False vacuum decay around black holes

Ratmir Gazizov

Moscow State University & Institute for Nuclear Research of RAS

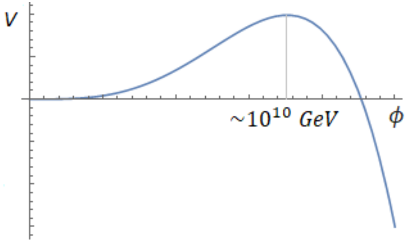
February 17, 2025

The Higgs vacuum

- The effective Higgs potential has a false vacuum:

$$V_{\text{eff}}(\phi) = \frac{1}{4}\lambda_{\text{eff}}(\phi)\phi^4$$

- The field can decay due to quantum tunneling.

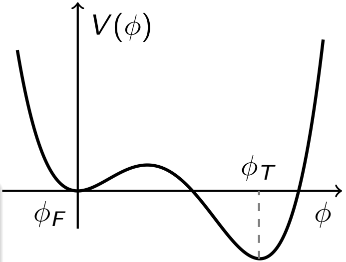


The effective Higgs potential

D. Buttazzo et al (2013)

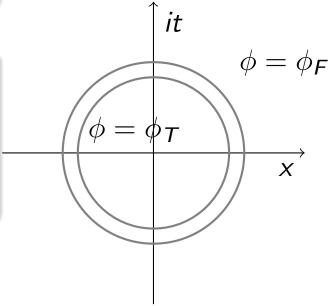
Coleman instantons

$$S = \int d^4x \left(\frac{1}{2}(\partial_t \phi)^2 - \frac{1}{2}(\partial_x \phi)^2 - V(\phi) \right)$$



The probability of false vacuum decay in flat space-time:

$$P \sim e^{-S_E}$$



Periodic instantons in a thermal bath (for the potential $V(\phi)$): $\tau_{\text{Period}} = 1/T$.
For a very high temperature T:

$$P \sim e^{-E_{sph}/T}$$

Sidney R. Coleman (1977)

False vacuum decay in the presence of BH

Hawking temperature:

$$T_H = \frac{M_{Pl}^2}{8\pi M_{BH}}$$

Conjecture: small black holes have high temperatures $\Longrightarrow$ significantly increase the decay probability.

P. Burda et al (2016)

A calculation from first principles is needed.

(see also A. Shkerin, S. Sibiryakov (2021))

Formulation from first principles

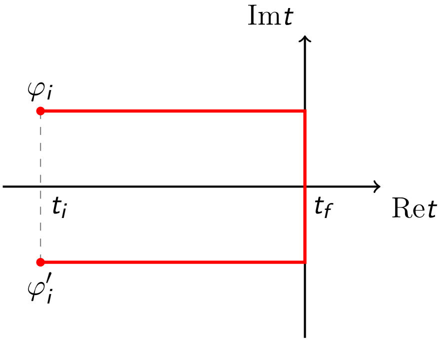
Decay probability from functional integral:

$$P = \int D\phi_f D\phi_i D\phi_i' \langle \phi_f | \hat{S} | \phi_i \rangle \langle \phi_i | \hat{\rho} | \phi_i' \rangle \langle \phi_i' | \hat{S}^\dagger | \phi_f \rangle$$

Fields ϕ and ϕ' can be written as a united field on the double-bent time contour.

Saddle-point approximation:

$$P \sim e^{iS[\phi_{cl}] + iB[\phi_{cl}]}$$



S.Yu. Khlebnikov et al (1991)

V.A. Rubakov, M.E. Shaposhnikov (1996)

Scalar field in Schwarzschild metric

$$S = \int \sqrt{-g} d^4x \left(\frac{1}{2} g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi - V(\phi) \right)$$

$$ds^2 = f(r)\,dt^2 - \frac{dr^2}{f(r)} - r^2 d\Omega^2, \quad f(r) = 1 - \frac{2M}{r}, \quad M_{Pl} = 1$$

Substitution:

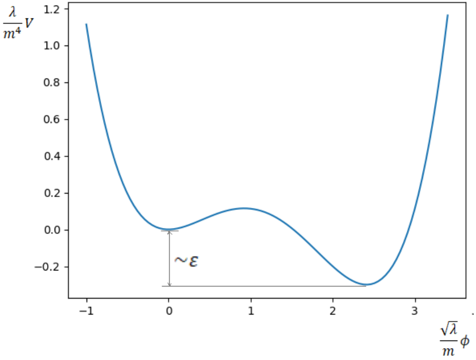
$$\phi = \frac{\varphi}{r}, \quad x = r + 2M \ln(r - 2M),$$

$$S = 4\pi \int dt dx \left(\frac{1}{2} (\partial_t \varphi)^2 - \frac{1}{2} (\partial_x \varphi)^2 - \frac{1}{2} U(x) \varphi^2 - r^2 f(r) V\left(\frac{\varphi}{r} \right) \right),$$

The toy potential:

$$V(\phi) = \frac{m^2}{2}\phi^2 - \frac{m\sqrt{\lambda}}{2}\phi^3 + \frac{\lambda}{8}(1-\varepsilon)\phi^4$$

If the parameter $\varepsilon \ll 1$, then thin wall approximation is valid.



The potential V used in this work

Boundary conditions

The boundary conditions can be imposed on the Fourier coefficients

$$\phi_i(x) = \int_0^\infty \frac{d\omega}{\sqrt{4\pi\omega}} \sum_{I=R,L} \left(a_{I,\omega} f_{I,\omega}(x) e^{-i\omega t_i} + b_{I,\omega} f^*_{I,\omega}(x) e^{i\omega t_i} \right)$$

$$a_{I,\omega} = b^*_{I,\omega} e^{-\omega \beta_I}$$

$$\beta_I = \begin{cases} \beta_H, I = R \\ \beta_E, I = L \end{cases}$$

where $\beta_H = 8\pi M_{BH}/M_{Pl}^2$.

Period for the case of thermal equilibrium:

$$\beta = \beta_H = \beta_E$$

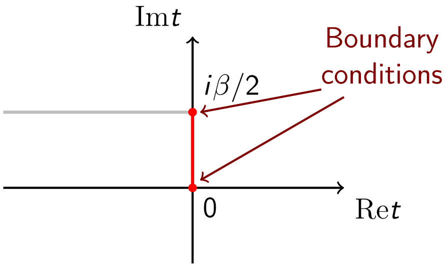
Simplified boundary conditions:

$$\partial_\tau \phi(0, x) = \partial_\tau \phi(\beta/2, x) = 0$$

$$\partial_x \phi(t, -\infty) = \partial_x \phi(t, \infty) = 0$$

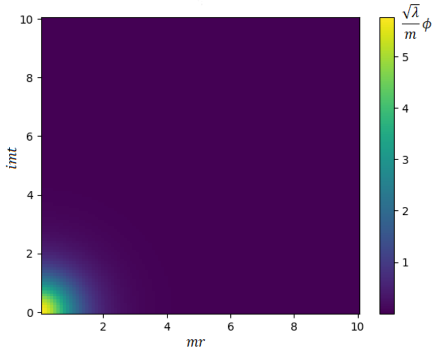
The decay probability:

$$P \sim e^{-S_E}$$

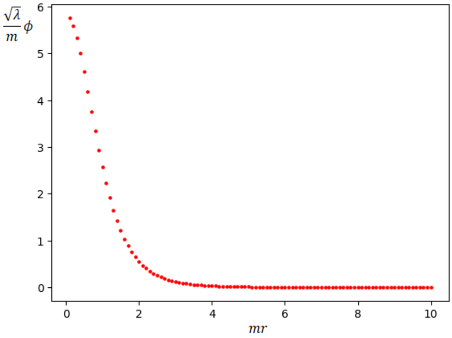


Numerical results

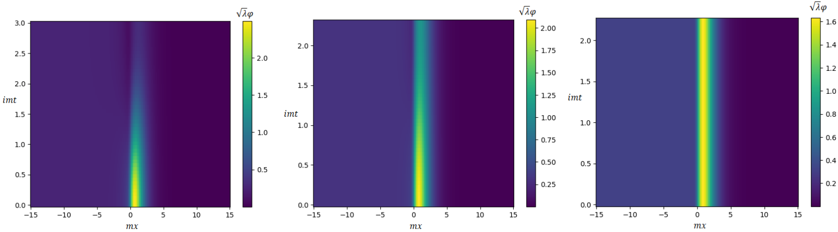
We use the Newton-Raphson method to solve the system of nonlinear equations.



The instanton in flat space-time with $\varepsilon = 1$, $(N_t \times N_x = 100 \times 100)$

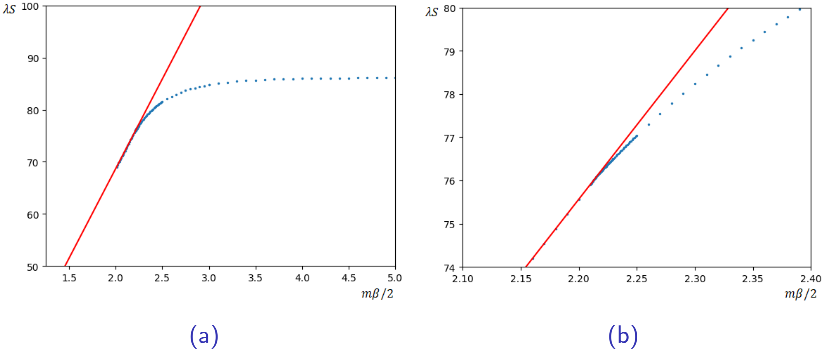


The profile at $t = 0$, $(N_t \times N_x = 100 \times 100)$

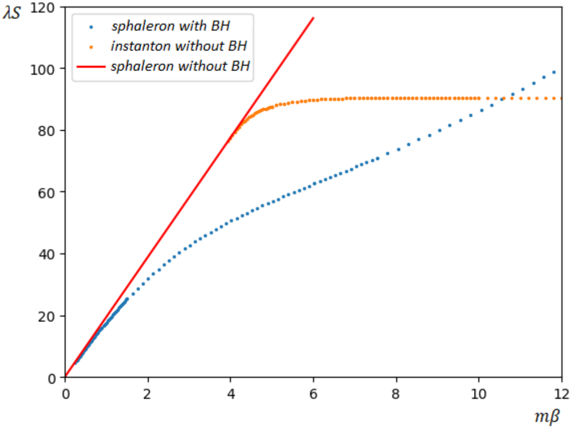


(a) $m\beta/2 = 3$ (b) $m\beta/2 = 2.3$ (c) $m\beta/2 = 2.2$

Periodic instantons with at different β in the presence of BH $mr_h = \frac{2Mm}{M_{Pl}^2} = 0.1$, $(N_t \times N_x = 50 \times 300)$. Period and mass of BH are independent parameters here.



Euclidean action S as a function of period β for non-trivial instantons (blue) and sphalerons (red), $mr_h = \frac{2Mm}{M_{Pl}^2} = 0.1$ ($m\beta/2 \neq m\beta_H/2 = 8\pi Mm/2M_{Pl}^2 \simeq 0.63$).



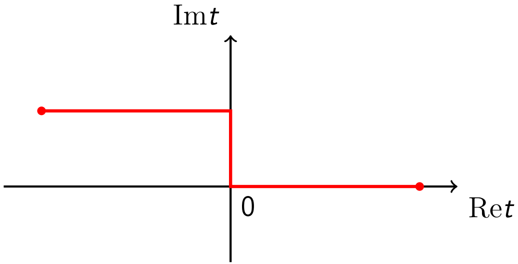
Euclidean action S_E of shalerons as a function of the inverse temperature β. Blue and orange dotted lines correspond to presence of BH with $m\beta_H = m\beta = 8\pi mM/M_{Pl}^2$ and to decay in flat space-time respectively. Red line corresponds to sphalerons in flat space-time.

Conclusions

- the physical solutions $\beta = \beta_H$ describing the process are static sphalerons.
- $\beta \to 0$: sphalerons with BH approach flat-space sphalerons. It means that small BHs $mr_h \ll mr_b$ do not significantly change sphalerons in a very hot environment.
- $\beta \to \infty$: a large massive BH $mr_h \not\ll mr_b$ changes the geometry of space $\implies$ sphalerons change too.

Further research

- Nonequilibrium case $\beta \neq \beta_H \implies$ a new contour and new boundary conditions



- More realistic potentials: $V(\phi) = -\lambda\phi^4/4$

Thank you for your attention

The project was done with the support of the scientific program

of the National Center for Physics and Mathematics, sector 5 "Particle Physics and Cosmology", stage 2023-2025