{x} - \tilde{x}_{cl}$

$$= \frac{\sqrt{2\pi}\, g}{\sqrt{a_1}} \cdots \frac{\sqrt{2\pi}\, g}{\sqrt{a_N}}\, e^{-\frac{f(\tilde{x}_{cl})}{g^2}}$$

$$= \frac{(2\pi)^{N/2} g^N}{\sqrt{\det(f''(\tilde{x}_{cl}))}}\, e^{-\frac{f(\tilde{x}_{cl})}{g^2}}$$

$$\langle x_f = 0 | e^{-\hat{H}\tau_{fi}} | x_i = 0 \rangle = \int \mathcal{D}\delta x\, e^{-\int d\tau \left(\frac{1}{2} \delta x \left(-\frac{d^2}{d\tau^2} + \omega^2 \right) \delta x \right)}$$

$\delta x|_{\tau_i} = \delta x|_{\tau_f} = 0$

$$\begin{cases} \frac{d^2 x}{d\tau^2} - \omega^2 x = 0 \\ x(\tau_0) = x(\tau_f) = 0 \end{cases} \qquad \begin{cases} x_{cl} = 0 \\ x = x_{cl} + \delta x \end{cases}$$

$$= \frac{N}{\sqrt{\det\left(-\frac{d^2}{d\tau^2} + \omega^2 \right)}}$$

а) Определение:

$$\begin{cases} x_n(\tau_f) = x_n(\tau_i) = 0 \\ \hat{L} \cdot x_n(\tau) = \lambda_n x_n(\tau) \end{cases}$$

$\Downarrow$

$\det L = \prod \lambda_n$

б) Теорема Гельфанда–Яглома $\qquad \hat{L} = -\frac{d^2}{d\tau^2} + W(\tau)$, $\quad \psi(\tau)$

$$\begin{cases} \hat{L}\psi(\tau) = 0 \\ \psi(\tau_i) = 0 \\ \frac{d\psi}{d\tau}\big|_{\tau_i} = 1 \end{cases} \Rightarrow \det \hat{L} = N' \psi(\tau_f)$$

не зависит от W

Осциллятор:
$$\begin{cases} \left(-\frac{d^2}{d\tau^2} + \omega^2 \right) \psi = 0 \\ \psi(\tau_i) = 0 \\ \frac{d\psi}{d\tau}(\tau_i) = 1 \end{cases} \qquad \psi = \frac{1}{\omega} \operatorname{sh}(\omega(\tau - \tau_i))$$

$$\det\left(-\frac{d^2}{d\tau^2} + \omega^2 \right) = \frac{N'}{\omega} \operatorname{sh}(\omega(\tau_f - \tau_0))$$

$$\langle x = 0 | e^{-H\tau_{fi}} | x = 0 \rangle = \frac{N N'^{-1/2} \sqrt{\omega}}{\sqrt{\operatorname{sh}(\omega(\tau_f - \tau_i))}} \Big|_{\omega \to 0} = \frac{N N'^{-1/2}}{\sqrt{\tau_f - \tau_0}}$$

$N N'^{-1/2}$ не зависит от ω $\qquad m = 1 = \hbar$

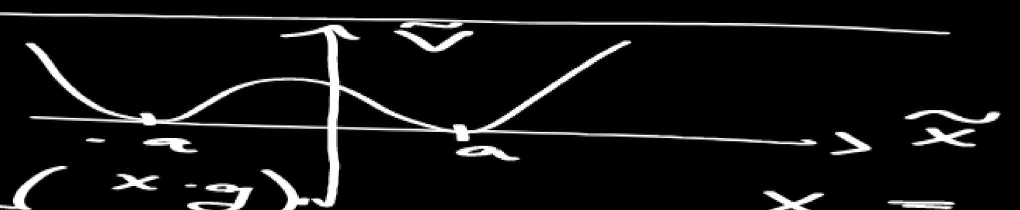
$$\langle x_f | e^{-H\tau_{fi}} | x_i \rangle \Big|_{\omega = 0} = \frac{e^{-\frac{(x_f - x_i)^2}{2(\tau_f - \tau_0)}}}{\sqrt{2\pi(\tau_f - \tau_i)}} \qquad N N'^{-1/2} = \frac{1}{\sqrt{2\pi}}$$

$$\langle x = 0 | e^{-H\tau_{fi}} | x = 0 \rangle = \sqrt{\frac{\omega}{2\pi}} \frac{1}{\sqrt{\operatorname{sh}(\omega(\tau_f - \tau_i))}} \xrightarrow{\tau_{fi} \to \infty} |\langle 0 | x = 0 \rangle|^2 e^{-\frac{\omega}{2}\tau_{fi}} + \dots$$

$$\longrightarrow \sqrt{\frac{\omega}{\pi}}\, e^{-\frac{\omega}{2}\tau_{fi}}$$

$$V = \frac{g^2}{2}\left(x^2 - \frac{a^2}{g^2} \right)^2$$

$$= \frac{1}{2g^2}\left((xg)^2 - a^2 \right)^2 = \frac{\tilde{V}}{g^2}(x \cdot g)$$



$x = \frac{\tilde{x}}{g}$

$$S_E = \int d\tau \left(\frac{\dot{x}^2}{2} + V(x) \right) = \frac{1}{g^2} \int d\tau \left(\frac{1}{2}\left(\frac{d\tilde{x}}{d\tau}\right)^2 + \tilde{V}(\tilde{x}) \right).$$

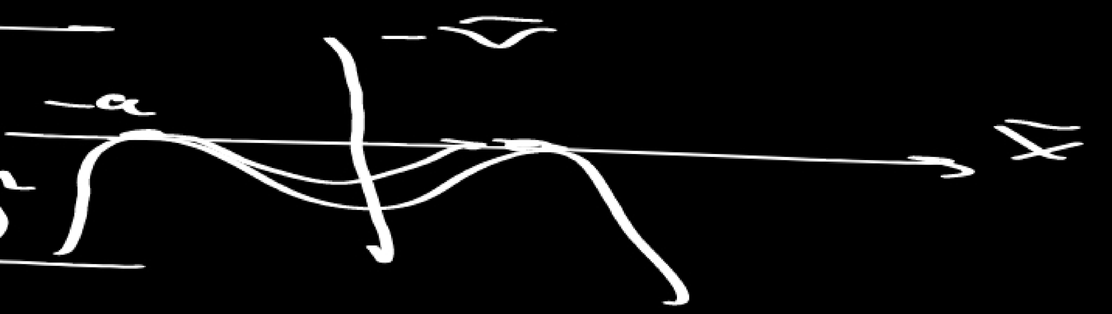
$V(x) = \frac{1}{g^2}\tilde{V}(x \cdot g)$

$$\langle x = +a | e^{-H\tau_{fi}} | x = -a \rangle = N \int \mathcal{D}\tilde{x}(\tau)\, e^{-\frac{\tilde{S}_E[\tilde{x}]}{g^2}}$$

$\tau_{fi} \to \infty$ $\qquad$ седловая точка.

седло $\longrightarrow \frac{d^2}{d\tau^2}\tilde{x}(\tau) = \tilde{V}'(\tilde{x}) = -(-\tilde{V})'$

$\tilde{x}_{cl} = -a$, $+a$ $\qquad \longleftarrow \tilde{V} = \frac{1}{4}(\tilde{x}^2 - a^2)^2$



$$\boxed{\tilde{x}_{cl} = a\, \operatorname{th}\left(\frac{a}{\sqrt{2}}\tau \right)}$$

$q = i m_g$

$$= \frac{8\pi e^{?}}{\tau} e^{-m_g z} = \rho(0) e^{-z/\alpha}$$

$$\frac{1}{2} 4\pi e\, i \,\mathrm{Im}\, \lim_{q \to i m_g} \frac{d}{dq} \left[(1+q^2\alpha)^2 \frac{e^{iqz} q}{(1+q^2\alpha)^2} \right]$$

$$f(z_0) = \frac{1}{1} \lim_{z \to a} \frac{d}{dz} \left((z-a) f(z) \right)$$

$$\int_{-\infty}^{\infty} \frac{q e^{iqz} dq}{\left(1+\frac{q^2}{m^2}\right)^2} = \int \frac{q e^{iqz} dq}{m^2 \left(1+\frac{iq}{m}\right)^2 \left(1-\frac{iq}{m}\right)^2} =$$

$$= 2\pi i \cdot \lim_{q \to im} \frac{d}{dq} \left(\frac{q e^{iqz}}{\left(1-\frac{iq}{m}\right)^2} \right) m^2 =$$

$$= -2\pi m^2 i \lim_{q \to im} \left(\frac{e^{iqz}}{\left(1-\frac{iq}{m}\right)^2} + \frac{q i z e^{iqz}}{\left(1-\frac{iq}{m}\right)^2} + \frac{2 q e^{iqz} \frac{i}{m}}{\left(1-\frac{iq}{m}\right)^3} \right) =$$

$$= -2\pi m^2 i \left(\frac{e^{-mz}}{4} - \frac{m z e^{-mz}}{4} - \frac{e^{-mz}}{4} \right) = i \frac{\pi}{2} m^3 z e^{-mz}$$

$$\frac{2\pi e^2}{2} m^3 e^{-mz} = \rho(0) e^{-mz}$$

$q = p' - p$ $\quad M \to m_g$

$$\langle p' | J_\mu(q) | p \rangle =$$

$$= \bar{u}_{p'} \left\{ \gamma^\mu F_1(q^2) - \frac{F_2(q^2)}{2M} \sigma^{\mu\nu} q_\nu \right\} u_p$$

$p^\mu \gamma_\mu u = m u$

$$= \bar{u}_{p'} \left\{ \gamma^\mu F_m(q^2) + 2M \left[F_e(q^2) - F_m(q^2) \right] \frac{P^\mu}{P^2} \right\} u_p$$

$$\gamma_\nu (\gamma_\mu p^\mu - m) u^p = 0 \qquad \{\gamma_\mu, \gamma_\nu\} = 2 g_{\mu\nu}$$

$\gamma_\nu \gamma_\mu = 2 g_{\mu\nu} - \gamma_\mu \gamma_\nu$

$$\sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu] = \frac{i}{2} (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)$$

$$\sigma_{\mu\nu} q^\nu = \frac{i}{2} [\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu] (p'^\nu - p^\nu) = \frac{i}{2} \gamma_\mu \hat{p}' - \frac{i}{2} \gamma_\mu m$$

$$q^\nu = p'^\nu - p^\nu \quad \frac{i}{2} (\gamma_\mu \hat{p}' - m \gamma_\mu + p_\mu) \quad - \frac{i}{2} (2 g_{\mu\nu} - \gamma_\mu \gamma_\nu)(p'^\nu - p^\nu)$$

$$= \frac{i}{2} \gamma_\mu \hat{p}' - \frac{i}{2} \gamma_\mu m - \frac{i}{2} (m - \hat{p})$$

$\gamma_\mu =$

$$\frac{i}{2} \hat{p} \gamma_\mu = p^\nu (2 g_{\mu\nu} - \gamma_\mu \gamma_\nu) = \frac{i}{2} (p_\mu - \gamma_\mu m)$$

$P^\mu = p'^\mu + p^\mu$

$$\frac{i}{2} \gamma_\mu \hat{p}' = \frac{i}{2} (2 g_{\mu\nu} - \gamma_\nu \gamma_\mu) p'^\nu = i p_\mu - \frac{i}{2} \gamma_\mu m$$

$$\sigma^{\mu\nu} q^\nu = \frac{1}{2} (p_\mu + p_\mu - 2m \gamma_\mu)$$

$$\gamma^\mu F_m + 2M [F_e - F_m] \frac{P^\mu}{P^2}$$

$$\gamma_\mu F_1(q^2) - \frac{F_2(q^2)}{2m} \sigma^{\mu\nu} q_\nu \qquad P^2 = (p^\mu + p'^\mu)^2$$

$P_\mu = p_\mu + p'_\mu$

$q_\mu = p'_\mu - p_\mu$

$$\gamma_\mu F_1(q^2) - m \frac{F_2(q^2)}{2m^2} (p'_\mu + p_\mu - 2m \gamma_\mu)$$

$P^2 = 2m^2 + 2(p'p)$

$q^2 = 2m^2 - 2(p'p)$

$$\gamma_\mu (F_1 + F_2) + \frac{P^2}{2m} \qquad P^2 = p^2 + p'^2 + 2(p p')$$

$P^2 + q^2 = 4m^2$

$P^2 = 4m^2 - q^2$

$$- \frac{4m F_2}{2(P^2 + q^2)} P^\mu + \gamma_\mu (F_1 + F_2)$$

$$m^2 = \frac{P^2 + q^2}{4}$$

$$\bar{u}_{p'} \left\{ \gamma^\mu F_m(q^2) + 2M (F_e - F_m) \frac{P^\mu}{P^2} \right\} u_p \quad (-1) \frac{(p^\mu + p'^\mu)}{2M^2 + 2M^2 - q^2}$$

$F_m = F_1 + F_2$

$F_e = F_1 + F_2 \frac{q^2}{4M^2}$

$$\frac{2M F_2}{4M^2\, 2M} (q^2 - 4M^2)$$

$\bar{u}_{p'} \gamma^\mu q_\mu u_p = 0$

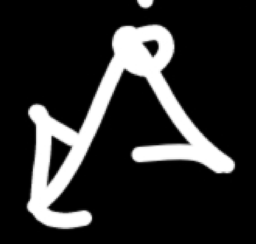
$$q_\mu \langle p' | J_\mu(q) | p \rangle = \ldots$$

$f(x) = e^{iPx}$

$$\int \langle p' | J^{\mu}_{\mu}(x) | p \rangle \, dx \, e^{iqx}$$

$$= \int \langle p' | e^{i\hat{Q}x} J_\mu(0) e^{-i\hat{P}x} | \vec{p} \rangle \, dx \, e^{ipx}$$

iq



$\langle p' | j^{em}_\mu(x) | p \rangle$

$F(0)=0$

$\langle \quad\quad |p\rangle$

$$\int dx \, e^{iqx} e^{ip'x - ipx}$$

$\langle p' | J_\mu(x=0) | p \rangle$

$j_\mu(q)$

$F_E(0)=1$

$J_\mu(q)$