Сессия ОФН РАН, посвященная 70-летию В.А. Рубакова
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Nonlinear acoustic waves in neutron stars with quark (~~and mirror matter~~) admixtures

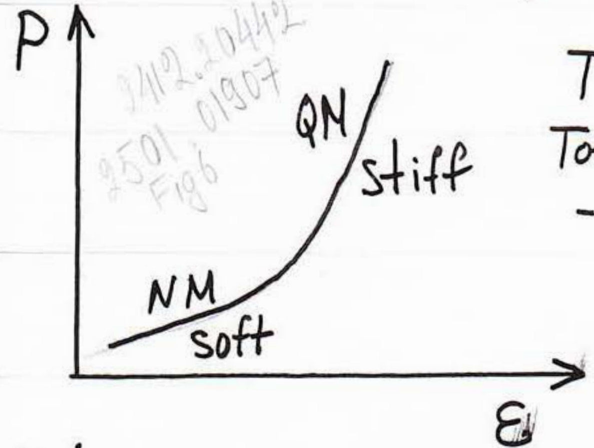
Б.О. Кербиков, ФИАН им. П.Н. Лебедева

① Introduction in Brief

Neutron stars (NSs) in the Multimessenger Era.
A plethora of new M-R measurements. Massive $M \lesssim 2M_\odot$
The heaviest NS observed (Black Widow) PSR J0952-0607
$M = 2.35 \pm 0.17\, M_\odot$.
The NS Equation of State (EoS) is still an open question
Quark Matter (QM) presence in NSs with $M \lesssim 2M_\odot$ – – Hybrid Stars (HNSs). The putative mixed neutron- – mirror neutron stars (MNSs).

② EoS: Stiff, Soft, or both?

EoS: $P = P(\varepsilon, T) \to NS \to P = P(\varepsilon)$



TOV $\to$ M-R $\leftrightarrow \frac{P}{\varepsilon} \to \frac{dp}{d\varepsilon} = c_s^2$

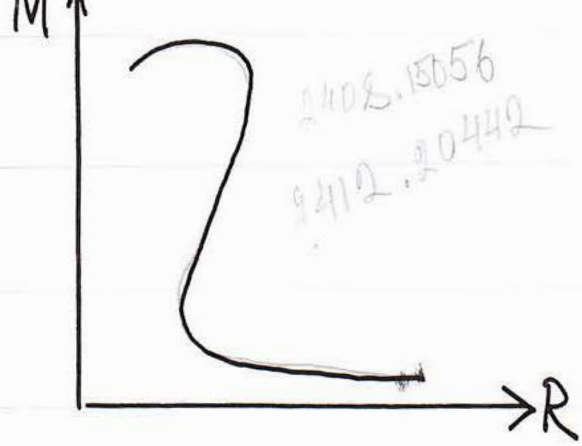
Tolman-Oppenheimer-
- Volkoff 1939

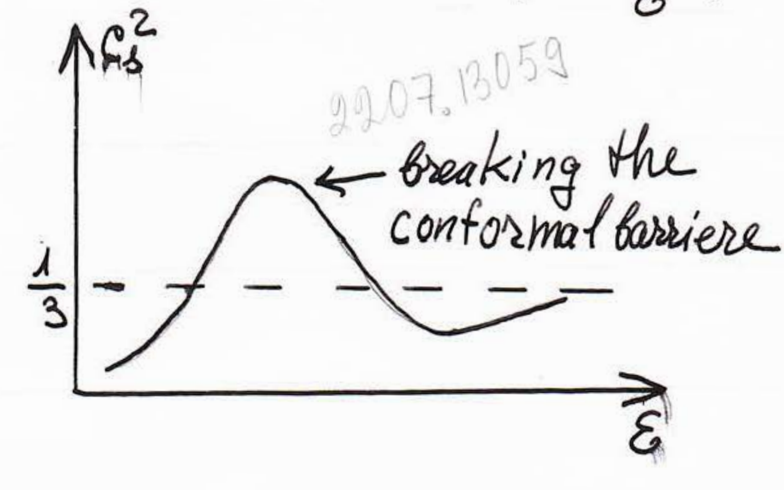
$$\gamma = \frac{d\ln p}{d\ln \varepsilon} = \frac{\varepsilon}{p}\frac{dp}{d\varepsilon} = \frac{\varepsilon}{p} c_s^2$$

γ – polytropic index

Γ – adiabatic index

$$p = k \rho^{\Gamma}, \quad \Gamma = \frac{\varepsilon + p}{\varepsilon}\gamma$$





(3) Essential Acoustic

$$c_s^2 = \frac{dp}{d\varepsilon} = \frac{p}{\varepsilon}\gamma = \frac{1}{3} - \Delta - \varepsilon \frac{d\Delta}{d\varepsilon},$$

$$\gamma = \frac{d\ln p}{d\ln \varepsilon}, \quad \Delta = \frac{1}{3} - \frac{p}{\varepsilon} \text{ trace anomaly } \frac{g_{\mu\nu}T^{\mu\nu}}{3\varepsilon}$$

$\Delta = 0 \to c_s^2 = 1/3$ measure of conformality

$$c_s^2 = \frac{1}{\rho\varkappa}, \quad \varkappa = -\frac{1}{V}\frac{dV}{dp} \text{ - compressibility}$$

The sound attenuation $u = A e^{i(\omega t - kx)} = A e^{i\omega(t - \frac{x}{c})} e^{-\alpha x}$

$\alpha = \frac{\omega^2}{2\rho c_s^3}\left(\frac{4}{3}\eta + \zeta\right)$ attenuation coefficient, η - shear viscosity, ζ - bulk viscosity

Nonlinearity is a property of a two-phase (TP) medium (gas-liquid), NM - QM. Giant frequency dependence of compressibility and sound speed.

(4) Hybrid NS

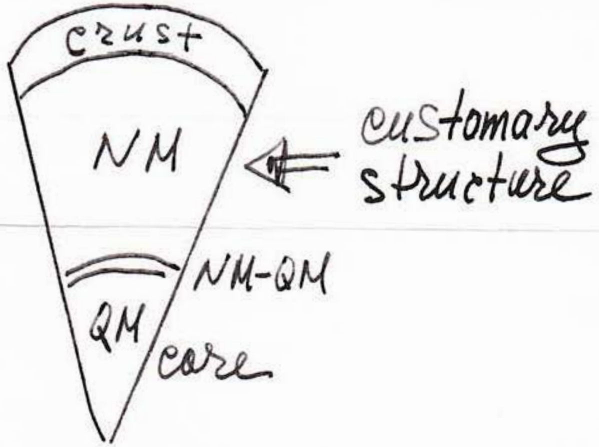
$M \gtrsim 2 M_\odot$ → soft EoS → collapse to BH → very stiff EoS →
→ NM-QM hybrid NS → $c_s^2 > 1/3$

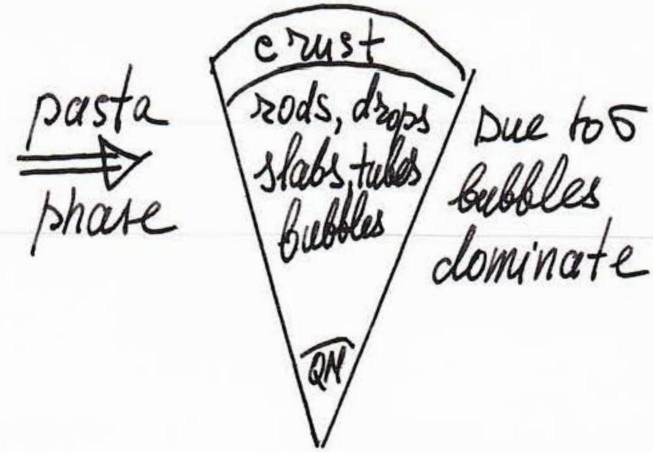
The two phases (TP), nonlinearity. The location and order (1-st or crossover) are poorly known

$n \simeq (2-5)\, n_0$, $n_0 = 0.16\ fm^{-3}$; 1 st order or crossover)

Surface tension $\sigma \left[\frac{MeV}{fm^2}\right]$ at the phase boundary

$\sigma > \sigma_c \sim (50-100)\ MeV/fm^2$ — 1 st order





(5) Hybrid NS EoS

NM EoS, QM EoS, PT matching?

Alternatively — A model-independent, model-agnostic polytropic EoS: E. Annala, T. Gorda, A. Kurkela, J. Nattila, A. Vuorinen, Nature Physics, 16, 907 (2020) >600 citations

Input: Astrophysical observations, a great number of theoretical ab-initio calculations, ensemble ~ 10^5 EoSs

Maximally massive NSs have a sizable QM cores

$M \simeq 1.4 M_\odot$ NM EoS, no need for QM

$$\gamma = \frac{d(\ln p)}{d(\ln \varepsilon)} = \frac{\varepsilon}{p} c_s^2, \quad \gamma = \begin{cases} 1, \ \varepsilon = 3p, \ c_s^2 = 1/3 \ \text{conformal} \\ \simeq 2.5 \ \text{NM around and above } n_0 \\ \gamma < 1.75 \ \text{QM} \\ \gamma \to 1 \quad n \gtrsim 40 n_0 \end{cases}$$

$\gamma = 1.75$ - the deviding line between NM and QM

⑥ Bubbly HS

Liquid containing gas bubbles – a long history of studies

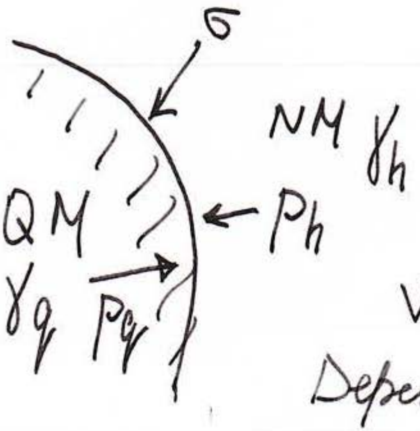
Lord Rayleigh 1917: cavitation damage of the ship propellers

Rayleigh Equation (RE)

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = 0 \quad \text{pulsation and collapse}$$

$$R(t) \sim (t_c - t)^{2/5}, \quad \dot{R}(t) \sim (t_c - t)^{-3/5}$$ divergent wall velocity at $t \to t_c$

RE is oversimplified → Rayleigh-Plesset eq-n (RPE)



$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho_h}\left\{P_q - P_h - \frac{2\sigma}{R} - 4\eta\frac{\dot{R}}{R} - P(t)\right\}$$

$P(t)$ ↑ acoustic pressure

$$R = R_0\left[1 + x(t)\right]$$

Weakly oscillating or strongly collapsing

Depending on R_0, σ, P

Sonoluminescence – flash of light

⑦ RPE solution

Sound speed and frequency dispersion.

C_h, C_q, C_m ~ NM, QM, bubbly medium

$$\frac{C_h^2}{C_m^2} = 1 + \frac{3\beta C_h^2}{R_0^2}\left(\omega_0^2 - \omega^2 + ig\omega\right)^{-1}$$

$\rho_m = (1-\beta)\rho_h + \beta\rho_q$ – the average density,

β - the volume fraction of N bubles

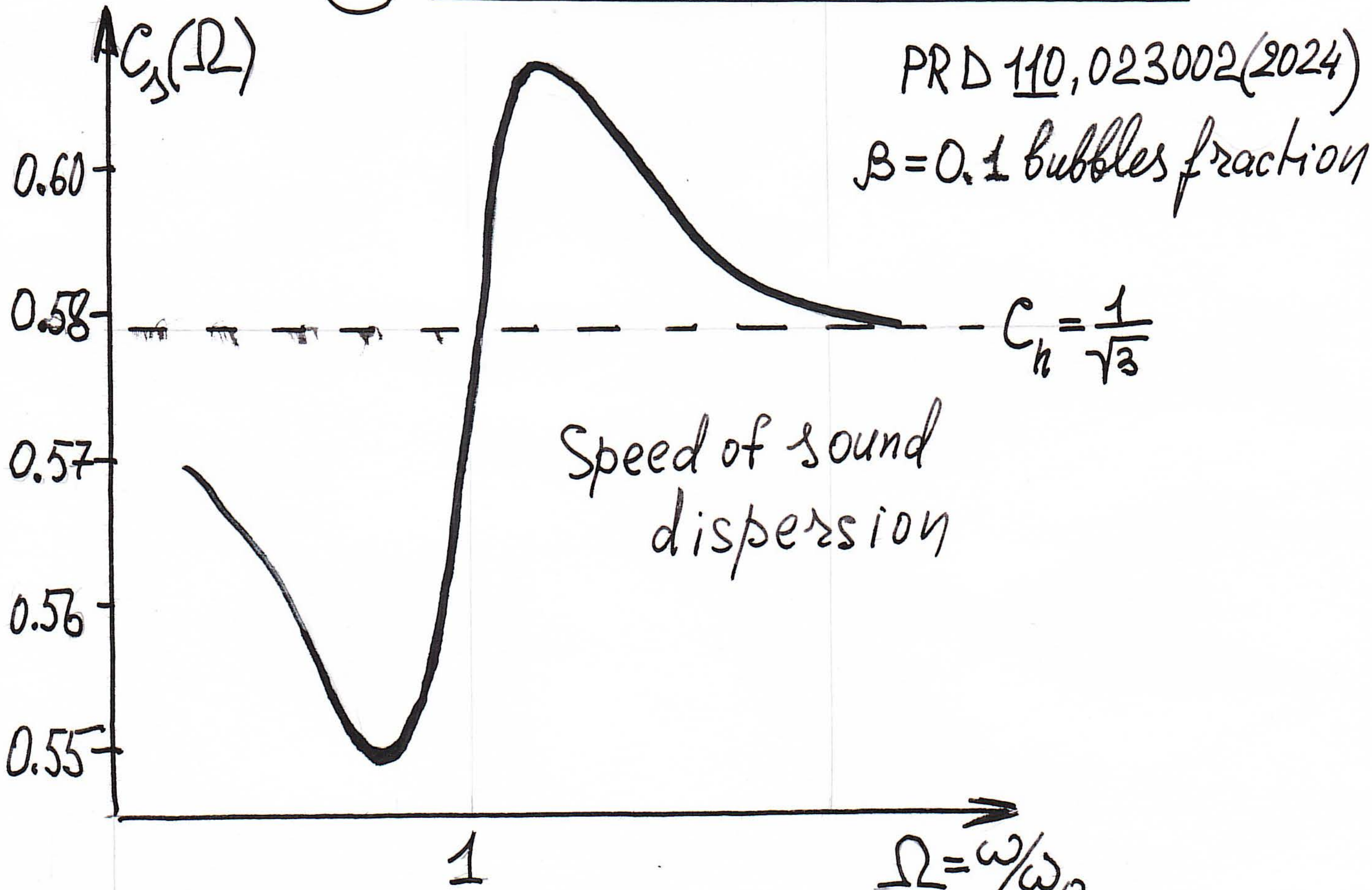
$$\omega_0^2 = \frac{1}{\rho_h R_0^2}\left(3\gamma_q p_q - \frac{2\sigma}{R_0}\right)$$ – bubble eigenfrequency

$$g = \frac{4\eta}{\rho_h R_0^2}$$ – viscous friction damping

$$Q = \frac{\omega_0}{g}$$ – Q-factor, quality factor, non-linearity parameter

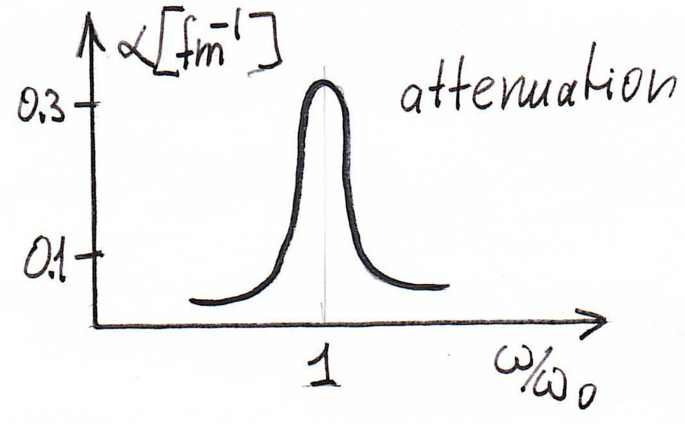
$$f_r = \frac{\omega_0}{2\pi}$$ – resonance sound wave frequency

⑧ Results and Conclusions



$$u = Ae^{i(\omega t - k_m x)} =$$

$$= Ae^{i\omega\left(t - \frac{x}{c_{ph}}\right)} e^{-\alpha x}$$

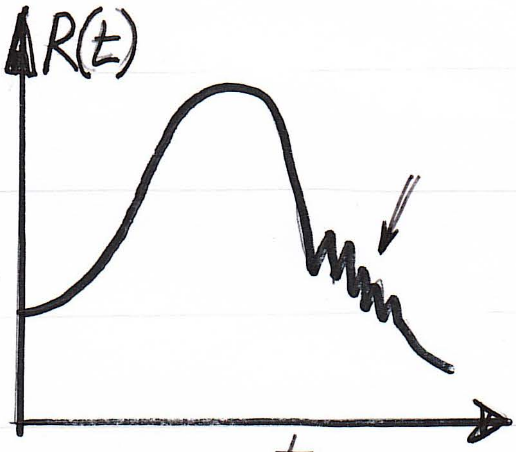


(8') Results and Conclusions

(1) Even a small fraction ($\beta < 0.1$) of QM bubbles can lead to a giant sound wave nonlinearity

(2) Physical parameters $R_0, \sigma, \eta, P_{h,q}, \rho_{h,q}$ are interrelated and loosely known

(3) The resonance frequency is in the Mega-Hertz (MHz) range rather than in the usual kilo-Hertz (kHz). The possible source of MHz GW may be the 1-st order bubble PT in binary NS mergers (2210.03171), or rapid bubble oscillations in the vicinity of the collapse.



Sound in neutron-mirror neutron star is a big issue for the future talk. I am sorry for this

Thanks for your attention