ЧЕРНЫЕ ДЫРЫ В ПОЛУКЛАССИЧЕСКОЙ f(R) ТЕОРИИ ГРАВИТАЦИИ С ДОПОЛНИТЕЛЬНЫМИ ИЗМЕРЕНИЯМИ

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Newtonian equation of gravity

$$\nabla\phi = 4\pi\rho\,, \tag{1}$$

$$\phi = -\frac{M}{r} \quad \text{– Newtonian potential of a point mass} \tag{2}$$

c = G = 1

$(\mathcal{M}, g)$ - Riemannian manifold

$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu, \qquad \mu, \nu = 1, 2, 3, 4 \tag{3}$$

Einstein-Hilbert action :

$$S \sim \int d^4x \sqrt{|g(x)|} \left(R(x) - 2\Lambda + L_{matter} \right) \qquad \Rightarrow \tag{4}$$

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}. \tag{5}$$

$g(x)$ - determinant of $g_{\mu\nu}(x)$,
Λ - cosmological constant,
$R(x)$ - scalar curvature of space,
$R_{\mu\nu}$ - Ricci tensor.

Schwarzschild black hole

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \frac{dr^2}{\left(1 - \frac{2M}{r}\right)} - r^2\left(d\theta^2 + \sin^2(\theta) d\varphi^2\right), \qquad (6)$$

where M is the mass of the black hole.
Away from the black hole

$$r \gg 2M \qquad (7)$$

$$ds^2 \simeq \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 + \frac{2M}{r}\right) dr^2 - r^2\left(d\theta^2 + \sin^2(\theta) d\varphi^2\right). \qquad (8)$$

Here

$$\phi = -\frac{M}{r} \qquad \text{– Newtonian potential of a point mass} \qquad (9)$$

Einstein's equations are reduced to the Newtonian equation of gravity

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = 8\pi T_{\mu\nu} \quad \Rightarrow \quad \nabla\phi = 4\pi\rho\,. \qquad (10)$$

c = G = 1

$$S \sim \int d^4x \sqrt{|g(x)|} \left(R(x) - 2\Lambda \right) \quad (11)$$

Schwarzschild-de Sitter black hole

$$ds^2 = \left(1 - \frac{2m}{r} - \frac{\Lambda}{3} r^2\right) dt^2 - \frac{dr^2}{\left(1 - \frac{2m}{r} - \frac{\Lambda}{3} r^2\right)} - r^2 \left(d\theta^2 + \sin^2(\theta) d\varphi^2\right). \quad (12)$$

Some of the questions that Einstein's theory of gravity does not answer:

- why is the Universe homogeneous and isotropic (on a "large" scale) ?;
- why is the Universe expanding at an accelerated rate (what is "dark energy"?)?;
- why is the dimension of our space equal to 4?
- ...

The minimum set of ideas for solving the mentioned problems:

- modification of the theory of gravity
- introduction of additional dimensions

Quantum field theory in curved space $\Rightarrow$

$$S \sim \int d^4x \sqrt{|g(x)|} \left(R(x) - 2\Lambda + \alpha R^2 + \beta R^{\mu\nu} R_{\mu\nu} + \gamma R^{\mu\nu\lambda\kappa} R_{\mu\nu\lambda\kappa} + L_{matter} \right) \quad (13)$$

Λ - cosmological constant,
$R(x)$ - scalar curvature of space,
$R_{\mu\nu}$ - Ricci tensor,
$R_{\mu\nu\lambda\kappa}$ - Riemann tensor.

$f(R)$ theory in a space of dimension 4:

$$S \sim \int d^4x \sqrt{|g(x)|} \quad f(R) \Rightarrow \tag{14}$$

$$f_R R_{\mu\nu} - \frac{1}{2} f g_{\mu\nu} + \left[\nabla_\mu \nabla_\nu - g_{\mu\nu} \Box\right] f_R = 0\,, \tag{15}$$

$$\Box \equiv g^{\mu\nu} \nabla_\mu \nabla_\nu\,, \quad f_R = df/dR\,.$$

Example:

$$f(R) = aR^2 + R - 2\Lambda, \qquad a,\ \Lambda\ -\ \text{constants} \ \ \text{(Starobinsky A.)} \tag{16}$$

By performing a conformal rescaling

$$\tilde{g}_{\mu\nu} = e^{\phi/\sqrt{3}} g_{\mu\nu} \, , \qquad (17)$$

we transform to the Einstein frame:

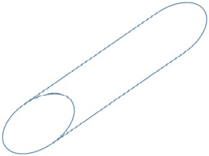
$$S \sim \int d^4x \sqrt{|\tilde{g}(x)|} \quad \left[\tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} (\nabla_\mu \phi)(\nabla_\nu \phi) + e^{-2\phi/\sqrt{3}} V \left(e^{\phi/\sqrt{3}} \right) \right] \qquad (18)$$

The Lagrangian containing the Ricci scalar in the form of specific function f(R) is transformed into the scalar-tensor model that strongly simplifies the subsequent calculations

- Why is the dimension of our space equal to 4?

Kaluza-Klein model

$$\mathcal{M}_5 = \mathcal{R}^4 \times \mathcal{S}^1$$



$$g_{BC} = \left|\begin{array}{cccc|c} & & & & -A_1 \\ & g_{\mu\nu} - A_\mu A_\nu & & & -A_2 \\ & & & & -A_3 \\ & & & & -A_4 \\ \hline -A_1 & -A_2 & -A_3 & -A_4 & -1 \end{array}\right|$$

$$B, C = 1, 2, 3, 4, 5, \qquad \mu, \nu = 1, 2, 3, 4$$

$$S \sim \int d^5x \sqrt{|g_{(5)}|} \quad \left[R_5 - 2\Lambda_5\right] \sim \int d^4x \sqrt{|g_{(4)}|} \quad \left[R_4 - 2\Lambda_4 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}\right], \quad (19)$$

$$F_{\mu\nu} = \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu}$$

The additional subspace is compact

$$10^{-33}\text{cm} < L < 10^{-18}\text{cm},$$

where L is the characteristic scale of the additional subspace

$$S = \frac{m_{\mathsf{D}}^{\mathsf{D}-2}}{2} \int_{M_{\mathsf{D}}} d^{\mathsf{D}} X \sqrt{|g_{\mathsf{D}}|} \quad f(R) \tag{20}$$

D-dimensional space $\mathcal{T} \times \mathcal{M}_3 \times \mathcal{M}_n$

The theory equations have the form

$$f_R R_A^B - \frac{1}{2} f \delta_A^B + \left[\nabla^B \nabla_A - \delta_A^B \Box\right] f_R = 0\,, \tag{21}$$

$$\Box \equiv g^{AB} \nabla_A \nabla_B\,, \quad f_R = df/dR\,,$$

$$c = \hbar = 1\,. \tag{22}$$

$$ds^2 = h(r)dt^2 - b(r)dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) - {L_0}^2 d\Omega_n^2. \quad (23)$$

We assume

$$l(r) \gg L_0 \gg l_{Pl}, \quad (24)$$

where $l(r)$ is the characteristic scale of curvature of the four-dimensional part of space-time

$$l(r)^{-1} \simeq \max\left\{\left|\frac{h'}{h}\right|, \left|\frac{b'}{b}\right|, \sqrt{\left|\frac{h''}{h}\right|}, \sqrt{\left|\frac{b''}{b}\right|}, \left|\frac{h'''}{h}\right|^{1/3}, \ldots\right\}, \quad (25)$$

Stress-energy tensor for quantized fields in a static spacetime when the fields is in the zero temperature vacuum state defined with respect to the timelike Killing vector which always exists in a static spacetime

$$\left\langle T_A^B \right\rangle = \sum_{k=1}^{N} \left\langle \overset{(k)}{T_A^B} \right\rangle = \frac{K_A^B}{{L_0}^{4+n}}\left(1 + O\left({L_0}^2/l(r)^2\right)\right), \quad (26)$$

where N is the number of mater fields,

$$K_A^B = \text{diag}\left(K_t^t, K_t^t, K_t^t, K_t^t, K_5^5, \ldots, K_5^5\right). \quad (27)$$

Example for a massless scalar field in spacetime (Popov A., Phys. Rev. D (2001))

$$ds^2 = dt^2 - dr^2 - {L_0}^2 \left(d\psi^2 + \sin^2\theta d\chi^2\right) \tag{28}$$

$$\begin{aligned}\langle T_t^t\rangle = \langle T_r^r\rangle &= \frac{1}{4\pi^2 {L_0}^4}\left\{\frac{3\xi^2}{8} - \frac{11\xi}{96} + \frac{79}{7680} + \left(-\frac{\xi^2}{2} + \frac{\xi}{6} - \frac{1}{60}\right)\ln\sqrt{\frac{2\xi - 1/4}{m_{\rm DS}^2 {L_0}^2}}\right. \\ &\left. + \left(2\xi^2 - \frac{\xi}{2} + \frac{1}{32}\right)\left[I_1\left(2\xi - \frac{1}{4}\right) - I_2\left(2\xi - \frac{1}{4}\right)\right]\right\},\end{aligned} \tag{29}$$

$$\begin{aligned}\left\langle T_\psi^\psi\right\rangle = \langle T_\chi^\chi\rangle &= \frac{1}{4\pi^2 {L_0}^4}\left\{\left(-\frac{\xi^2}{8} + \frac{\xi}{32} - \frac{1}{512}\right) + \left(\frac{\xi^2}{2} - \frac{\xi}{6} + \frac{1}{60}\right)\ln\sqrt{\frac{2\xi - 1/4}{m_{\rm DS}^2 {L_0}^2}}\right. \\ &\left. + \left(-2\xi^2 + \frac{\xi}{2} - \frac{1}{32}\right)\left[I_1\left(2\xi - \frac{1}{4}\right) - I_2\left(2\xi - \frac{1}{4}\right)\right]\right\},\end{aligned} \tag{30}$$

where

$$I_n(\mu) = \int_0^\infty \frac{x^{2n-1}\ln\left|1 - x^2\right|}{1 + e^{2\pi\mu x}}\,dx\,, \tag{31}$$

and $m_{\rm DS}$ is an arbitrary parameter due to the infrared cutoff.

$$S = \int_{M_{\mathrm{D}}} d^{\mathrm{D}} X \sqrt{|g_{\mathrm{D}}|} \quad \left(\frac{m_{\mathrm{D}}^{\mathrm{D}-2}}{2} f(R) + L_m \right) \tag{32}$$

D-dimensional space $\mathcal{T} \times \mathcal{M}_3 \times \mathcal{M}_n$

$$ds^2 = h(r)dt^2 - b(r)dr^2 - r^2 \left(d\theta^2 + \sin^2 \theta d\varphi^2 \right) - L_0{}^2 d\Omega_n^2 . \tag{33}$$

The theory equations have the form

$$f_R R_A^B - \frac{1}{2} f \delta_A^B + \left[\nabla^B \nabla_A - \delta_A^B \Box \right] f_R = -\frac{1}{m_{\mathrm{D}}^{\mathrm{D}-2}} \left\langle T_A^B \right\rangle , \tag{34}$$

$$f_R = df/dR,$$

$$c = \hbar = 1 \, . \tag{35}$$

$$ds^2 = h(r)dt^2 - b(r)dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) - L_0^2 d\Omega_{\mathbf{n}}^2, \tag{36}$$

Nontrivial equations of semiclassical f(R) theory can be written as

$$\frac{R'^2}{b} f_{RRR} + \frac{1}{b}\left[R'' + \left(-\frac{b'}{2b} + \frac{2}{r}\right)R'\right] f_{RR} + \frac{1}{b}\left(-\frac{h''}{2h} + \frac{h'^2}{4h^2} + \frac{h'b'}{4hb} - \frac{h'}{hr}\right) f_R - \frac{f}{2} = \frac{-1}{m_{\mathbf{D}}^{\mathbf{D}-2}}\left\langle T_t^t \right\rangle,$$

$$\left(\frac{h'}{2h} + \frac{2}{r}\right)\frac{R'}{b} f_{RR} + \frac{1}{b}\left(-\frac{h''}{2h} + \frac{h'^2}{4h^2} + \frac{h'b'}{4hb} + \frac{b'}{br}\right) f_R - \frac{f}{2} = \frac{-1}{m_{\mathbf{D}}^{\mathbf{D}-2}}\left\langle T_t^t \right\rangle,$$

$$\frac{R'^2}{b} f_{RRR} + \frac{1}{b}\left[R'' + \left(\frac{h'}{2h} - \frac{b'}{2b} + \frac{1}{r}\right)R'\right] f_{RR} + \left[\frac{1}{br}\left(-\frac{h'}{2h} + \frac{b'}{2b} - \frac{1}{r}\right) + \frac{1}{r^2}\right] f_R - \frac{f}{2} = \frac{-1}{m_{\mathbf{D}}^{\mathbf{D}-2}}\left\langle T_t^t \right\rangle,$$

$$\frac{R'^2}{b} f_{RRR} + \frac{1}{b}\left[R'' + \left(\frac{h'}{2h} - \frac{b'}{2b} + \frac{2}{r}\right)R'\right] f_{RR} + \frac{(n-1)f_R}{{L_0}^2} - \frac{f}{2} = \frac{-1}{m_{\mathbf{D}}^{\mathbf{D}-2}}\left\langle T_5^5 \right\rangle,$$

where

$$R(r) = \frac{1}{b}\left(-\frac{h''}{h} + \frac{h'^2}{2h^2} + \frac{h'b'}{2hb} - \frac{2h'}{hr} + \frac{2b'}{br} - \frac{2}{r^2}\right) + \frac{2}{r^2} + \frac{(n-1)n}{{L_0}^2},$$

and the prime denotes the derivative with respect to r, $\quad f_{RRR} = \dfrac{d^3 f(R)}{dR^3}$.

Some combination of these equations leads to

$$\left(R-\frac{(n-1)(n+3)}{{L_0}^2}\right)\frac{df(R)}{dR}-\frac{f(R)}{2}=\frac{1}{m_{\mathsf{D}}^{\mathsf{D}-2}}\left(4\left\langle T_t^t\right\rangle-3\left\langle T_5^5\right\rangle\right)$$
$$=\frac{(3K_5^5-4K_t^t)}{m_{\mathsf{D}}^{2+n}{L_0}^{4+n}}. \tag{37}$$

From this equation follows

$$R=R_0=\text{const.} \tag{38}$$

This significantly simplifies the solution of the equations under study.

$$ds^2 = h(r)dt^2 - b(r)dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) - L_0^2 d\Omega_{\mathsf{n}}^2, \tag{39}$$

$$h(r) = \frac{1}{b(r)} = \left(1 - \frac{2M}{r} - \frac{\Lambda_4}{3}r^2\right), \tag{40}$$

where

$$\Lambda_4 = \frac{f(R_0) - \dfrac{2}{m_{\mathsf{D}}^{2+\mathsf{n}}}\dfrac{K_t^t}{{L_0}^{4+\mathsf{n}}},}{2f_R(R_0)}, \tag{41}$$

and the values of L_0 and R_0 are determined from relations

$$R_0 = \frac{n(n-1)}{{L_0}^2} + \frac{2\left(f(R_0) - \dfrac{2}{m_{\mathsf{D}}^{2+\mathsf{n}}}\dfrac{K_t^t}{{L_0}^{4+n}}\right)}{f_R(R_0)}, \tag{42}$$

$$\frac{1}{{L_0}^2} = \frac{1}{2(n-1)}\frac{\left(f(R_0) - \dfrac{2}{m_{\mathsf{D}}^{2+\mathsf{n}}}\dfrac{K_5^5}{{L_0}^{4+\mathsf{n}}}\right)}{f_R(R_0)}. \tag{43}$$

$$ds^2 = h(r)dt^2 - b(r)dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) - L_0^2 d\Omega_{\mathsf{n}}^2, \tag{44}$$

$$\Lambda_4 = \frac{f(R_0) - \dfrac{2}{m_{\mathsf{D}}^{2+\mathsf{n}}}\dfrac{K_t^t}{{L_0}^{4+\mathsf{n}}},}{2f_R(R_0)} = 0, \tag{45}$$

$$h(r) = \frac{1}{b(r)} = \left(1 - \frac{2M}{r}\right), \tag{46}$$

$${L_0}^2 = \frac{\mathsf{n}(\mathsf{n}-1)}{R_0}, \tag{47}$$

and the value of R_0 is determined from the relation

$$f_R(R_0) = \frac{\left(K_t^t - K_5^5\right)}{m_{\mathsf{D}}^{2+\mathsf{n}}}\frac{R_0^{1+\mathsf{n}/2}}{\mathsf{n}^{1+\mathsf{n}/2}(\mathsf{n}-1)^{2+\mathsf{n}/2}}. \tag{48}$$

$f(R) = R + c$ and n $= 2$

$$ds^2 = h(r)dt^2 - b(r)dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) - L_0^2\left(d\psi^2 + \sin^2\psi d\chi^2\right), \quad (49)$$

$$h(r) = \frac{1}{b(r)} = \left(1 - \frac{2M}{r}\right), \quad (50)$$

$$R_0 = \frac{2}{{L_0}^2}, \quad (51)$$

$$L_0^2\,{m_6}^2 = \sqrt{K_t^t - K_5^5}, \quad (52)$$

$$\Lambda_4 = 0 \qquad \Rightarrow \qquad \frac{c}{{m_6}^2} = \frac{2K_5^5}{(K_t^t - K_5^5)^{3/2}}. \quad (53)$$

$$ds^2 = h(r)dt^2 - b(r)dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) - L_0^2 d\Omega_{\mathsf{n}}^2, \tag{54}$$

Schwarzschild-de Sitter Black Hole

$$h(r) = \frac{1}{b(r)} = 1 - \frac{2m}{r} - \frac{(\mathsf{n}-1)}{3L_0^2}r^2 = \left(1 - \frac{2M}{r} - \frac{\Lambda_4}{3}r^2\right). \tag{55}$$

$$R_0 = \frac{(\mathsf{n}+4)(\mathsf{n}-1)}{{L_0}^2}, \quad \left.\frac{f}{f_R}\right|_{R=R_0} = \frac{2(\mathsf{n}-1)}{{L_0}^2}. \tag{56}$$

Let's consider a 4+n-dimensional space

$$ds^2 = g^{(4)}_{\mu\nu}(x)dx^\mu dx^\nu + g^{(n)}_{ab}(x,y)dy^a dy^b, \quad \mu,\nu = 1,2,3,4, \quad a,b = 1,...,\mathsf{n}. \tag{57}$$

and

$$R_4 \ll R_{\mathsf{n}}. \tag{58}$$

Knowledge of the extra space metric allows us to integrate the action

$$S = \frac{(m_{4+\mathsf{n}})^{\mathsf{n}+2}}{2}\int f(R)\ \sqrt{|g^{(4)}|}d^4x\ \sqrt{|g^{(\mathsf{n})}|}d^n y, \tag{59}$$

over the coordinates y if we restrict ourselves to

$$f(R) \simeq f(R_2) + f_R(R_{\mathsf{n}})R_4 + ... \tag{60}$$

Comparing the action

$$S \simeq \frac{(m_{4+\mathsf{n}})^{\mathsf{n}+2}}{2} \int \sqrt{|g^4|} d^4x \ \sqrt{|g^{(\mathsf{n})}|} d^n y \, [f(R_\mathsf{n}) + f_R(R_\mathsf{n}) R_4 + \cdots] \,, \quad (61)$$

with its four - dimensional analogue

$$S_4 \simeq \frac{{m_4}^2}{2} \int \sqrt{|g^{(4)}|} d^4x \, [R_4 - 2\Lambda_4,] \,, \quad (62)$$

we obtain expressions for the Planck mass

$$ {m_4}^2 = \frac{1}{G_4} = (m_{4+n})^{n+2} \int \sqrt{|g^{(n)}|} d^n y \ f_R(R_\mathsf{n}). \quad (63)$$

$c = \hbar = 1$

$f(R) = R + c$ and n $= 2$

$$m_4{}^2 = m_6{}^4 \int \sqrt{|g^{(2)}|} d^2y \; f_R(R_0) = 4\pi L_0{}^2 m_6{}^4. \tag{64}$$

This allows the solution to be expressed in terms of the four-dimensional Planck mass

$$ds^2 = h(r)dt^2 - b(r)dr^2 - r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right) - L_0^2 \left(d\psi^2 + \sin^2\psi d\chi^2\right), \tag{65}$$

$$h(r) = \frac{1}{b(r)} = \left(1 - \frac{2M}{r}\right), \tag{66}$$

$$R_0 = \frac{2}{L_0{}^2}, \tag{67}$$

$$L_0 \; m_4 = \sqrt{4\pi(K_t^t - K_5^5)}, \tag{68}$$

$$\frac{c}{m_4{}^2} = \frac{1}{4\pi} \frac{2K_5^5}{(K_t^t - K_5^5)^2}. \tag{69}$$

$$ds^2 = h(r)dt^2 - b(r)dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) - {L_0}^2 d\Omega_n^2. \tag{70}$$

We assume

$$l(r) \gg L_0 \gg l_{Pl}, \tag{71}$$

where $l(r)$ is the characteristic scale of curvature of the four-dimensional part of space-time

$$l(r)^{-1} \simeq \max\left\{\left|\frac{h'}{h}\right|, \left|\frac{b'}{b}\right|, \sqrt{\left|\frac{h''}{h}\right|}, \sqrt{\left|\frac{b''}{b}\right|}, \left|\frac{h'''}{h}\right|^{1/3}, \dots\right\}, \tag{72}$$

Stress-energy tensor for quantized fields in a static spacetime when the fields is in the zero temperature vacuum state defined with respect to the timelike Killing vector which always exists in a static spacetime

$$\left\langle T_A^B\right\rangle = \sum_{k=1}^{N}\left\langle \overset{(k)}{T_A^B}\right\rangle = \frac{K_A^B}{{L_0}^{4+n}}\left(1 + \frac{P_A^B {L_0}^2 M}{r^3(1-2M/r)} + O\left({L_0}^4/l(r)^4\right)\right), \tag{73}$$

where N is the number of mater fields,

$$K_A^B = \text{diag}\left(K_t^t, K_t^t, K_t^t, K_t^t, K_5^5, \dots, K_5^5\right),$$
$$P_A^B = \text{diag}\left(P_t^t, P_r^r, P_\theta^\theta, P_\theta^\theta, P_5^5, \dots, P_5^5\right). \tag{74}$$

$$ds^2 = h(r)dt^2 - b(r)dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) - L(r)^2 d\Omega_2^2. \tag{75}$$

$$h(r) = 1 - \frac{2M}{r} + \delta h(r), \quad b(r) = \frac{1}{1 - \dfrac{2M}{r}} + \delta b(r), \quad L(r) = L_0 + \delta L(r) \qquad \Rightarrow$$

The only non-trivial equation

$$\nabla_B \left\langle T_r^B \right\rangle = \frac{1}{(K_t^t - K_5^5)}\left(2{m_6}^3 (K_t^t - K_5^5)^{1/4}\frac{d(\delta L)}{dr} - \frac{M(P_r^r + 2P_\theta^\theta)}{r^4(1-2M/r)^2}\right) = 0$$

of the system $\nabla_B \left\langle T_A^B \right\rangle = 0$ gives for $r \gg 2M$

$$\delta L(r) \simeq -\frac{(P_r^r + 2P_\theta^\theta)}{12(K_t^t - K_5^5)^{1/4}}\frac{2M}{{m_6}^3 r^3}\,. \tag{76}$$

$$\begin{aligned} {m_4}^2 = \frac{1}{G} = {m_6}^4 \int d^2y\sqrt{|g^{(2)}|} = 4\pi\Big(L_0 + \delta L(r)\Big)^2 {m_6}^4 \\ \simeq 4\pi {m_6}^2\sqrt{K_t^t - K_5^5} \; -\frac{\mathbf{2\pi}}{\mathbf{3}}\left(\mathbf{P_r^r} + \mathbf{2P_\theta^\theta}\right)\frac{\mathbf{2M}}{\mathbf{r^3}}. \end{aligned} \tag{77}$$

$c = \hbar = 1$

- Учет поляризации вакуума квантованных полей принципиально меняет статические сферически симметричные в четырехмерной части пространства решения $f(R)$ теории гравитации с дополнительной n-мерной сферой.
- Многомерная ($D = 4 + n$) полуклассическая $f(R)$ теория гравитации допускает эйнштейновский (и, соответственно, ньютоновский) предел.

Памяти Валерия Рубакова