

**Baryonic matter
under
Extreme Conditions**

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JINR

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Outline:

- ▶ Introduction
- ▶ Baryonic matter at finite temperature
 - ▶ Theoretical perspective to phase diagram
 - ▶ Lattice study of dense QCD
- ▶ QCD under extreme conditions
- ▶ The phase diagram from astrophysics
- ▶ Baryonic matter at low temperature
 - ▶ Theoretical perspective
 - ▶ QCD-like theories
- ▶ Conclusion

Baryon density and chemical potential

- ▶ For **Helmholtz free energy** the number of particles is fixed

$$F = F_N(T, V) = -T \log Z_N, \quad Z_N = \text{Tr} \exp\left(-\frac{\hat{H}}{T}\right)$$

- ▶ Grand canonical ensemble

$$Z = \text{Tr} \exp\left(-\frac{\hat{H}}{T}\right) \rightarrow \text{Tr} \exp\left(-\frac{\hat{H} - \mu \hat{N}}{T}\right)$$

$$Z = \sum_N \sum_i e^{-\frac{E_{i,N}}{T}} \times e^{-\frac{\mu N}{T}} = \sum_N e^{-\frac{F_N}{T}} \times e^{-\frac{\mu N}{T}}$$

- ▶ **Gibbs free energy**: $\Omega(\mu) = -T \log Z(\mu)$

$$n = -\frac{1}{V} \frac{\partial \Omega(\mu)}{\partial \mu}$$

- ▶ For u, d, s -quarks one can introduce μ_u, μ_d, μ_s :

$$Z = \text{Tr} \exp\left(-\frac{\hat{H} - \mu_u \hat{N}_u - \mu_d \hat{N}_d - \mu_s \hat{N}_s}{T}\right)$$

$$n_u = -\frac{1}{V} \frac{\partial \Omega}{\partial \mu_u}, \quad n_d = -\frac{1}{V} \frac{\partial \Omega}{\partial \mu_d}, \quad n_s = -\frac{1}{V} \frac{\partial \Omega}{\partial \mu_s}$$

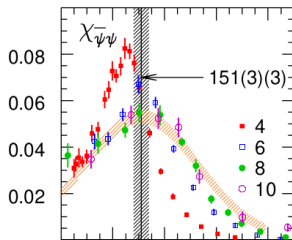
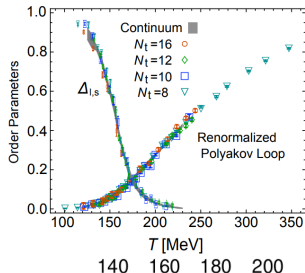
- ▶ Instead of the μ_u, μ_d, μ_s one uses μ_B, μ_I, μ_S

$$n_B = \frac{1}{3}(n_u + n_d + n_s), \quad n_I = n_u - n_d,$$

$$n_B = -\frac{1}{V} \frac{\partial \Omega}{\partial \mu_B}, \quad n_I = -\frac{1}{V} \frac{\partial \Omega}{\partial \mu_I}$$

Chiral symmetry breaking in QCD

- ▶ Massless QCD Lagrangian
 $\mathcal{L} = \bar{\Psi}i\hat{D}\Psi = \bar{\Psi}_Ri\hat{D}\Psi_R + \bar{\Psi}_Li\hat{D}\Psi_L$
- ▶ For N_f quarks chiral symmetry is $SU_L(N_f) \times SU_R(N_f) \times U_V(1) \times U_A(1)$
- ▶ Order parameter: **chiral condensate**
 $\langle \bar{\Psi}\Psi \rangle = \langle \bar{\Psi}_L\Psi_R \rangle + \langle \bar{\Psi}_R\Psi_L \rangle$
 - ▶ CS is broken: $\langle \bar{\Psi}\Psi \rangle \neq 0, M_q \neq 0$
 - ▶ CS is restored: $\langle \bar{\Psi}\Psi \rangle = 0, M_q = 0$
- ▶ Dynamical chiral symmetry breaking
 $SU_L(N_f) \times SU_R(N_f) \rightarrow SU_V(N_f)$
- ▶ In massless QCD the first or the second order phase transition
- ▶ But for $m_q \neq 0$ CS symmetry is explicitly broken. $M_q \sim 10 \rightarrow 300$ MeV
- ▶ **CS breaking/restoration is crossover**
 $T_c = (151 \pm 4)$ MeV
 Z. Fodor, Phys.Lett.B 643 (2006)

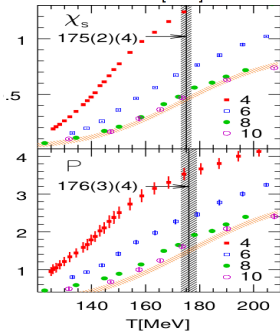
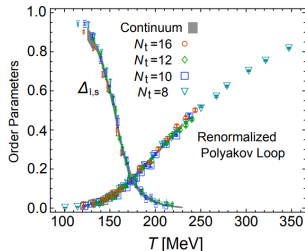


Confinement/deconfinement transition in QCD

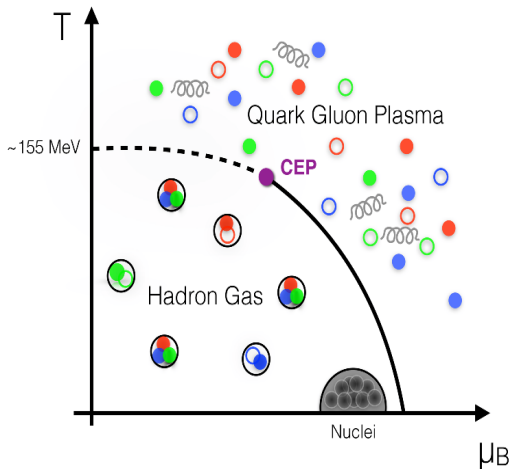
- ▶ Order parameter **Polyakov line**:

$$P(\vec{x}) = \langle \text{Tr} P \exp (i g \int_0^T dx^4 \hat{A}_4(\vec{x}, x^4)) \rangle$$

$$P = e^{-F_Q/T}$$
 - ▶ Confinement: $F_Q = \infty \Rightarrow P = 0$
 - ▶ Deconfinement: $F_Q < \infty \Rightarrow P \neq 0$
- ▶ Z_3 symmetry of gluodynamics
 but $P \rightarrow e^{2\pi k/3i} P$, $k = 0, 1, 2$
- ▶ P is similar to magnetization in ferromagnetic
- ▶ First order phase transition in $SU(3)$ gluodynamics
- ▶ Quarks violate Z_3 symmetry
- ▶ Confinement/deconfinement is crossover
 $T_c = (176 \pm 5) \text{ MeV}$
 Z. Fodor, Phys.Lett.B 643 (2006)



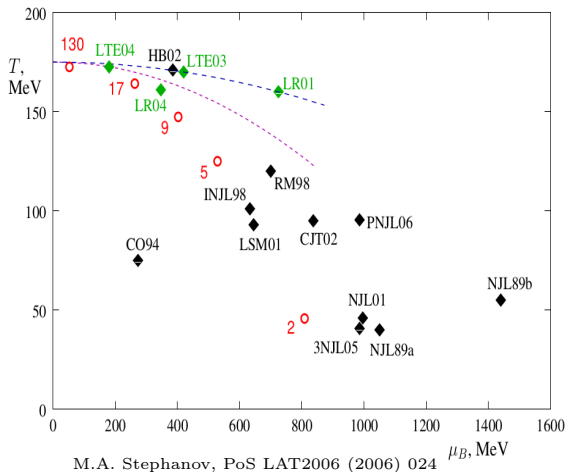
QCD phase diagram



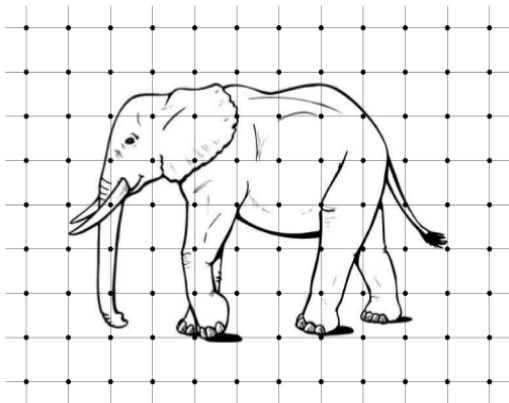
- ▶ The other point of view: transition at finite n_B is crossover

K. Fukushima, e-Print: 2501.01907

Critical point: theory



- ▶ Large systematic uncertainty of theoretical models
- ▶ QCD is nonlinear theory with large coupling constant
- ▶ Too complicated for first-principles theoretical study



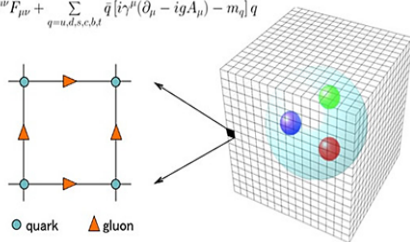
Lattice simulation

- ▶ Allows to study strongly interacting systems
- ▶ Based on the first principles of quantum field theory
- ▶ Powerful due to modern supercomputers and algorithms

Lattice QCD

QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q} [i\gamma^\mu(\partial_\mu - igA_\mu) - m_q] q$$



- ▶ Introduce regular cubic four dimensional lattice
 $N_s \times N_s \times N_s \times N_t = N_s^3 \times N_t$
- ▶ Lattice spacing a
- ▶ Degrees of freedom
 - ▶ **Gluon fields:** 3×3 matrices $U \in SU(3)$, live on links
 - ▶ **Quarks fields:** column q, \bar{q} , live on sites

Lattice QCD

- ▶ We study QCD in thermodynamic equilibrium
- ▶ QCD partition function
$$Z = \int DU \exp(-S_G(U)) \times \prod_{i=u,d,s,\dots} \det(\hat{D}_i(U) + m_i)$$
- ▶ In continuum lattice partition function exactly reproduces QCD partition function
 - ▶ Gluon contribution: $S_G(U) \Big|_{a \rightarrow 0} = -\frac{1}{4} \sum_{a=1}^8 F_a^{\mu\nu} F_{\mu\nu}^a$
 - ▶ Quark contribution:
$$\bar{q}(\hat{D}(U) + m)q \Big|_{a \rightarrow 0} = \bar{q}(\gamma^\mu \partial_\mu + ig\gamma^\mu A_\mu + m)q$$
- ▶ Carry out continuum extrapolation $a \rightarrow 0$
- ▶ Uncertainties (discretization and finite volume effects) can be systematically reduced
- ▶ **The first principles based approach. No assumptions!**
- ▶ Parameters: coupling constant $g(a)$ and masses of quarks $m_q(a)$

Lattice QCD

- ▶ We calculate partition function

$$Z \sim \int DU e^{-S_G(U)} \prod_{i=u,d,s,\dots} \det(\hat{D}_i(U) + m_i) = \int DU e^{-S_{eff}(U)}$$

- ▶ 96×48^3
- ▶ Variables: $96 \cdot 48^3 \cdot 4 \cdot 8 \sim 300 \cdot 10^6$
- ▶ Matrices: $100 \cdot 10^6 \times 100 \cdot 10^6$
- ▶ Simulations at physical quark masses and $a \sim 0.05$ fm
- ▶ Stochastic process: Hybrid Monte Carlo(HMC) generates $\{U_1\} \rightarrow \{U_2\} \rightarrow \{U_3\} \rightarrow \dots$
- ▶ For sufficiently large n : $p(U) \sim e^{-S_{eff}(U)}$
- ▶ Calculation of observable $O(U)$
 $\langle O \rangle = \frac{1}{N} \sum_{i=1}^N O(U_i)$

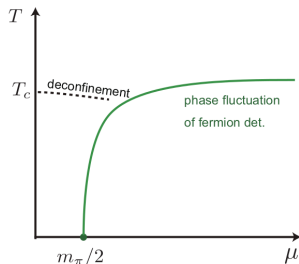
Lattice simulations at finite baryon density

Zero baryon density

- ▶ For MC the probability distribution should be positive
 $p(U) \sim e^{-S_G(U)} \times \det(\hat{D} + m), \quad \hat{D} = \gamma^\mu(\partial_\mu + ig\hat{A}_\mu)$
- ▶ Eigenvalues of the \hat{D} are imaginary and form pairs $(i\lambda, -i\lambda)$
 $\gamma_5 \hat{D}^\dagger \gamma_5 = \hat{D} \Rightarrow \gamma_5 \hat{D} \gamma_5 = -\hat{D}$
 $\hat{D}\psi_\lambda = i\lambda\psi_\lambda \Rightarrow \hat{D}\gamma_5\psi_\lambda = -i\lambda\gamma_5\psi_\lambda$
- ▶ $\det(\hat{D}(U) + m) = \prod_n (i\lambda_n + m)(-i\lambda_n + m) = \prod_n (\lambda_n^2 + m^2) \geq 0$

Nonzero baryon density

- ▶ $\hat{D} = \gamma^\mu(\partial_\mu + ig\hat{A}_\mu) + \mu\gamma_4$
- ▶ Complex eigenvalues and complex determinant
- ▶ Strongly oscillating at large volume V
 $\det(\hat{D}(U) + m) = |\det(\hat{D}(U) + m)| \times e^{i\theta(U)},$
 $e^{i\theta(U)} = e^{i\mu V f(U)}$
- ▶ Monte Carlo methods are not applicable -
Sign problem



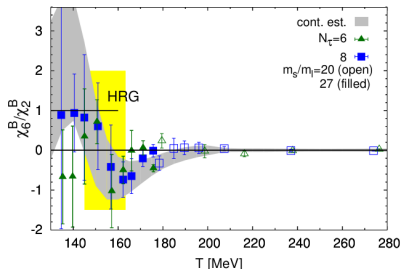
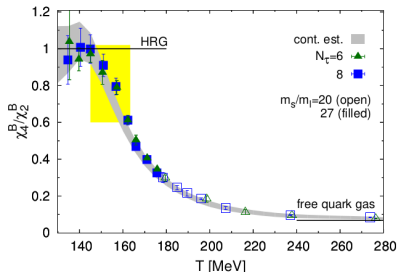
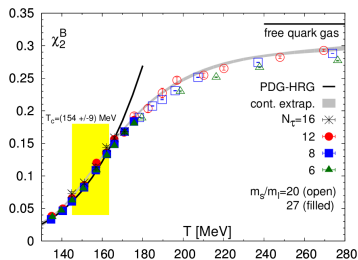
Possible solutions to the sign problem

- ▶ Taylor expansion
- ▶ Reweighting method
- ▶ Analytic continuation from imaginary chemical potential
- ▶ Canonical method
- ▶ Complex Langevin method
- ▶ Dual formulations
- ▶ Lefschetz thimbles

Taylor expansion

- ▶ $O(T, \mu) = \sum_n c_n(T) \left(\frac{\mu}{T}\right)^n$,
 $c_n = \frac{1}{n!} \left(\frac{\partial^n O}{\partial \mu^n}\right)_{\mu=0}$
- ▶ c_n can be calculated on lattice
- ▶ Larger uncertainty at larger n
 manifestation of the sign problem
- ▶ **Small baryon densities can be studied**
- ▶ Example: equation of state

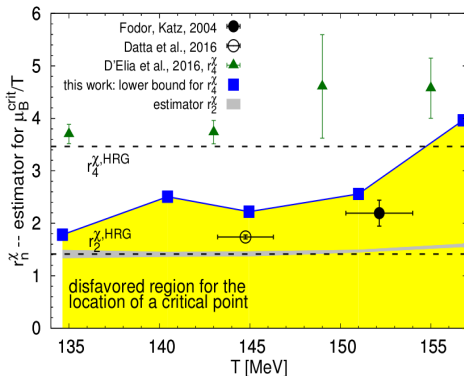
$$\frac{p(T, \mu_B) - p(T, 0)}{T^4} = \sum_k \frac{\chi_{2k}(T)}{(2k!)} \left(\frac{\mu_B}{T}\right)^{2k}$$



A. Bazavov, Phys.Rev.D 95 (2017) 5, 054504

Convergence of Taylor expansion

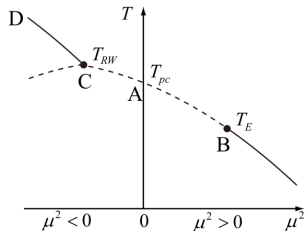
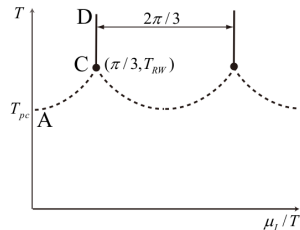
- ▶ $O(T, \mu) = \sum_n c_n(T) \left(\frac{\mu}{T}\right)^n$
- ▶ Slow convergence in hadronic phase
- ▶ Few terms are enough in QGP phase
- ▶ Radius of convergence $r = \lim_{n \rightarrow \infty} \left| \frac{c_n}{c_{n+1}} \right|$



A. Bazavov, Phys.Rev.D 95 (2017) 5, 054504

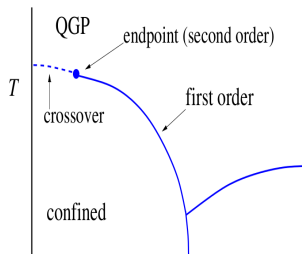
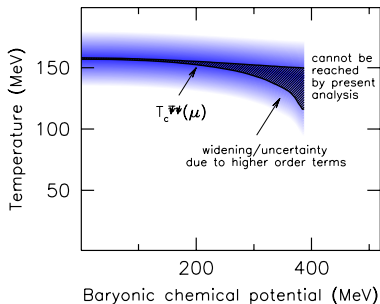
Simulations at imaginary chemical potential

- ▶ At $\mu = i\mu_i$ HMC can be applied
 $\hat{D}(i\mu_I) = \gamma^\mu (\partial_\mu + ig\hat{A}_\mu) + i\mu_i\gamma_4$
 $\det(\hat{D}(U) + m) = \prod_n (\lambda_n^2 + m^2) \geq 0$
- ▶ Roberge-Weiss transition
 At $\mu_i = \pi T$ Polyakov loop changes phase $P \rightarrow e^{-2\pi/3i} P$
- ▶ II order transition at C
 $T_{RW} = 208(5)$ MeV, $\mu_i = \pi T$
 Phys.Rev.D 93 (2016) 7, 074504
- ▶ One can study QCD in the region
 $\mu_i < \pi T$



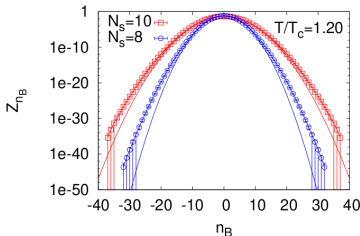
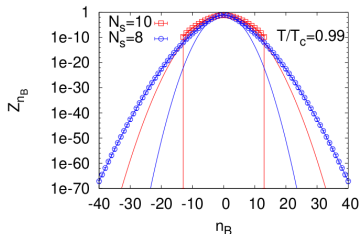
Critical temperature at finite baryon density

- ▶ $\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T_c(0)} \right)^2$
- ▶ $\kappa = 0.0180(40)$
Phys. Rev. D89 no. 7, (2014) 074512
- ▶ $\kappa = 0.0149(21)$
Phys. Lett. B751 (2015) 559–564
- ▶ $\kappa = 0.0135(20)$
Phys. Rev. D90 no. 11, (2014) 114025



Canonical method

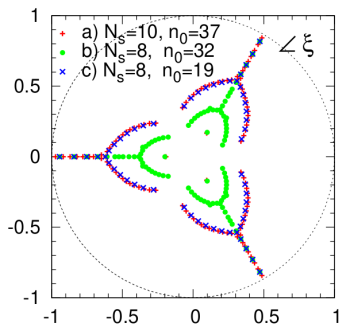
- ▶ Gibbs free energy Ω
 $\Omega = -T \log Z(\mu),$
 $Z(\mu) = Tr \exp\left(-\frac{\hat{H} - \mu \hat{N}}{T}\right)$
- ▶ Helmholtz free energy F_n
 $F_n = -T \log Z_n,$
 $Z_n = Tr \exp\left(-\frac{\hat{H}}{T}\right)$
- ▶ Fugacity expansion
 $Z(\mu) = \sum_n Z_n e^{n\mu/T}$
- ▶ Z_n can be calculated on lattice
 $Z_n = \int_{-\pi}^{\pi} \frac{d\varphi}{2\pi} Z(i\mu_i) e^{-in\varphi}, \varphi = \frac{\mu_i}{T}$



K. Nagata e-Print: 2108.12423

Canonical method

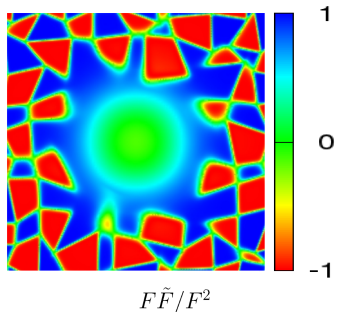
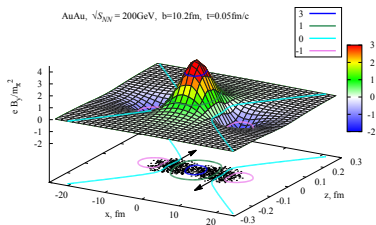
- ▶ $Z(\xi) = \sum_{n=-N}^N Z_n \xi^n, \xi = e^{\mu/T}$
- ▶ Lee-Yang zeros $Z(\xi) = 0$
- ▶ The distribution of Lee-Yang zeros approaches the real axis in the thermodynamic limit and causes the singularity in thermodynamic quantities.
- ▶ Perspective approach:
 $N_B \sim 10, L_s \sim 2 \text{ fm}, n_B \sim 1 \text{ fm}^{-3} > 0.17 \text{ fm}^{-3}$
- ▶ But very complicated: small volumes, large lattice spacing, heavy pion, beyond double precision
- ▶ Recent studies:
[V. G. Bornyakov, Phys. Rev. D 95 \(2017\), Phys. Rev. D 107 \(2023\)](#); [HotQCD, Phys.Rev.D105 \(2022\)](#); [S. Borsányi e-Print: 2502.03211,](#)



K. Nagata e-Print: 2108.12423

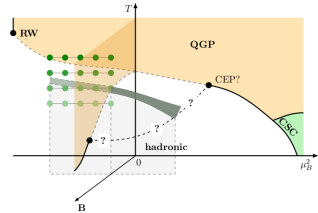
Strong magnetic field in heavy ion collisions

- ▶ Heavy ion collision creates huge magnetic field $eB < 1 \text{ GeV}^2$
D.Kharzeev, L.McLerran, H.Warringa, Nucl.Phys.A803 (2008);
V.Skokov, A.Illarionov, V.Toneev, Int. J. Mod. Phys. A 24 (2009)
V.Voronyuk, V.Toneev, W.Cassing, E.Bratkovskaya, V.Konchakovski, S.Voloshin, Phys. Rev C 84 (2011)
- ▶ Strong eB modifies vacuum properties
Magnetic catalysis
V.Gusynin, P.Miransky, V.Shovkovy, Phys.Rev.Lett. 73 (26)
- ▶ Catalyzing role of electromagnetic field on deconfinement
B.Galilo, S.Nedelko, Phys. Rev. D 84 (2011)
S.Nedelko, V.Voronin, Eur. Phys. J A (2015)
- ▶ Inverse magnetic catalysis
F.Bruckmann, G.Endrodi, T.Kovacs, JHEP 04 (2013)

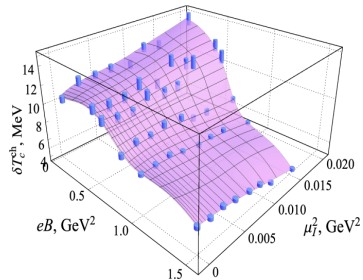
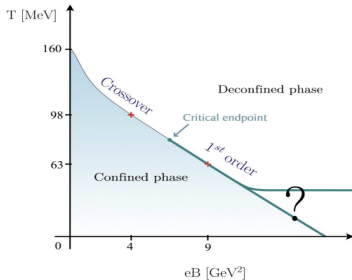


Dense QCD at nonzero magnetic field

- ▶ Idea: consider the phase diagram in space (T, μ_B, eB)
- ▶ Crossover at $eB = 4 \text{ GeV}^2$ and first order at $eB = 9 \text{ GeV}^2$
M. D'Elia, Phys.Rev.D 105 (2022) 3, 034511
- ▶ Phase diagram at $\mu_B \neq 0, eB < 1.5 \text{ GeV}^2$
Inverse magnetic catalysis at finite μ_B
V. Braguta Phys.Rev.D 100 (2019) 11, 114503
- ▶ $\delta T_c(eB, \mu_B) = \delta T_c(eB, \mu_B) - A_2(eB)\mu_B^2$
 $(T^{CEP}, \mu_B^{CEP}) = (100(25), 800(140)) \text{ MeV}$
V. Braguta, Phys.Rev.D 100 (2019) 11

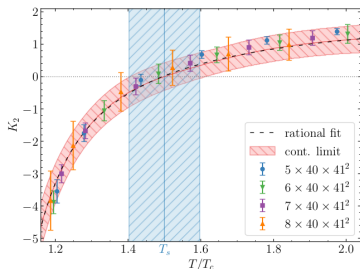


S. Borsanyi, PoS LATTICE2023 (2024) 164

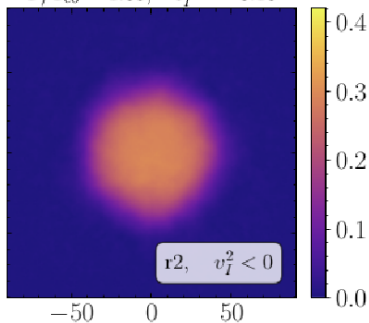


Relativistic rotation in heavy ion collisions

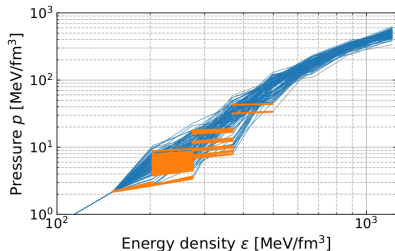
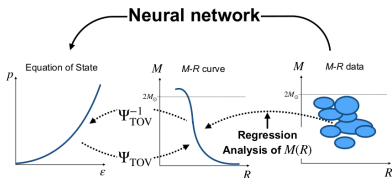
- ▶ Characteristic $\Omega \sim 10$ MeV
STAR, Nature 548, 62 (2017)
- ▶ $v \sim c$ at $r \sim 10 - 20$ fm ($\sim 10^{22}$ s $^{-1}$)
- ▶ Relativistic rotation of baryon matter
General relativity effects must be accounted
- ▶ Rotation influences both quark and gluon degrees of freedom
- ▶ Surprising results from lattice simulations at $\mu_B = 0$
V.Braguta, Phys.Rev.D 103 (2021) 9, 094515;
PoS LATTICE2022 (2023) 190; Phys.Lett.B 855 (2024); Phys.Rev.D 110 (2024);
Phys.Lett.B 852 (2024); Phys.Lett.B 852 (2024)
- ▶ Competition between quark and gluon sectors
- ▶ Negative moment of inertia
- ▶ Inhomogeneous phase transitions
talk of Artem Roenko, on 19 February at 16:15



$$T/T_{c0} = 1.05, \quad v_I^2 = -0.16$$



QCD phase diagram from astrophysics



- ▶ Machine-learning analysis
Fujimoto, Y., Fukushima, K., Murase, K., JHEP03(2021)
- ▶ Neutron star data on the radius-mass $R - M$
- ▶ Data $R - M \Rightarrow$ EoS (maximum likelihood method)
- ▶ Likely crossover but the first-order phase transitions is not excluded

Dense QCD at low temperature

- ▶ No good solution. The sign problem is manifested in all approaches
- ▶ One can conduct lattice studies for $\mu \leq \text{few} \times T$
- ▶ At low or zero temperature these approaches do not work
- ▶ The only possibility: QCD-like theories
Without sign problem and common properties to real QCD

QCD-like theories

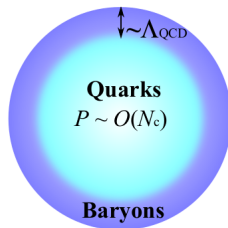
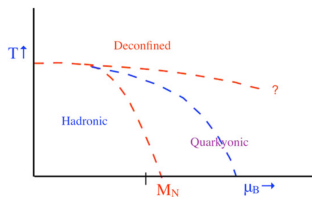
- ▶ Two-color QCD
Symmetry $(\gamma_5 C \tau_2) \cdot D^* = D \cdot (\gamma_5 C \tau_2) \Rightarrow$ pairs (λ, λ^*)
- ▶ QCD at finite isospin density: $\mu_u = -\mu_d = \mu_I \Rightarrow \mu_B = 0$
 $\det(\hat{D}(\mu_I) + m) \times \det(\hat{D}(-\mu_I) + m) = |\det(\hat{D}(\mu_I) + m)|^2$
- ▶ ...

Sketch of dense QCD phase diagram

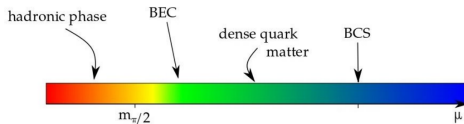
Phases of dense quarks at large N_c

L. McLerran, R.D. Pisarski, Nuclear Physics A796 (2007)

- ▶ Hadronic phase $\mu_B < M_N$
- ▶ Dilute baryonic matter $\mu_B > M_N$
- ▶ Liquid-gas transition
 $\mu_B - M_N \sim \frac{\Lambda_{QCD}}{N_c^2} \sim \frac{\Lambda_{QCD}}{N_c^2} \sim 30 \text{ MeV}$
- ▶ Quarkyonic phase ($\mu \sim \Lambda_{QCD}$)
 - ▶ Quarks form Fermi sphere
 - ▶ Baryon on Fermi surface
 - ▶ Chiral symmetry is restored
 - ▶ Confinement
- ▶ Deconfinement: $\mu_B \gg \Lambda_{QCD}$



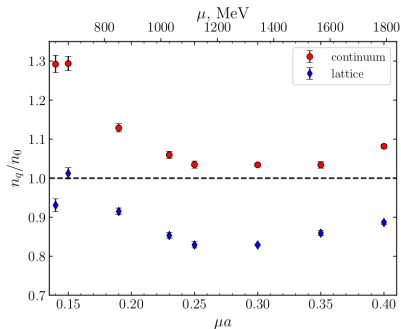
Phases in two-color QCD



Similar to Quarkyonic phase

- ▶ Fermi sphere is formed: $n_q \simeq n_{SB}$
- ▶ Baryons on the surface $\Sigma \sim \mu^2$
- ▶ Chiral symmetry is restored
- ▶ The system is in confinement

V.Braguta, Phys.Rev.D 94 (2016),
Phys.Rev.D 102 (2020); Etsuko Itou,
JHEP 10 (2024); S. Hands, PoS
LATTICE2024 (2025)

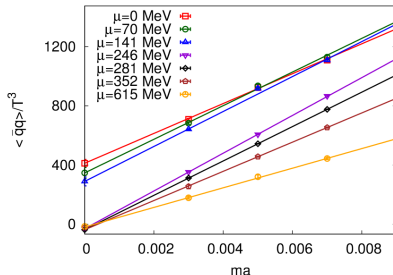
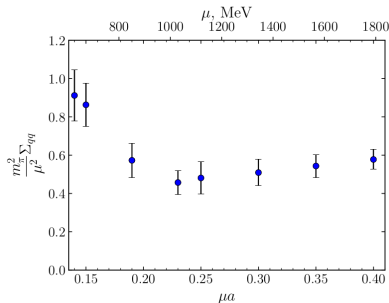


Phases in two-color QCD

Similar to Quarkyonic phase

- ▶ Fermi sphere is formed: $n_q \simeq n_{SB}$
- ▶ Baryons on the surface $\Sigma \sim \mu^2$
- ▶ Chiral symmetry is restored
- ▶ The system is in confinement

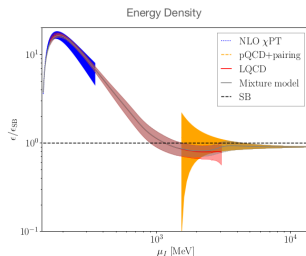
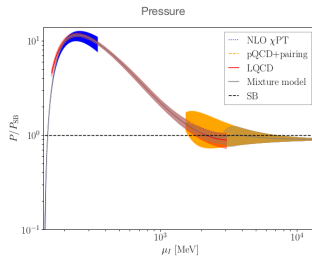
V.Braguta, Phys.Rev.D 94 (2016),
Phys.Rev.D 102 (2020); Etsuko Itou,
JHEP 10 (2024); S. Hands, PoS
LATTICE2024 (2025)



Phases of QCD at finite isospin density

Similar to Quarkyonic phase

- ▶ Fermi sphere is formed: $n_I \simeq n_{SB}$
 - ▶ π^+ mesons on the surface $\Sigma \sim \mu_I^2$
 - ▶ Chiral symmetry is restored
 - ▶ The system is in confinement
- NPLQCD, Phys.Rev.D 108 (2023),
Phys.Rev.Lett. 134 (2025), G. Endrodi
JHEP 07 (2023)



Quark matter at high baryon density

- ▶ The Fermi sphere is formed and $\alpha_s(\mu) \ll 1$
- ▶ One gluon exchange potential is attractive in the colour anti-triplet state
- ▶ Instability towards the formation of Cooper pairs
- ▶ Colour-superconductivity (CSC) with diquark condensate
 - M. Iwasaki and T. Iwado, Phys. Lett. B350 (1995)
 - M. G. Alford, K. Rajagopal, and F. Wilczek, Phys. Lett. B422 (1998)
 - R. Rapp, T. Schafer, E. V. Shuryak, and M. Velkovsky, Phys. Rev. Lett. 81 (1998)
- ▶ Scalar diquark condensate: $d_{\alpha i} \sim \epsilon_{\alpha\beta\gamma}\epsilon_{ijk}\langle\psi_{\beta j}^T C\gamma_5\psi_{\gamma k}\rangle$
 - ▶ Colour-flavour locked phase: $\epsilon_{l\beta\gamma}\epsilon_{ljk}\langle\psi_{\beta j}^T C\gamma_5\psi_{\gamma k}\rangle$
 - F. Wilczek, Nucl.Phys. B 537 (1999)
 - ▶ Two-flavour superconducting phase: $\epsilon_{3\beta\gamma}\epsilon_{jk}\langle\psi_{\beta j}^T C\gamma_5\psi_{\gamma k}\rangle \neq 0$
 - F. Wilczek, Phys. Lett. B 422 (1998)
- ▶ Diquark condensates $d_{\alpha i}$: $[ud]$, $[us]$, $[ds]$ -diquarks
 - Nucl. Phys. A743 (2004) 127, Phys. Rev. Lett. 93 (2004) 132001, Phys. Rev. D71 (2005) 034002, Nucl. Phys. B558 (1999) 219–242, arXiv:hep-ph/0407257

Quark matter at high baryon density

- ▶ Spin-one diquarks

Phys. Rev. D66 (2002) 114010, Phys. Rev. Lett. 91 (2003) 242301,
arXiv:hep-ph/0407257, Phys. Lett. B350 (1995) 163, Phys. Rev. D71 (2005) 054016

- ▶ CSC at finite temperature

Melting pattern: CFL $\rightarrow ([du], [ds]) \rightarrow [ud] \rightarrow$ NQM

Phys. Rept. 107 (1984) 325, Phys. Rev. D63 (2001) 074018, Phys. Rev. Lett. 93 (2004) 132001, Phys. Rev. D71 (2005) 034002

- ▶ $U_A(1)$ -anomaly and chiral symmetry breaking

Phys. Rev. Lett. 97 (2006) 122001, Phys. Rev. D76 (2007) 074001, JHEP 12 (2008) 060,
Phys. Rev. D81 (2010) 125010

- ▶ Meson masses: $M_{\pi^\pm} > M_{K^\pm} \simeq M_{K^\pm}$: $\pi^+ ([\bar{d}\bar{s}][su])$, $\pi^+ ([\bar{d}\bar{s}][ud])$

Phys. Lett. B464 (1999) 111–116, Phys. Rev. D61 (2000) 074012, Phys. Rev. D62 (2000) 059902

- ▶ Kaon condensation in QM

Nucl. Phys. A697 (2002) 802–822, Phys. Rev. D65 (2002) 054042, Phys. Rev. D71 (2005) 034004, Phys. Rev. D81 (2010) 054033, Phys. Rev. D72 (2005) 094032

- ▶ Spatially modulated chiral condensate: $\langle \bar{\psi}(x)\psi(x) \rangle = \sigma \cos 2\vec{q}\vec{x}$, $|\vec{q}| = \mu_q$

D. Deryagin, D. Grigoriev, V. A. Rubakov, *Int. J. Mod. Phys. A7* (1992)

- ▶ ...

Quark matter at $\mu_B \gg \Lambda_{QCD}$

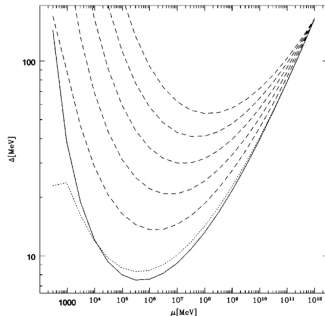
Dense QCD

- ▶ Typical mass gap in BCS:
$$\Delta \sim \exp\left(-\frac{C}{\mu^2 g^2}\right)$$
- ▶ Mass gap in CSC: $\Delta \sim g^{-5} \mu \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right)$
D. T. Son Phys. Rev. D59
T. Schäfer, F. Wilczek, Phys. Rev. D60
- ▶ Debye screening of chromoelectric field
- ▶ Landau damping of chromomagnetic field
- ▶ Typical scale is $\Delta \sim 100$ MeV
- ▶ $T_c \sim \Delta$

QCD-like theories

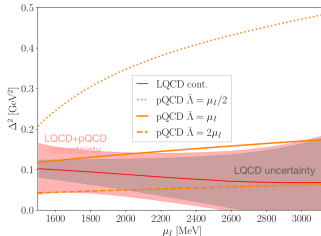
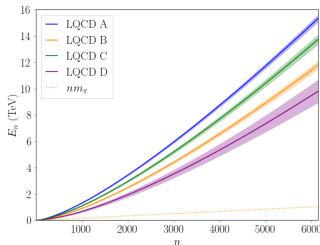
- ▶ Two-color QCD: $\Delta \sim g^{-5} \mu \exp\left(-\frac{2\pi^2}{g}\right)$
T. Schäfer Nucl. Phys. B 2000, 575, 269-284
- ▶ QCD with isospin density:
$$\Delta \sim g^{-5} \mu \exp\left(-\frac{3\pi^2}{2g}\right)$$

D. T. Son, M. A. Stephanov, Phys.Rev.Lett. 86 (2001) 592

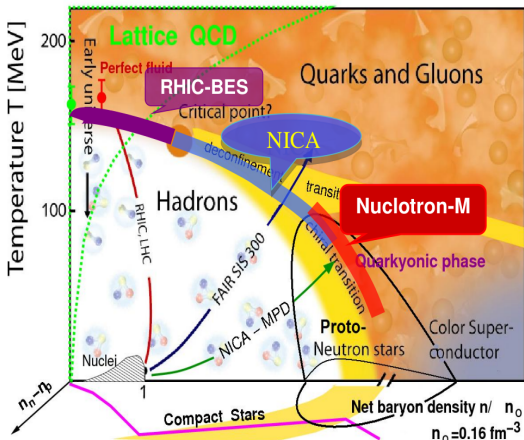


Quark matter at high isospin density

- ▶ Correlation function of many pions
 $C(t) = \langle (\pi^+(\vec{x}, 0))^n (\pi^-(\vec{y}, t))^n \rangle$
- ▶ Late time behaviour $C_n(t) \rightarrow e^{-E_n t}$
- ▶ $N_f = 2 + 1$ and physical quark masses
- ▶ $n = 6144$
- ▶ C_{6144} varies by 10^5 orders of magnitude
2-double and 3-double precision
NPLQCD, Phys.Rev.D 108 (2023), Phys.Rev.Lett.
134 (2025)
- ▶ Mass gap $\Delta \sim 100$ MeV
- ▶ Similar magnitude of Δ in two-color QCD



Conclusion



- ▶ Poor understanding of QCD at finite baryon density
- ▶ Rich physics due to extreme conditions
Large baryon density, strong electromagnetic fields, relativistic rotation...
- ▶ **NICA creates unique conditions for studying completely new physics**