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# Ограничения на теории гравитации с помощью астрономических данных

С.О.Алексеев, О.И.Зенин

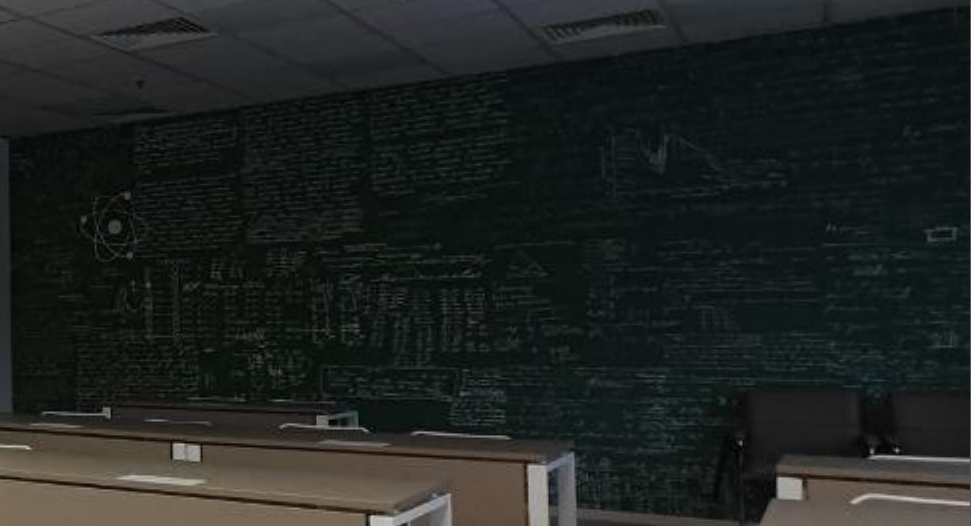
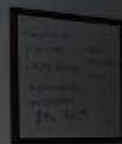
*ГАИШ МГУ & Физфак МГУ*

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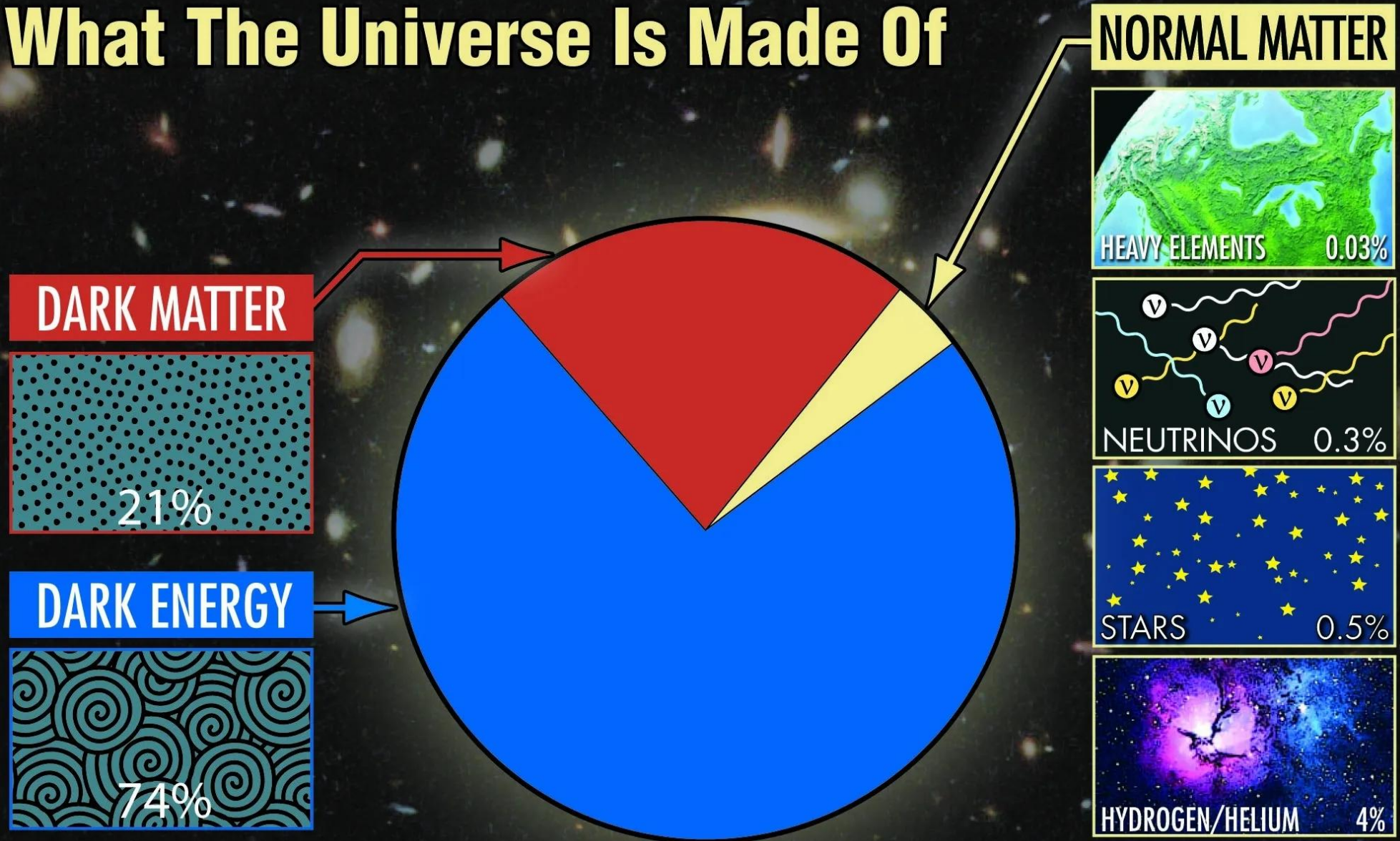


**Б 213**

**УЧЕБНАЯ  
АУДИТОРИЯ**



# What The Universe Is Made Of



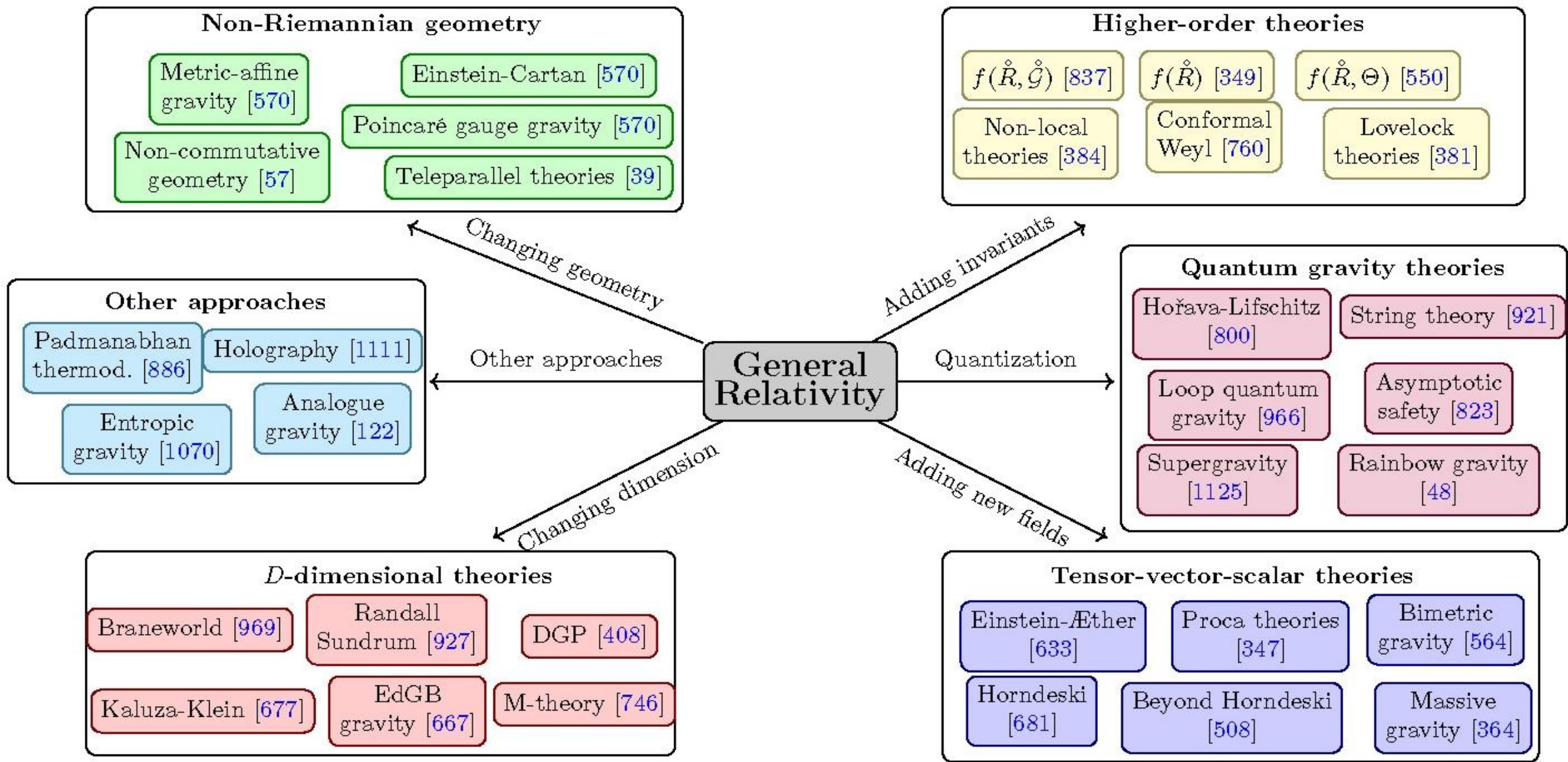


Figure 8: Representation of some possible ways of modifying GR through breaking the Lovelock's theorem along with some examples.

**Extending PPN to different energy ranges**



**A system of tests to constrain an extended gravity theory on different energy scales with astronomical data**

Extended Gravity Constraints at Different Scales

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† This paper is an extended version of our paper published in An extended version of a conference paper The paper represents an extended version of the lecture presented by SA at XXII International Meeting "Physical Interpretations of Relativity Theory-2021" (5–9 July 2021), held at Bauman Moscow State Technical University.

Simple Summary: Simple summary We review a set of the possible ways to constrain extended gravity models at Galaxy clusters scales (the regime of dark energy explanations and comparison with ΛCDM), for black hole shadows, gravitational wave astronomy, binary pulsars, the Solar system and a Large Hadron Collider (consequences for high-energy physics at TeV scale).

Abstract: We review a set of the possible ways to constrain extended gravity models at Galaxy clusters scales (the regime of dark energy explanations and comparison with ΛCDM), for black hole shadows, gravitational wave astronomy, binary pulsars, the Solar system and a Large Hadron Collider (consequences for high-energy physics at TeV scale). The key idea is that modern experimental and observational precise data provide us with the chance to go beyond general relativity.

Keywords: general relativity; extended gravity; black hole; turnaround radius; shadow of black hole; gravitational waves; binary pulsars

PACS: 04.50.+h; 04.50.Gh; 04.80.Cc

1. Introduction

The theory of General Relativity (GR) is confirmed in all projects of experimental astronomy. However, the problems of dark energy, dark matter, the evolution of the early Universe, and the quantum theory of gravity remain open. For example, the theoretical description of the Universe’s accelerated expansion (i.e., dark energy) is realised by adding the cosmological constant to the GR action L as

L\_GRA = sqrt(-g)(R + Lambda), (1)

where R is Ricci scalar and Lambda is the cosmological constant. The problem is that Lambda-term is the best fit for the observational data. On the other hand, from the fundamental point of view, it appears to be a pure fine-tuning parameter. The next step is to consider an additional scalar field phi in the form of Brans–Dicke model

L\_BD = sqrt(-g)(phi R + omega/phi^2 \* d\_mu phi d^mu phi + V(phi)). (2)

Such a model can reproduce the cosmological constant with the help of taking the appropriate form of V(phi). Now, one has to find the origin of the scalar field in Equation (2). The same problem occurs with the inflation stage: accelerated expansion of

Horndeski instabilities (Horndeski 1974). It represents a covariant generalization of Coulton gravity. Horndeski gravity suggests solutions for some GR’s problems. For example, the scalar field can play the role of DE and explain the accelerating expansion of the Universe (de Felice & Trnkef 2012). Therefore, during last few years in connection with all these circumstances, the Horndeski gravity attracts a large number of researchers. This theory has recently been studied extensively in the context of cosmology (Gemati & Martin-Moruno 2017; Kennedy, Louhe & Taylor 2017; Nunez, Marty-Morero & Lebe 2017) and physics of black holes (Trnkef 2017; Trnkef & Lebe 2019). Taking into account the generality and importance of Horndeski model, it is natural to ask the question: how can we perform different observational tests and impose restrictions on its parameters. The Horndeski gravity has already been tested in many experiments (Jordan-Lensing (Nakano et al. 2013), the cosmic microwave background (CMB) data (Sabauelli, Piazza & Martinoli 2016; Reik, Zumalacaran & Moutard 2016), and so on). Special attention should be paid to the recent works of Fagnano & Zampieri (2017) and Baber et al. (2017) related to the verification of the Horndeski theory using LIGO data for event GW17017 (Abbott et al. 2017) and the constant gamma-ray burst GRB 170817A (Abbott et al. 2017b). In these papers, authors investigate the speed of gravitational waves in various theories and show that the data of the binary neutron star merger GW17017 (Abbott et al. 2017a) and the cosmological gamma-ray burst GRB 170817A (Abbott et al. 2017b) allow us to restrict the parameters of the Horndeski gravity. The most general form of Horndeski gravity predicts the existence of a 100 Hz band that is strongly constrained by results from Solar system scales. If a theory involves a scalar field for description of DE, it should contain a mechanism for suppressing of the scalar interaction with matter on small scales, that is, reduce only

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ТЕНИ ЧЕРНЫХ ДЫР КАК ИСТОЧНИК ОГРАНИЧЕНИЙ НА РАСШИРЕННЫЕ ТЕОРИИ ГРАВИТАЦИИ 2: SGR A\*

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1. ВВЕДЕНИЕ
Черная дыра (ЧД) в центре нашей Галактики Sgr A\* была тем в течение ряда лет крупнейшей черной дырой в центре галактики MS17. Таким образом, наше понимание излучаемой тени от черной дыры (ЧД) расширяется: ранее стало возможным сравнить из-друг с другим, а также использовать эти, на более тонкой проверке предсказаний различных расширенных теорий гравитации. Отметим, что оценка массы Sgr A\* от Event Horizon Telescope (EHT) совпадает с оценкой, полученной по результатам наблюдений за транзитом звезды, произошедшего перед Sgr A\*, что дает возможность более точно проверить применимость различных расширенных теорий гравитации, включая дополнительные ограничения на них. Напомним, что ограничения на радиус тени ЧД при наблюдениях Sgr A\* составляют (4.3M < D < 5.3M) [2].

2. ОБЩИНАЯ МЕТРИКА МОДЕЛИ ВАМБЛИБИ
Напомним, что в модели Вамблби размер тени не зависит от космологического масштаба B(r) > 0 и не зависит от скорости движения наблюдателя [1]. Метрика Вамблби обобщает стандартную метрику только компонентой B(r), поэтому рассмотрим предельный вариант альтернативного обобщения [1]:

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Horndeski gravity without screening in binary pulsars
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Abstract—We test the subclasses of Horndeski gravity without Vainshtein mechanism in the strong field regime of binary pulsars. We find the rate of energy losses via the gravitational radiation produced by such theories and compare our results from quasi-circular binaries PSR J1738+0333, PSR J0737–3039, and PSR J1012+5307. In addition, we consider few specific cases: the hybrid metric–Palatini f(R)-gravity and massive Brans–Dicke theory.
Key words: gravitation—gravitational waves—methods: analytical—pulsars: general.
1. INTRODUCTION
The general relativity (GR) is the universally recognized theory of gravity. It successfully describes a wide range of scales and gravitational regimes (weak field limit) in Solar system and strong field regime of binary black holes). Together with Standard model, they represent two pillars of modern physics. Unfortunately, some phenomena cannot be explained completely in the frameworks of these two approaches. The accelerated expansion of our Universe has been found from the supernovae type Ia (SN Ia) observations (Riess et al. 1999, 2004; Perlmutter et al. 1999; Spergel et al. 2007). So an extra component called “dark energy” (DE) has been introduced by Turner (1999), but the nature of this phenomenon is not fully understood. The other problem is dark matter (DM) (Oort 1932; Zwicky 1933). It is the invisible matter, which fills up galaxies and maintains their equilibrium via the gravitational interaction. Also, this phenomenon can be described (part from “new physics”) by changing the gravitational theory to galaxy scales (Capozziello et al. 2013; Bouda Jorjovic et al. 2016; Katsuragawa & Matsumoto 2017; Shi, Li & Han 2017). Furthermore, there is no any complete self-consistent quantum theory of gravity. All these facts lead to an increasing number of modified gravitational theories. One of the most widespread approaches to create the modified gravity is to extend GR with higher order curvature corrections and additional degrees of freedom (Alexeyev & Ponomarev 1997; Alexeyev & Ruzin 2012). But the simplest way to modify GR remains adding of a scalar field. The Horndeski gravity is the most general tensor-torsion theory providing the second-order field equations which evades DE.

trochodski instabilities (Horndeski 1974). It represents a covariant generalization of Coulton gravity. Horndeski gravity suggests solutions for some GR’s problems. For example, the scalar field can play the role of DE and explain the accelerating expansion of the Universe (de Felice & Trnkef 2012). Therefore, during last few years in connection with all these circumstances, the Horndeski gravity attracts a large number of researchers. This theory has recently been studied extensively in the context of cosmology (Gemati & Martin-Moruno 2017; Kennedy, Louhe & Taylor 2017; Nunez, Marty-Morero & Lebe 2017) and physics of black holes (Trnkef 2017; Trnkef & Lebe 2019). Taking into account the generality and importance of Horndeski model, it is natural to ask the question: how can we perform different observational tests and impose restrictions on its parameters. The Horndeski gravity has already been tested in many experiments (Jordan-Lensing (Nakano et al. 2013), the cosmic microwave background (CMB) data (Sabauelli, Piazza & Martinoli 2016; Reik, Zumalacaran & Moutard 2016), and so on). Special attention should be paid to the recent works of Fagnano & Zampieri (2017) and Baber et al. (2017) related to the verification of the Horndeski theory using LIGO data for event GW17017 (Abbott et al. 2017) and the constant gamma-ray burst GRB 170817A (Abbott et al. 2017b). In these papers, authors investigate the speed of gravitational waves in various theories and show that the data of the binary neutron star merger GW17017 (Abbott et al. 2017a) and the cosmological gamma-ray burst GRB 170817A (Abbott et al. 2017b) allow us to restrict the parameters of the Horndeski gravity. The most general form of Horndeski gravity predicts the existence of a 100 Hz band that is strongly constrained by results from Solar system scales. If a theory involves a scalar field for description of DE, it should contain a mechanism for suppressing of the scalar interaction with matter on small scales, that is, reduce only

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НЕЛОКАЛЬНЫЕ ГРАВИТАЦИОННЫЕ ТЕОРИИ И ИЗОБРАЖЕНИЯ ТЕНЕЙ ЧЕРНЫХ ДЫР

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<sup>1</sup> Государственный астрономический институт им. П. К. Штернберга, Московский государственный университет им. М. В. Ломоносова, 119884, Москва, Россия

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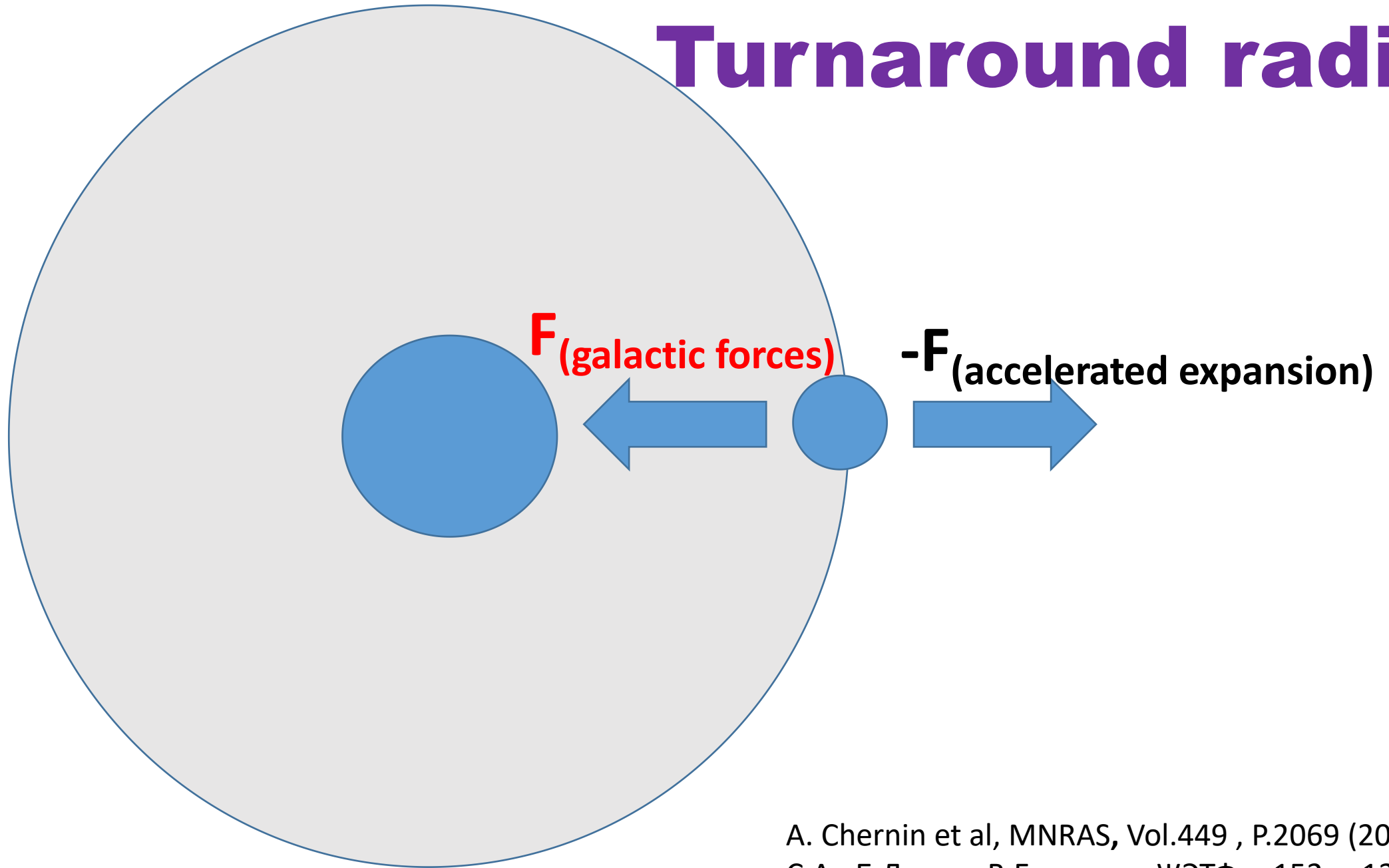
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- **Galaxy clusters scales: ways to explain dark energy & comparing with  $\Lambda$ CDM.**
- **Shadows of black holes: deviations from GR.**
- **Gravitational wave astronomy: deviations from GR.**
- **Binary pulsars: deviations from GR.**
- **Solar system: Newtonian limit and deviations from it.**
- **Large Hadron Collider: gravity at TeV scale.**



# Turnaround radius



A. Chernin et al, MNRAS, Vol.449 , P.2069 (2015),  
С.А., Б.Латош, В.Ечеистов, ЖЭТФ, т.152, с.1271 (2017),  
S.A., K.Kovalkov, IJMP A, v.35, p.204057 (2020).

## Idea:

to calculate turnaround radius using 2 independent methods

- To calculate gravitational potential  $\phi$ . At turnaround radius  $d\phi/dr=0$ ,  $d^2\phi/dr^2 > 0$ .
- *To use astronomical data on gravitational lensing for the experimental estimation of turnaround radius value.*
- At 1st step to use  $\Lambda$ CDM asymptote for estimation.

**NOTE:** Calculations of  $\phi$  are based on metrics  $\implies$  one can compare different models.

# Horndeski theory (*see O.Zenin's talk for details*)

Constrains from turnaround radius (*О.Зенин, СА, ЖЭТФ, принято к печати (2025)*)

+

Constrains from GW200115 (*E.Babichev, C. Charmousis, M. Hassaine and N. Lecoer, PRD 108, 024019 (2023)*)

+

Constrains from PSR J1915+1606 (*P. Dyadina, N. Avdeev, SA, MNRAS 483, 947 (2019)*)



Additional separate constrains on  $\alpha_4$ ,  $\alpha_5$ ,  $\eta$

# DGP model (*see O.Zenin's talk for details*)

Constrains from turnaround radius (*О.Зенин, СА, ЖЭТФ, принято к печати (2025)*)

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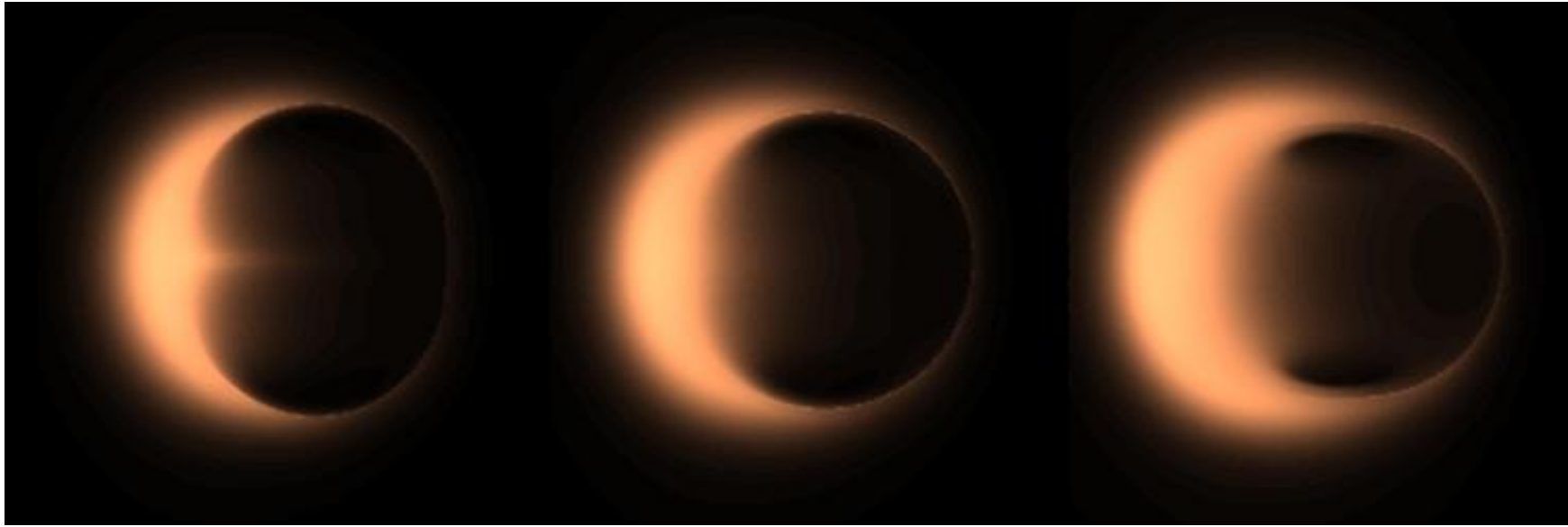
Constrains from solution by itself (*R. Gannouji, EPJ C 78, 318 (2018)*)



Confirmation of DGP (4+)D regime at high scale (near Vainstein radius)

- **Galaxy clusters scales: ways to explain dark energy & comparing with  $\Lambda$ CDM.**
- **Shadows of black holes: deviations from GR.**
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- **Binary pulsars: deviations from GR.**
- **Solar system: Newtonian limit and deviations from it.**
- **Large Hadron Collider: gravity at TeV scale.**

# Constraints on gravity models from black hole shadows



Pic is taken from <https://www.eso.org/public/images/shadow-evt/>

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r}\right)} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \quad \longrightarrow \quad ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)} + r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

A. F. Zakharov, Sov. Phys. JETP, 64, 1 (1986).

A. F. Zakharov, A. A. Nucita, F. De Paolis, G. Ingrosso, New Astron. 10, 479 (2005)

A. F. Zakharov, IJMP D 54, 2340004 (2023)

A. Zakharov, Phys. Rev. D, Vol.90, P.062007 (2014)

V. Prokhorov, SA, O. Zenin, JETP, Vol.135, p.842 (2022) ...

С.А, А. Байдерин, А. Немтинова, О. Зенин, ЖЭТФ 165, 508 (2024)

# Idea:

- The general form of spherically-symmetric metrics:

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2).$$

- Equation of motion:  $\left(\frac{d\hat{r}}{d\tau}\right)^2 + \frac{L^2}{B(\hat{r})\hat{r}^2} = \frac{E^2}{A(\hat{r})B(\hat{r})}, \quad \frac{d\phi}{d\tau} = \frac{L}{\hat{r}^2},$

- Introduce:  $D = L/E$

- To calculate the shadow size one has to find maximal root of

$$u(r) = \left(\frac{d\hat{r}}{d\phi}\right)^2 = \frac{\hat{r}^4}{D^2 A(\hat{r})B(\hat{r})} - \frac{\hat{r}^2}{B(\hat{r})},$$

$$u(r) = 0, \quad \frac{du(r)}{dr} = 0, \quad \frac{d^2u(r)}{d^2r} > 0.$$

## Horndesky Model

$$A(r) = 1 - \frac{2M}{r} - \frac{2C_7}{7r^7}$$

$$B(r)^{-1} = 1 - \frac{2M}{r} - \frac{C_7}{r^7}$$

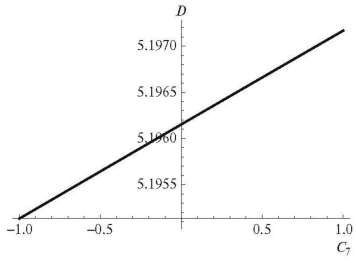


Fig. 3. The dependence of shadow size ( $D$ ) versus the combination of model constants  $C_7$  for Horndesky theory coupled with Gauss-Bonnet invariant (in the units of  $M$ ,  $M=1$ ).

E. Babichev, C. Charmousis, and A. Lehebel. JCAP, 2017. arXiv:1702.01938 V.Prokopov, SA, O.Zenin, JETP, Vol.135, P.91 (2022), *ibid*, p.842 (2022)

## Conformal gravity

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} [R - \alpha(\phi^2 R + 6\partial_\mu \phi \partial^\mu \phi)] - \frac{1}{2m_2^2} C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma}$$

$$A(r) = 1 - \frac{2M}{r} + \frac{Q_s^2}{r^2} + \frac{Q_s^2 \left( -M^2 + Q_s^2 + \frac{6}{m_2^2} \right)}{3r^4} + \dots,$$

$$B(r)^{-1} = 1 - \frac{2M}{r} + \frac{Q_s^2}{r^2} + \frac{2Q_s^2 \left( -M^2 + Q_s^2 + \frac{6}{m_2^2} \right)}{3r^4} + \dots,$$

## Bumblebee model

$$S_B = \int d^4x \sqrt{-g} \mathcal{L}_B = \int d^4x \sqrt{-g} (\mathcal{L}_g + \mathcal{L}_{gB} + \mathcal{L}_K + \mathcal{L}_V + \mathcal{L}_M)$$

$$\mathcal{L}_B = \frac{e}{2\kappa} R + \frac{e}{2\kappa} \xi B^\mu B^\nu R_{\mu\nu} - \frac{1}{4} e B_{\mu\nu} B^{\mu\nu} - e V(B^\mu) + \mathcal{L}_M$$

$$A(r) = \left( 1 - \frac{2M}{r} \right),$$

$$B(r) = \frac{1+l}{1 - \frac{2M}{r}},$$

## f(Q) gravity

$$S[g, \Gamma; \lambda, \rho] = \int_M d^4x \left( \frac{1}{2} \sqrt{-g} f(Q) + \lambda \alpha^{\beta\mu\nu} R_{\beta\mu\nu}^\alpha + \rho \alpha^{\mu\nu} T_{\mu\nu}^\alpha \right) + S_{matter}$$

$$A(r) = 1 - \frac{2M_{ren}}{r} - \alpha \frac{32}{r^2},$$

$$B(r)^{-1} = 1 - \frac{2M_{ren}}{r} - \alpha \frac{96}{r^2},$$

$$2M_{ren} = 2M - \alpha \left( \frac{32}{3M} + c_1 \right),$$

## Scalar Gauss-Bonnet gravity

$$S = \int d^4x \sqrt{-g} [\kappa R + \alpha_1 f_1(\theta) R^2 + \alpha_2 f_2(\theta) R_{ab} R^{ab} + \alpha_3 f_3(\theta) R_{abcd} R^{abcd} + \alpha_4 f_4(\theta) R_{abcd} * R^{abcd}] - \frac{\beta}{2} (\nabla_a \theta \nabla^a \theta + 2V(\theta)) + \mathcal{L}_{mat}$$

$$A = -f(r) \left[ 1 + \frac{\zeta}{3r^3 f(r)} h(r) \right],$$

$$B = \frac{1}{f(r)} \left[ 1 - \frac{\zeta}{r^3 f(r)} k(r) \right],$$

where

$$h(r) = 1 + \frac{26}{r} + \frac{66}{5r^2} + \frac{96}{5r^3} - \frac{80}{r^4},$$

$$k(r) = 1 + \frac{1}{r} + \frac{52}{3r^2} + \frac{2}{r^3} + \frac{16}{5r^4} - \frac{368}{3r^5},$$

$$f(r) = 1 - \frac{2}{r},$$

## Bumblebee model

$$A(r) = \left( 1 - \frac{2M}{r} \right),$$

$$B(r) = \frac{1+l}{1 - \frac{2M}{r}}$$

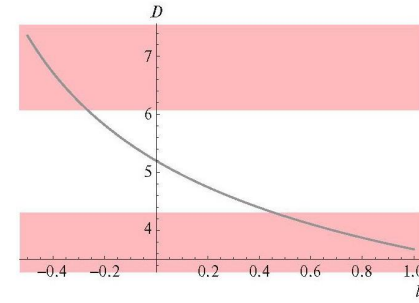


Fig. 6. The dependence of the shadow size  $D$  upon parameter  $l$  in alternative Bumblebee generalization with Schwarzschild approximation (in the units of  $M$ ,  $M=1$ ).

R. Casana, A. Cavalcante, et al., PRD 2018 arXiv:1711.02273

V.Prokopov, SA, O.Zenin, JETP, Vol.135, P.91 (2022), *ibid*, p.842 (2022)

## Loop quantum gravity

$$A(r) = \left( 1 - \frac{2Mr^2}{r^3 + 2Mt^2} \right) \left( 1 - \frac{\alpha\beta M}{\alpha r^3 + \beta M} \right),$$

$$B(r)^{-1} = 1 - \frac{2Mr^2}{r^3 + 2Mt^2},$$

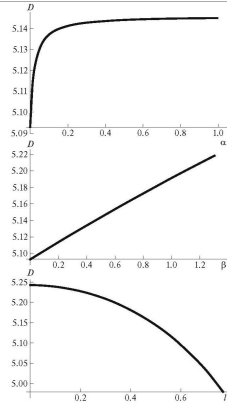


Fig. 4. The dependence of shadow size  $D$  upon the time delay  $\alpha$  when  $l=0.5M$  and  $\beta=0.5$  (top image), upon the 1-loop quantum corrections  $B$  when  $l=0.5M$ ,  $\alpha=0.5$  (central image), upon the central energy density  $l$  when  $\alpha=0.5$ ,  $\beta=0.5$  (bottom image) for BH in modified Hayward metric in the units of  $M$ ,  $M=1$ .

J. Hu, L. Shi, Y. Zhang, and P. Duan. Astrophysics and Space Science, 2018. arXiv:2104.07523 V.Prokopov, SA, O.Zenin, JETP, Vol.135, P.91 (2022), *ibid*, p.842 (2022)

## Conformal gravity

$$A(r) = 1 - \frac{2M}{r} + \frac{Q_s^2}{r^2} + \frac{Q_s^2 \left( -M^2 + Q_s^2 + \frac{6}{m_2^2} \right)}{3r^4} + \dots,$$

$$B(r)^{-1} = 1 - \frac{2M}{r} + \frac{Q_s^2}{r^2} + \frac{2Q_s^2 \left( -M^2 + Q_s^2 + \frac{6}{m_2^2} \right)}{3r^4} + \dots,$$

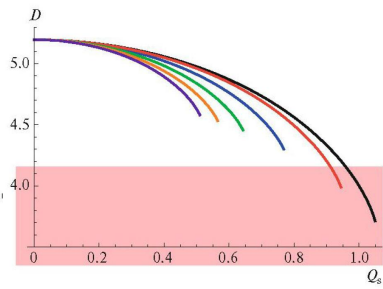


Fig. 5. The dependence of the shadow size  $D$  against the scalar charge  $Q_s$  for in new massive conformal gravity with different values of massive spin-2 mode  $m_2$  (in the units of  $M$ ,  $M=1$ ). Black line corresponds to  $m_2 \rightarrow \infty$ , red one corresponds to  $m_2 = 2$ , blue one corresponds to  $m_2 = 1$ , green one corresponds to  $m_2 = 0.707$ , orange one corresponds to  $m_2 = 0.577$ , purple one corresponds to  $m_2 = 0.5$ .

Y. S. Myung and D. Zou. PRD 2019. arXiv:1907.09676 V.Prokopov, SA, O.Zenin, JETP, Vol.135, P.91 (2022), *ibid*, p.842 (2022)

## f(Q) gravity

$$A(r) = 1 - \frac{2M_{ren}}{r} - \alpha \frac{32}{r^2},$$

$$B(r)^{-1} = 1 - \frac{2M_{ren}}{r} - \alpha \frac{96}{r^2},$$

$$2M_{ren} = 2M - \alpha \left( \frac{32}{3M} + c_1 \right),$$

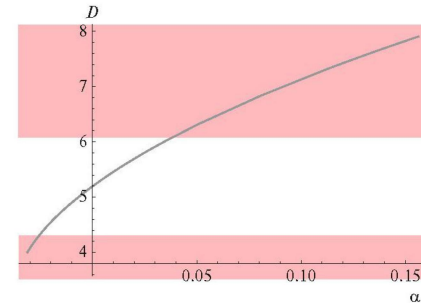


Fig. 7. The dependence of the shadow size  $D$  upon parameter  $\alpha$  in  $f(Q)$  gravity in  $M_{ren}$  units.

F. D'Ambrosio, S. D. B. Fell, L. Heisenberg, and S. Kuhn. PRD, 2022 arXiv:2109.03174

V.Prokopov, SA, O.Zenin, JETP, Vol.135, P.91 (2022), *ibid*, p.842 (2022)

## Scalar Gauss-Bonnet gravity

$$A = -f(r) \left[ 1 + \frac{\zeta}{3r^3 f(r)} h(r) \right],$$

$$B = \frac{1}{f(r)} \left[ 1 - \frac{\zeta}{r^3 f(r)} k(r) \right],$$

where

$$h(r) = 1 + \frac{26}{r} + \frac{66}{5r^2} + \frac{96}{5r^3} - \frac{80}{r^4},$$

$$k(r) = 1 + \frac{1}{r} + \frac{52}{3r^2} + \frac{2}{r^3} + \frac{16}{5r^4} - \frac{368}{3r^5},$$

$$f(r) = 1 - \frac{2}{r},$$

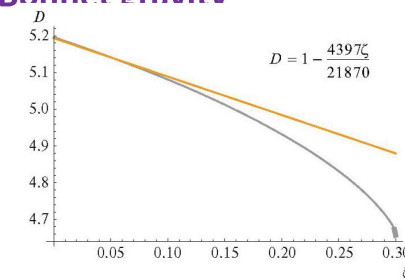


Fig. 8. The lower curve is the dependence of the shadow size  $D$  upon parameter  $\zeta$  in scalar Gauss-Bonnet gravity (in the units of  $M$ ,  $M=1$ ). The top line is the first order approximation.

N. Yunes and L. C. Stein. PRD 2011. arXiv:1101.2921

V.Prokopov, SA, O.Zenin, JETP, Vol.135, P.91 (2022), *ibid*, p.842 (2022)



# Constraints on gravity models from black hole shadows



Pic is taken from <https://www.eso.org/public/images/shadow-evt/>

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 - \frac{dr^2}{\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)} - r^2 d\Omega^2$$



$$ds^2 = \frac{\Delta}{\rho^2} (dt - a \sin^2 \theta d\tilde{\varphi})^2 - \frac{\sin^2 \theta}{\rho^2} ((r^2 + a^2) d\tilde{\varphi} - a dt)^2 - \frac{\rho^2}{\Delta} dr^2 - \rho^2 d\theta^2$$

# Newman-Janis algorithm (NJA) --> Approved NJA



## Note on the Kerr Spinning Particle Metric

E. T. Newman and A. I. Janis

Citation: *J. Math. Phys.* **6**, 915 (1965); doi: 10.1063/1.1704350

View online: <http://dx.doi.org/10.1063/1.1704350>

View Table of Contents: <http://jmp.aip.org/resource/1/JMAPAQ/v6/i6>

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*Eur. Phys. J. C* (2014) 74:2865

DOI 10.1140/epjc/s10052-014-2865-8

**THE EUROPEAN  
PHYSICAL JOURNAL C**

Regular Article - Theoretical Physics

## From static to rotating to conformal static solutions: rotating imperfect fluid wormholes with(out) electric or magnetic field

Mustapha Azreg-Aïnou<sup>a</sup>

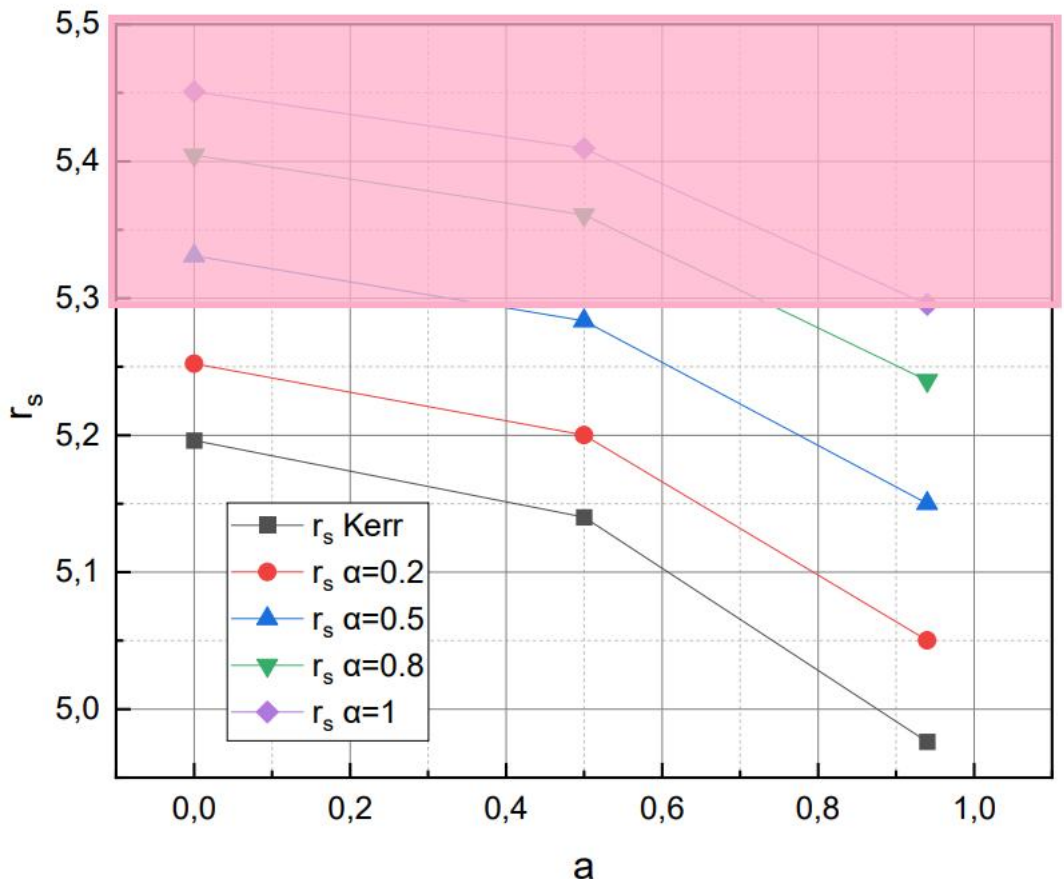
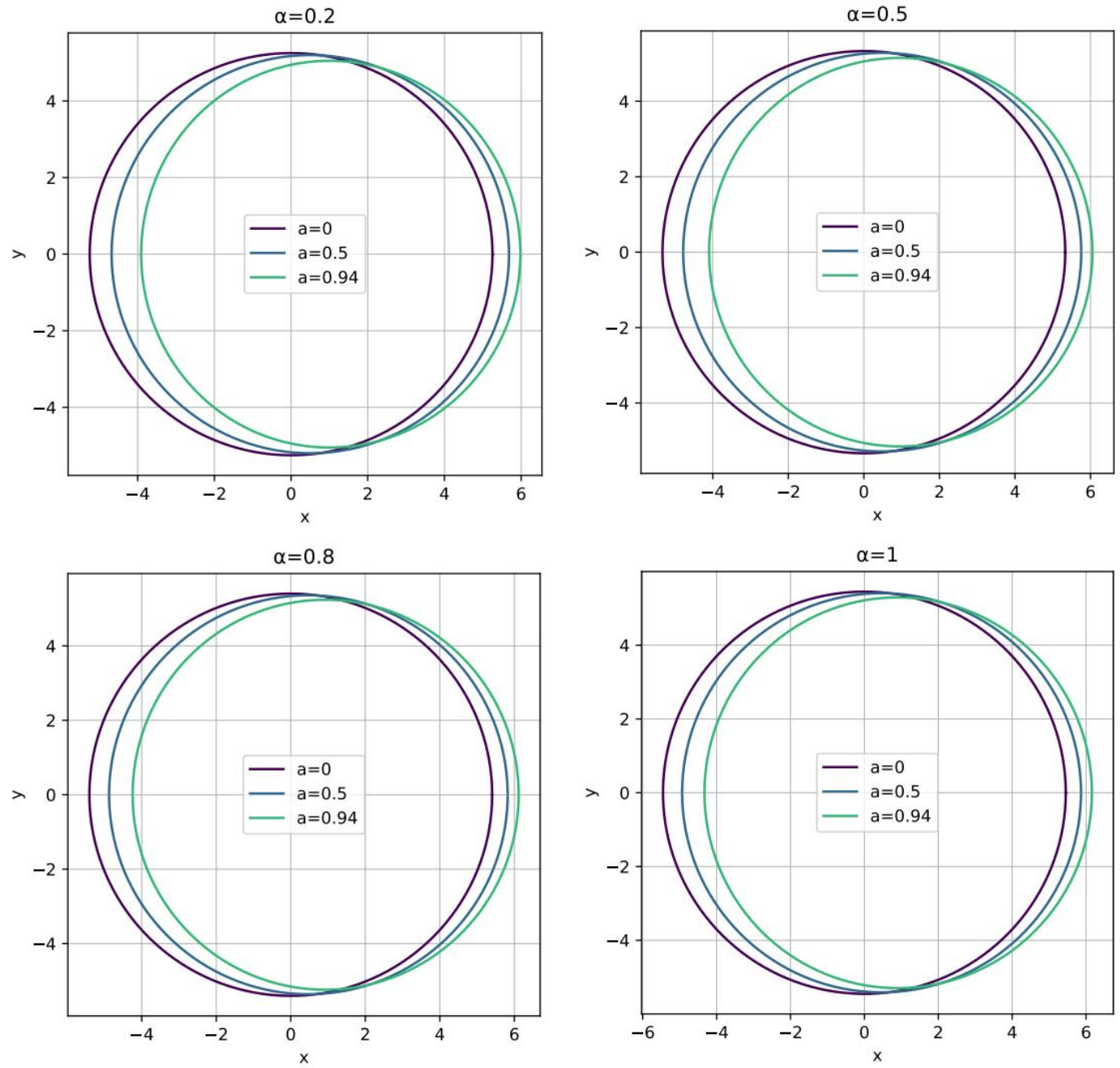
Department of Mathematics, Başkent University, Bağlıca Campus, Ankara, Turkey

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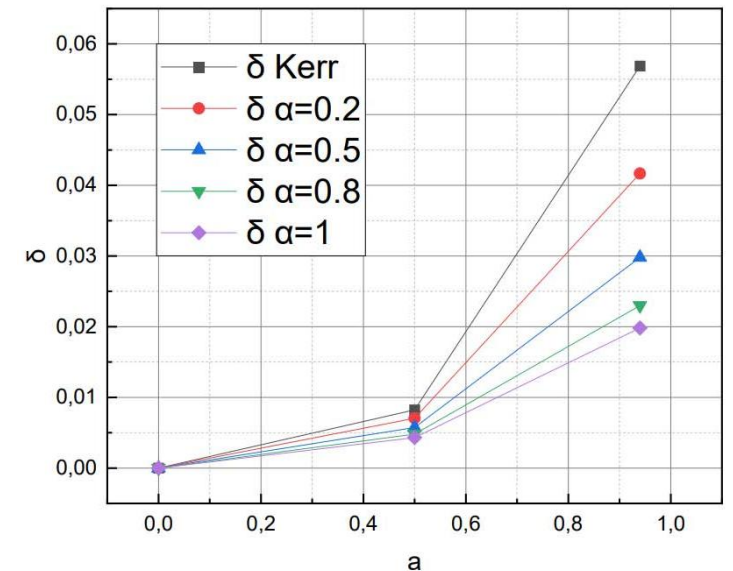
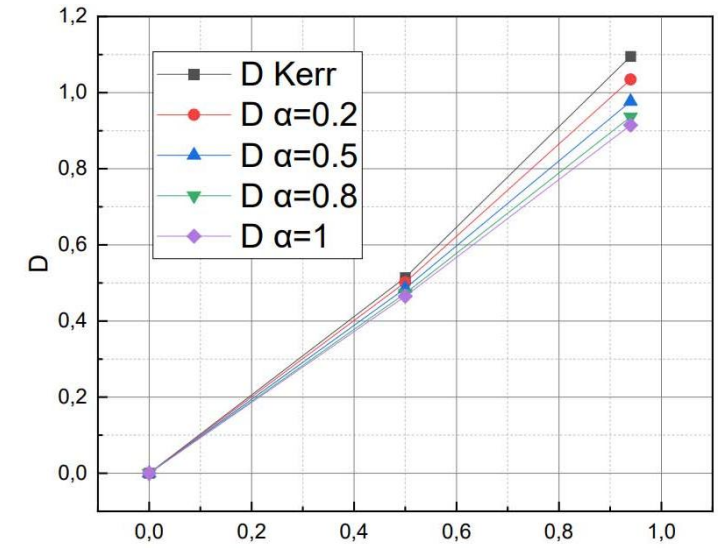
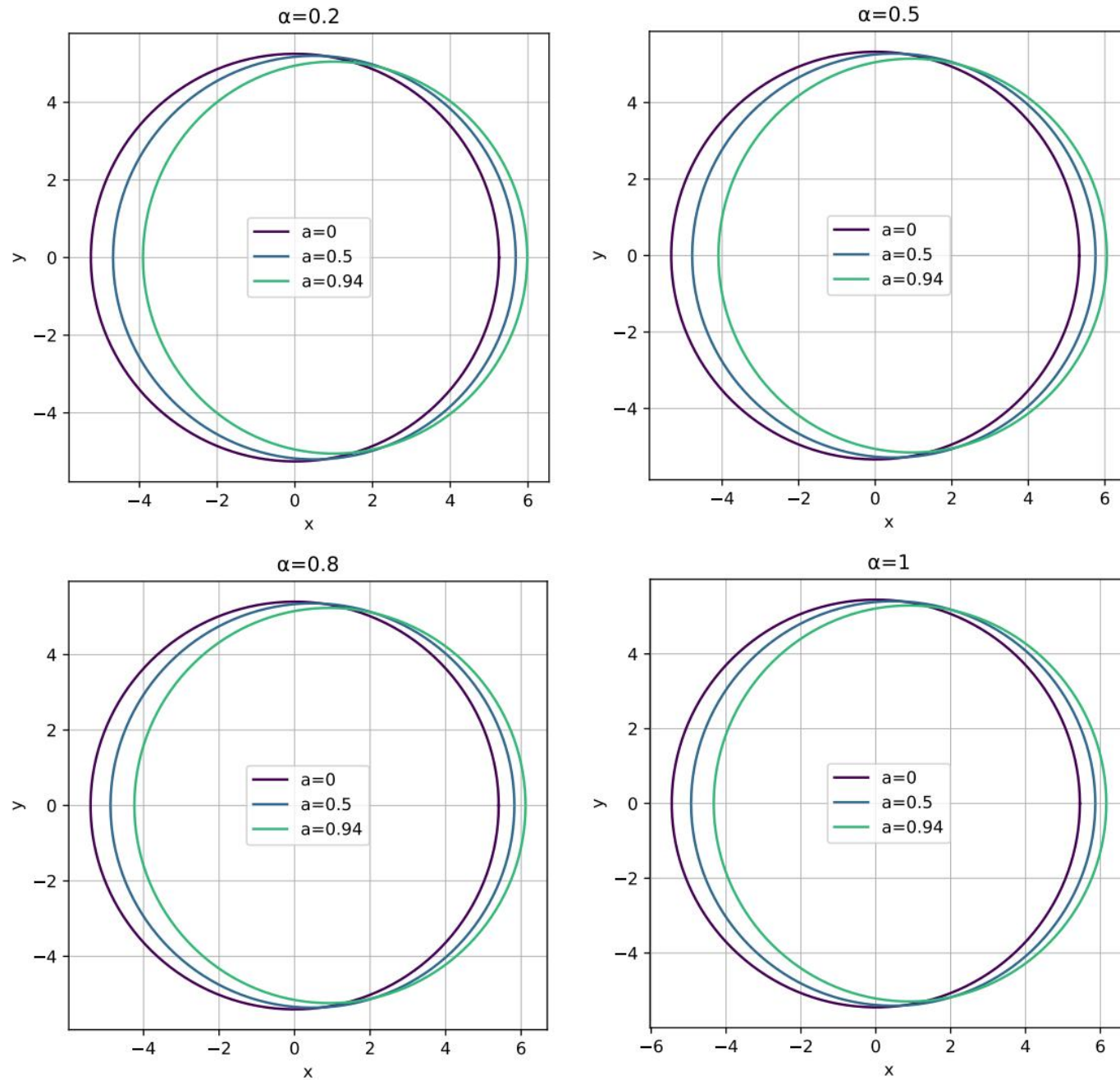
# Horndesky theory

## The dependence of the shadow size $r_s$ against rotation $a$



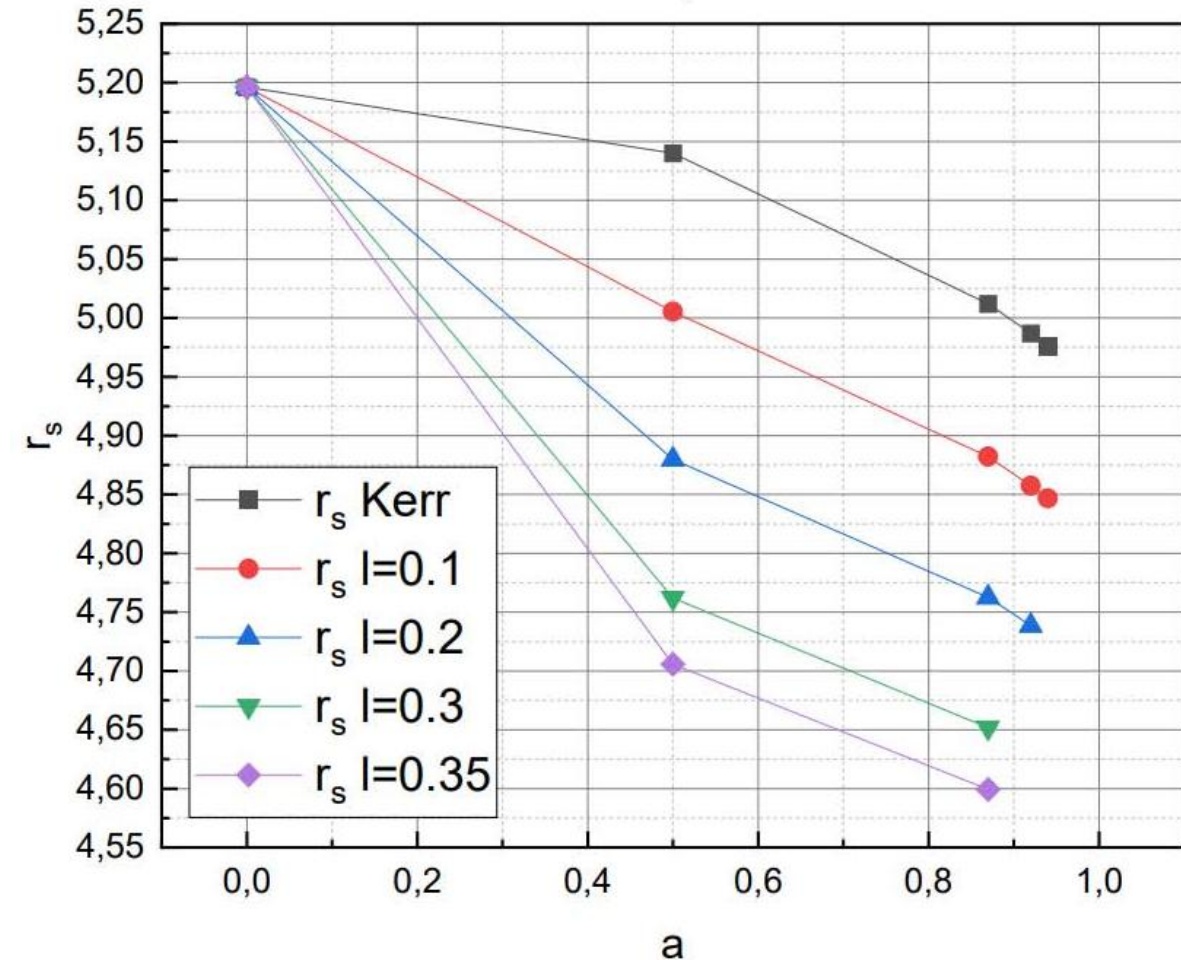
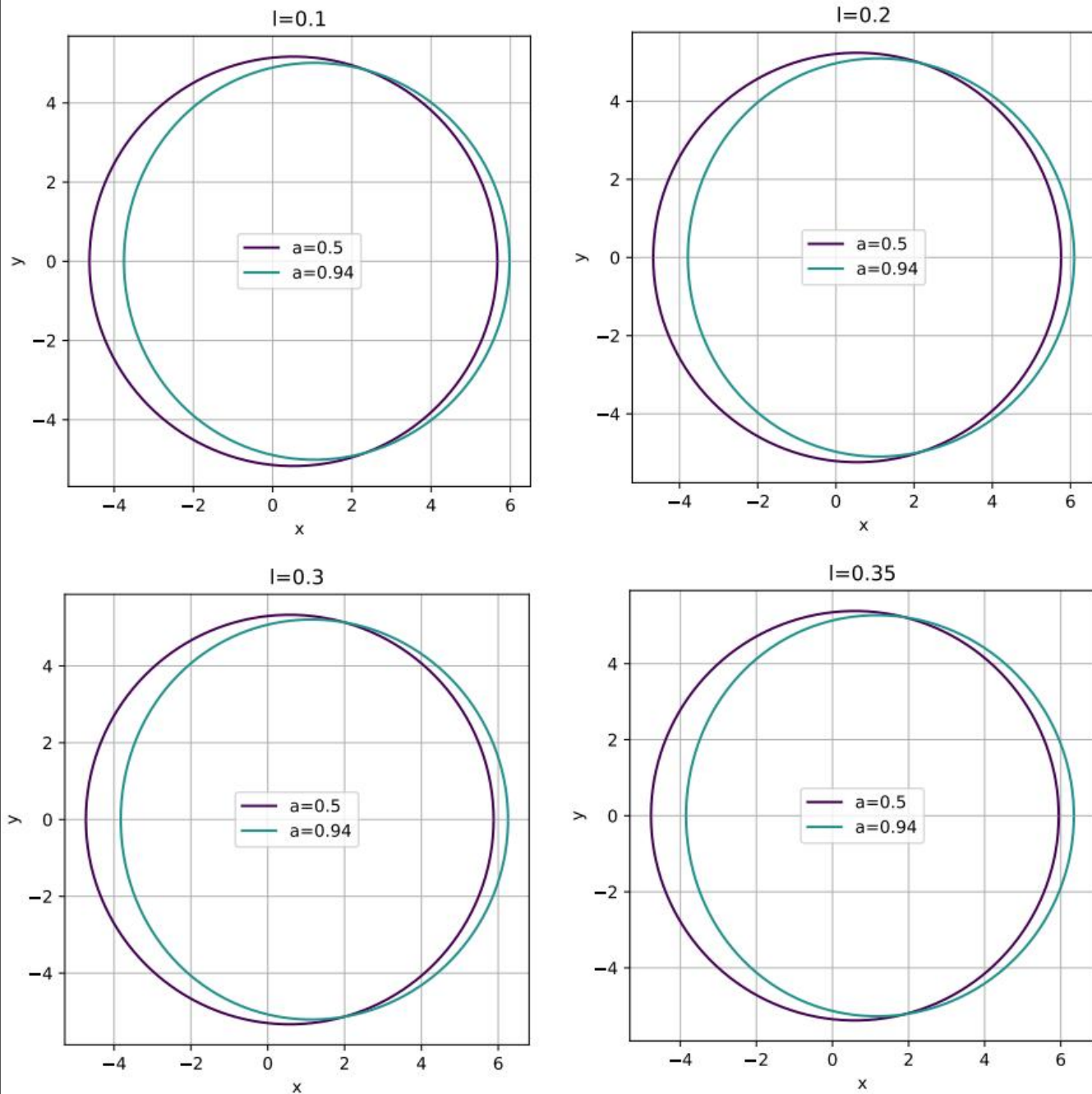
# Horndesky theory

The dependence of shift  $\mathbf{D}$  and distortion  $\delta$  against rotation  $\mathbf{a}$



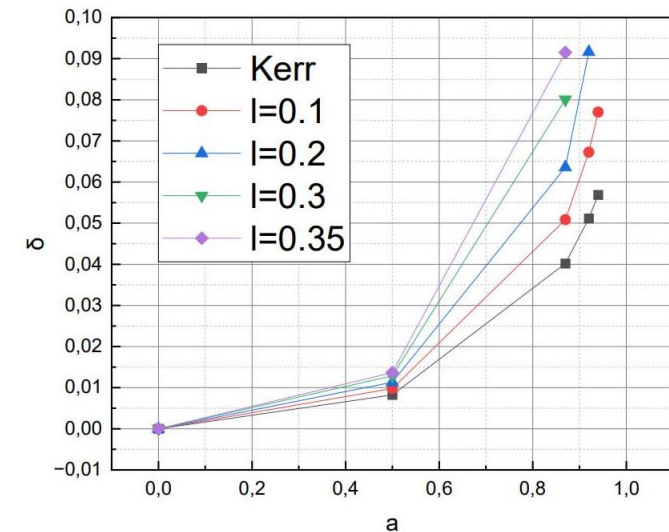
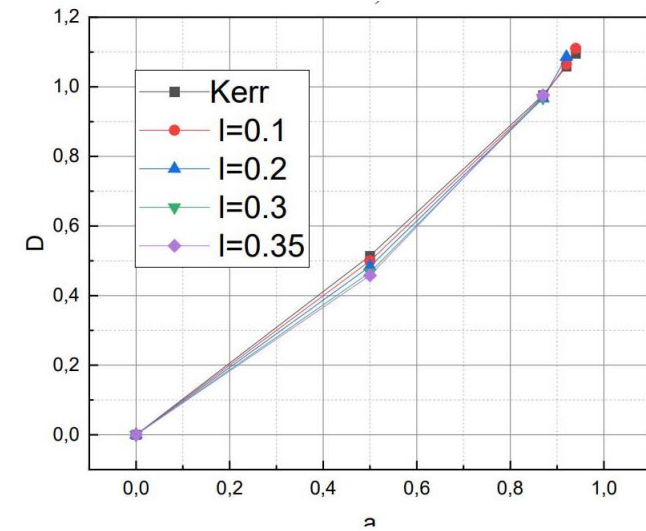
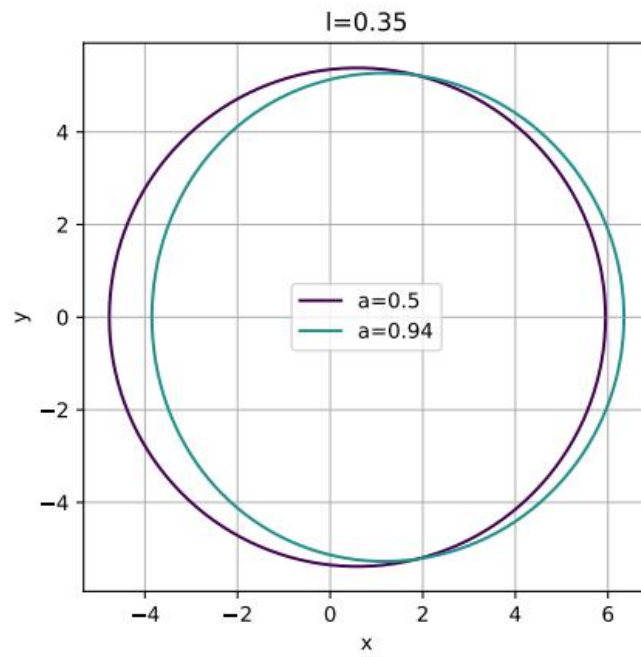
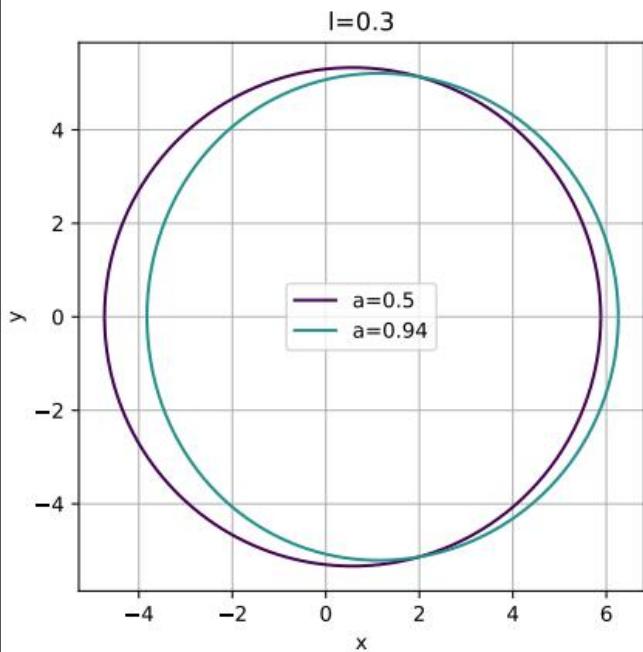
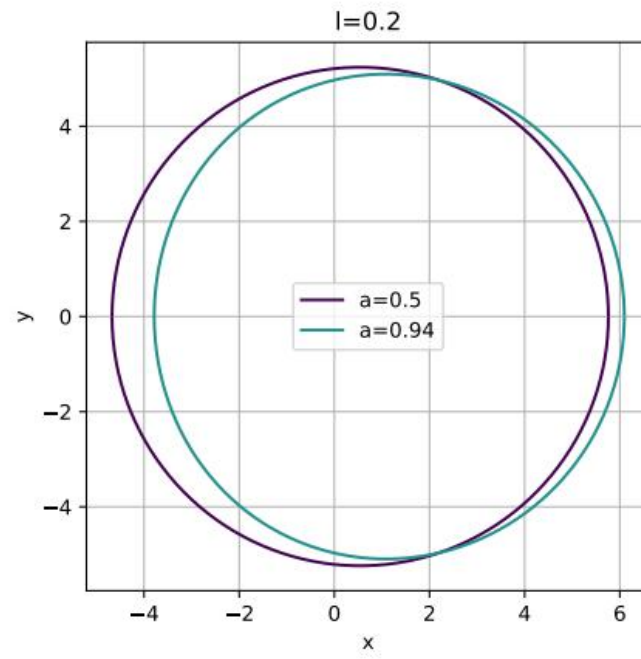
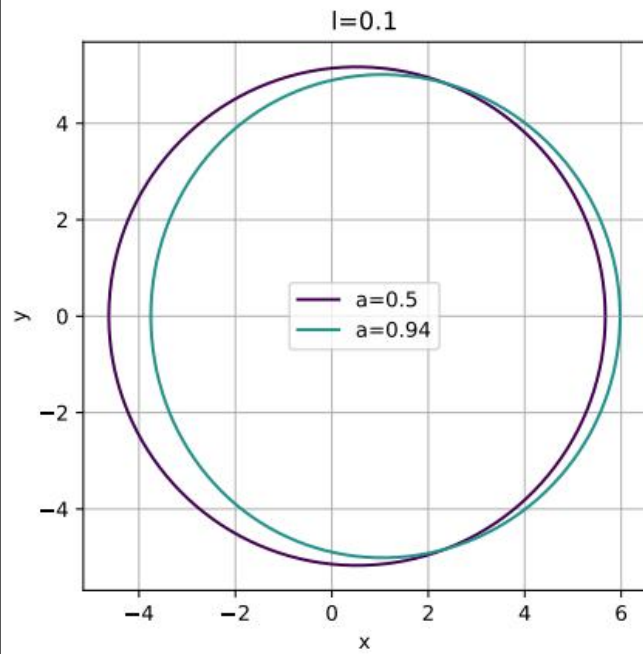
# Bumblebee model

## The dependence of the shadow size $r_s$ against rotation $a$



# Bumblebee model

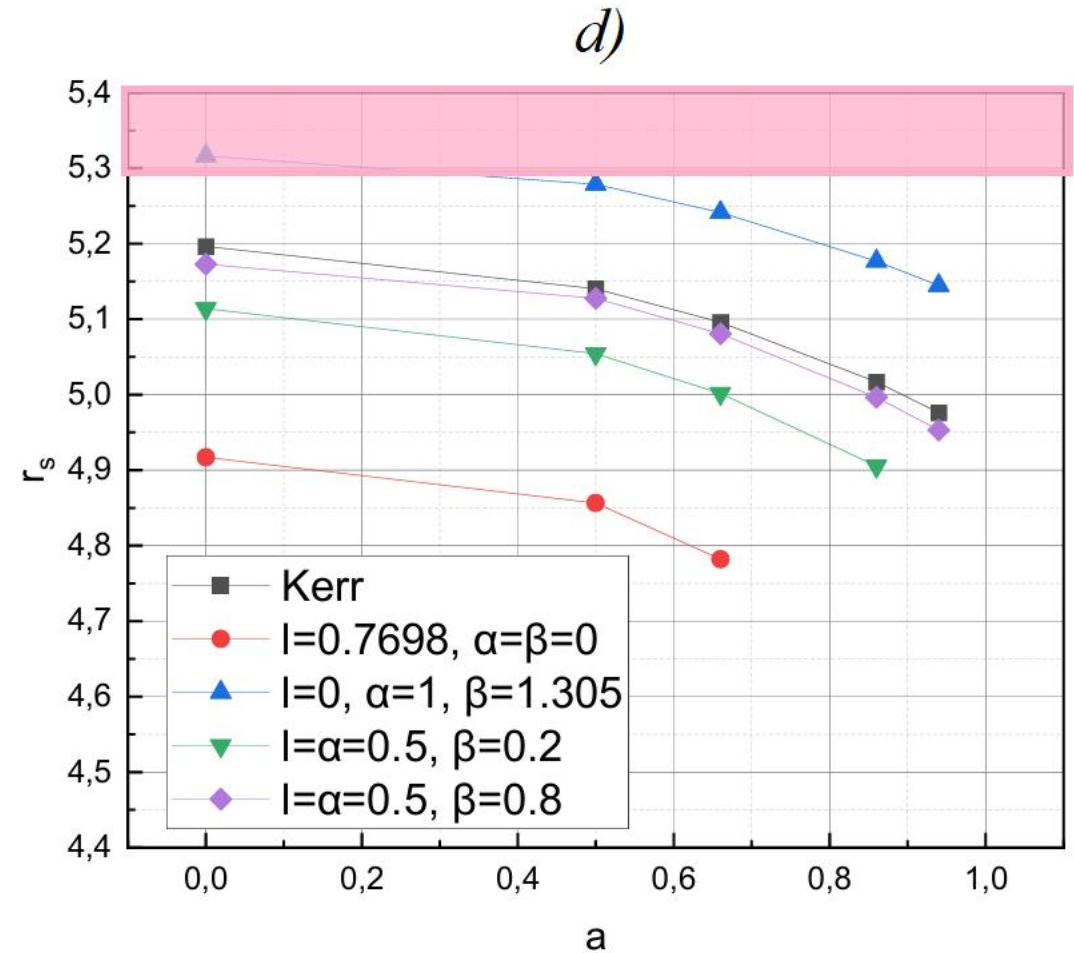
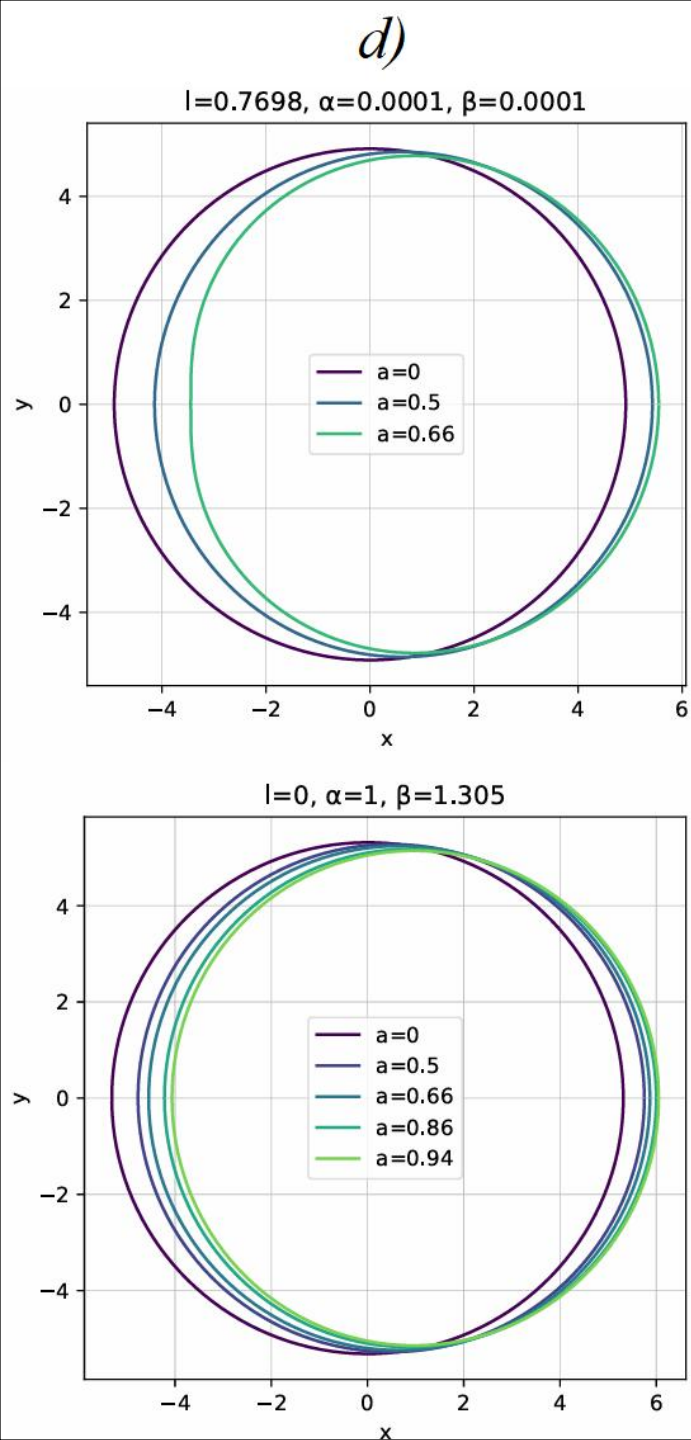
The dependence of shift  $D$  and distortion  $\delta$  against rotation  $a$



# Loop Quantum Gravity

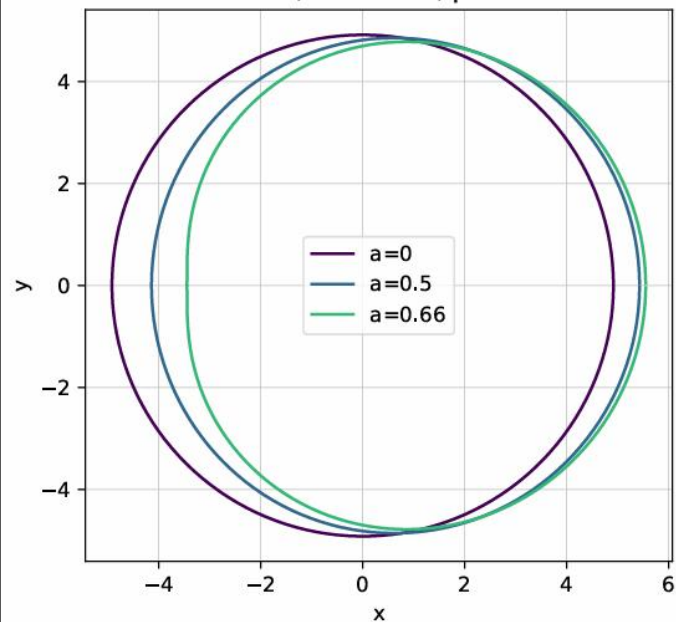
## The dependence of the shadow size

$r_s$  against rotation  $a$

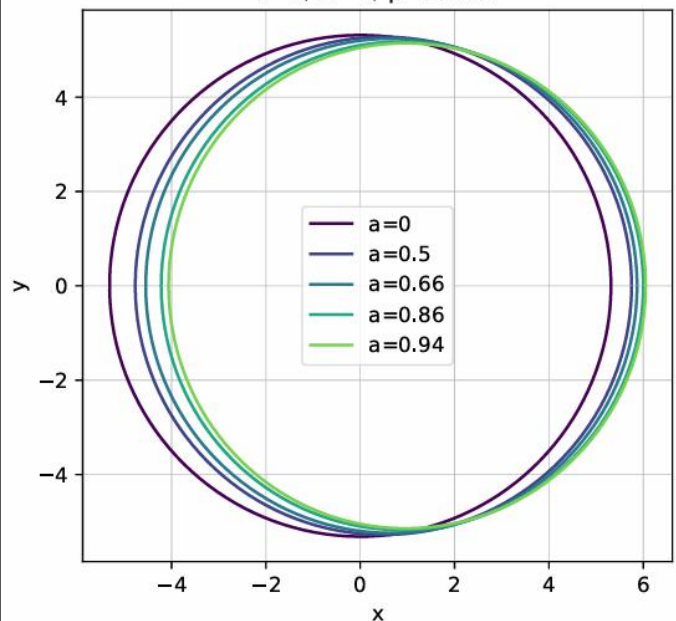


d)

$l=0.7698, \alpha=0.0001, \beta=0.0001$

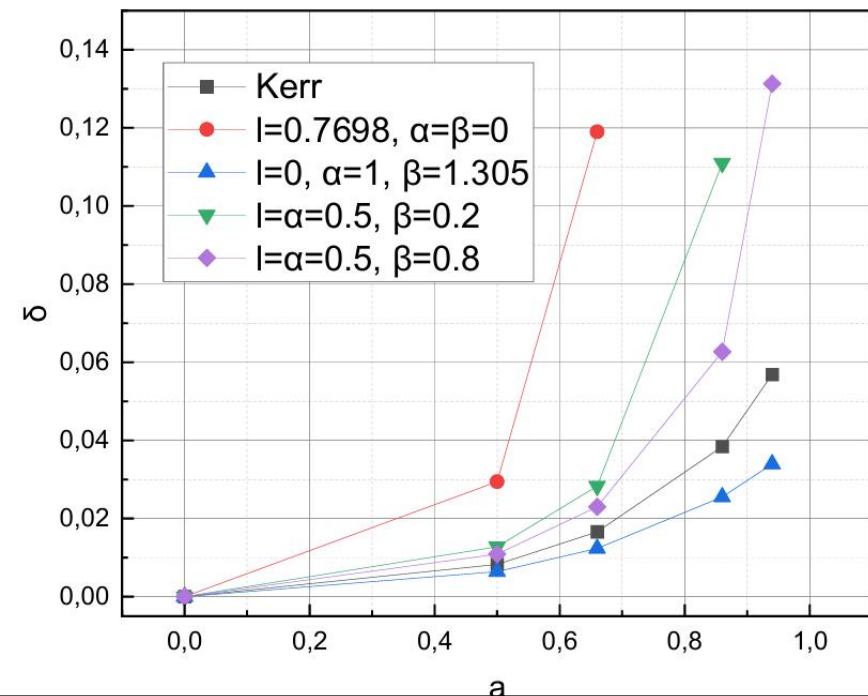
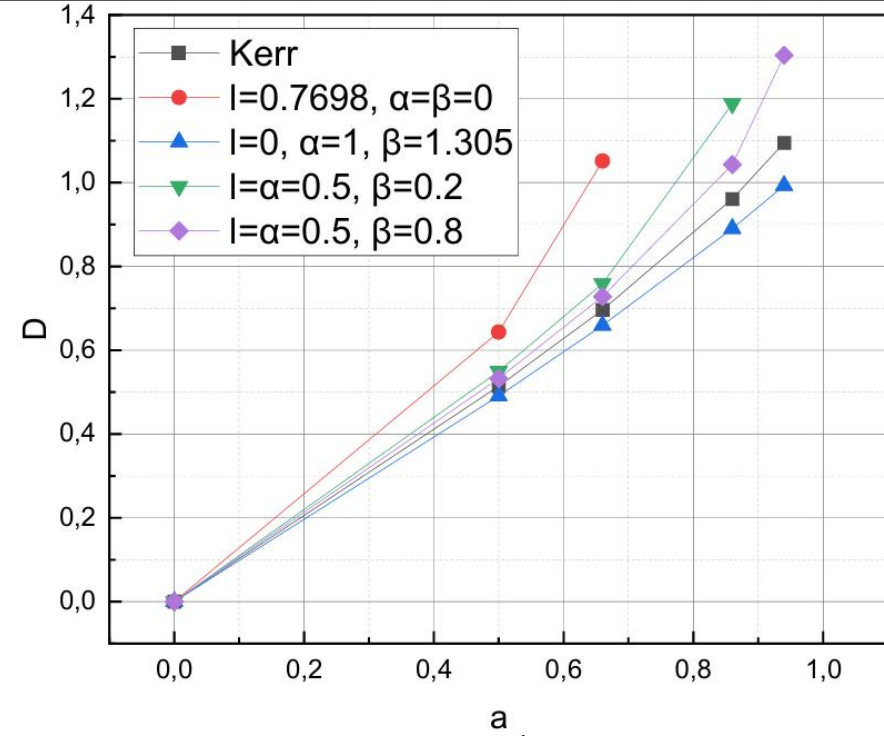


$l=0, \alpha=1, \beta=1.305$



# Loop Quantum Gravity

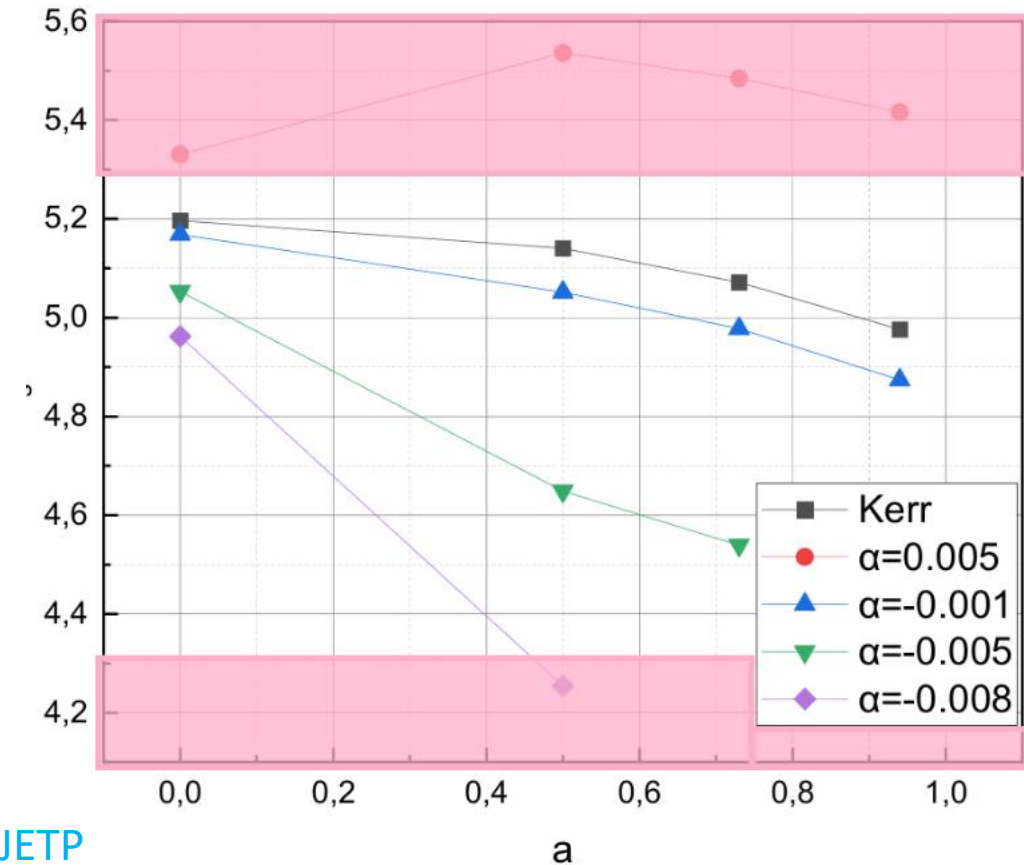
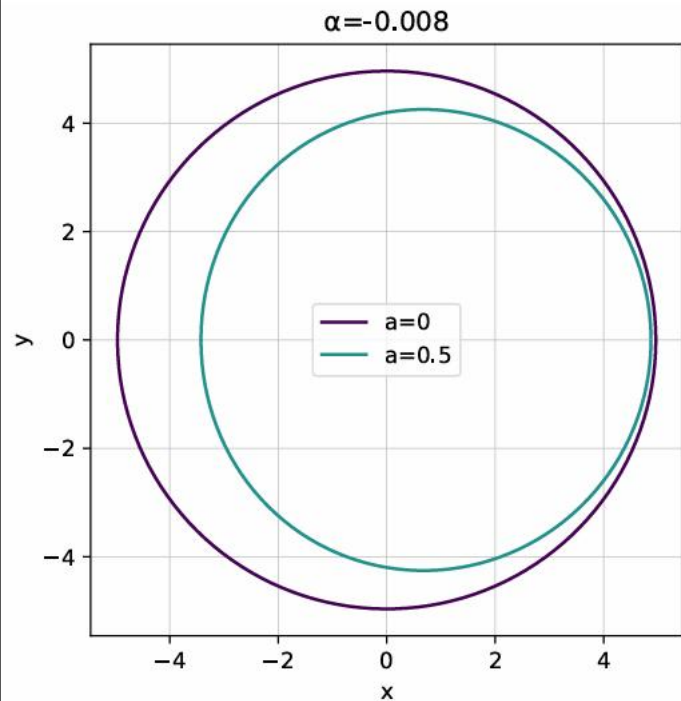
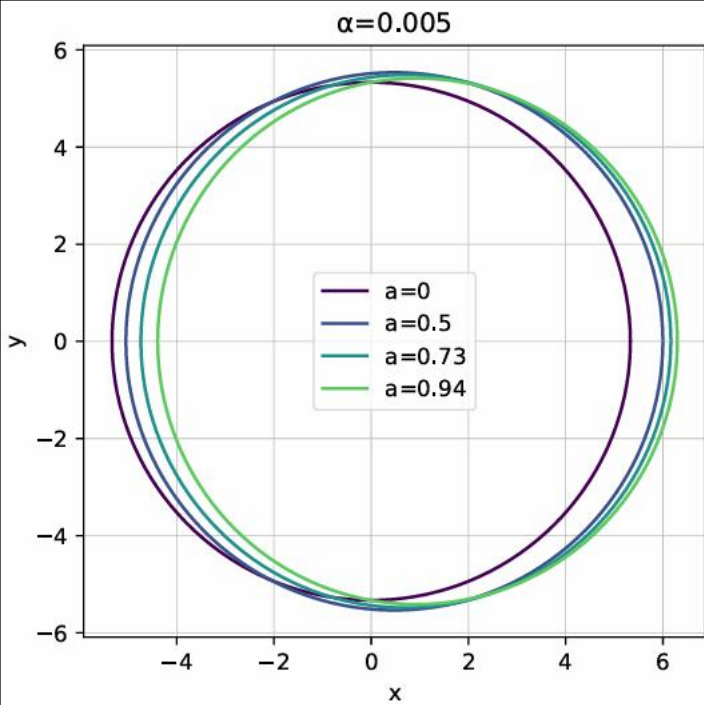
## The dependence of shift $D$ and distortion $\delta$ against rotation $a$

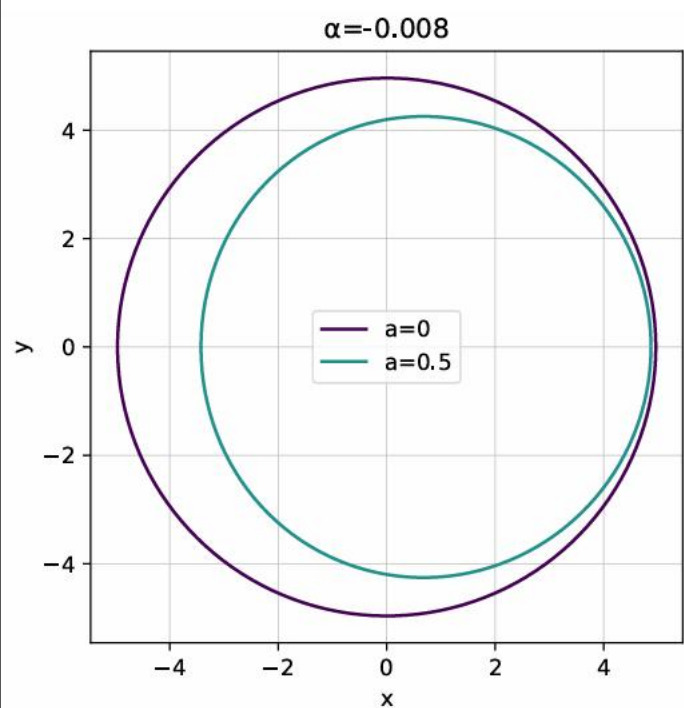
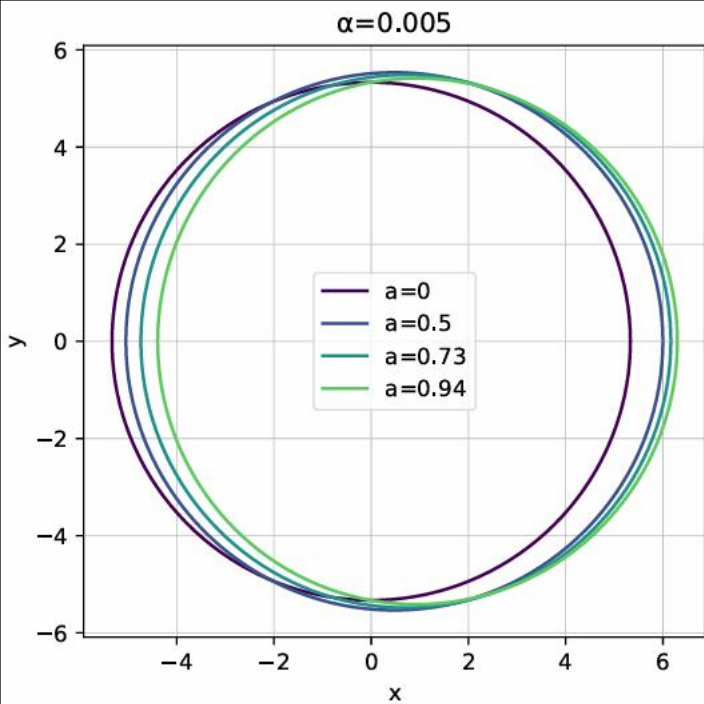




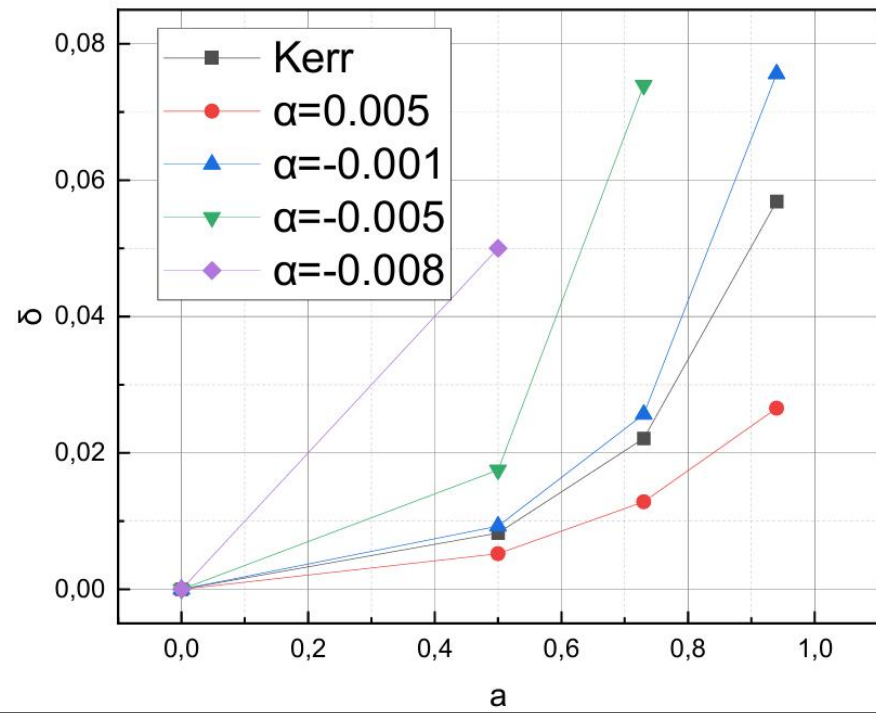
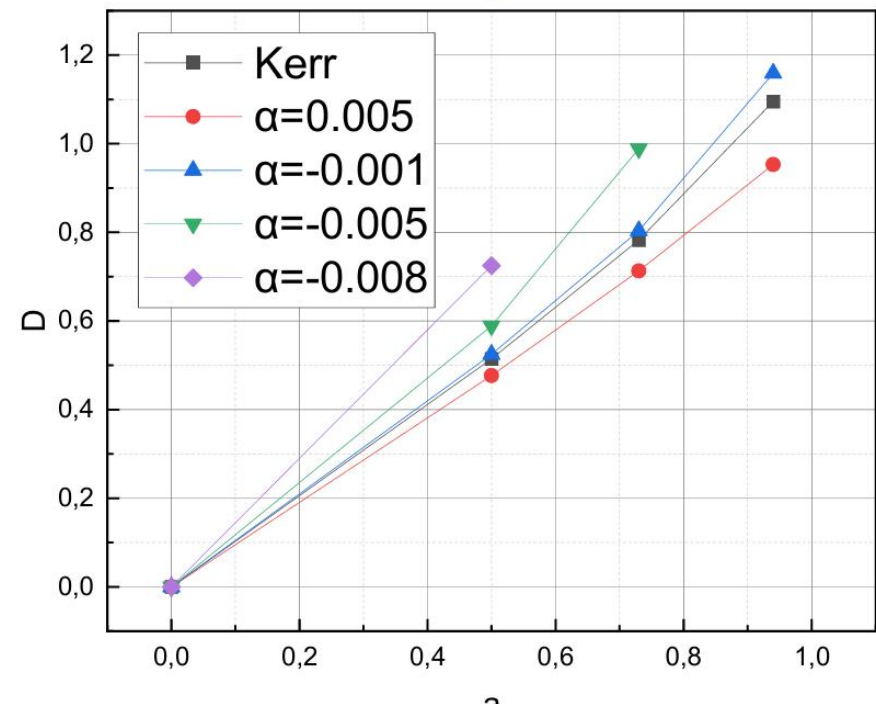
# f(Q) gravity

## The dependence of the shadow size $r_s$ against rotation $a$





**f(Q) gravity**  
 The dependence of  
 shift (up case) and  
 distortion (down  
 case) against  
 rotation **a**



- Generally for considered models some of them (Horndesky model and Gauss-Bonnet scalar gravity) weaken the effect of rotation and bumblebee model enhances it.
- This conclusion matches the previous one at non-local gravity models study: **extended gravity theories by themselves correct the effect of rotation in both directions.** This fact seems to be important as the accuracy of shadow images permanently increases.

# Comment on the Newman-Janis algorithm status

$$r^2 \rightarrow r^2 + a^2 \cos^2\theta$$

**partial symmetry group of rotating solution**

- **Galaxy clusters scales: ways to explain dark energy & comparing with  $\Lambda$ CDM.**
- **Shadows of black holes: deviations from GR.**
- **Gravitational wave astronomy: deviations from GR.**
- **Binary pulsars: deviations from GR.**
- **Solar system: Newtonian limit and deviations from it.**
- **Large Hadron Collider: gravity at TeV scale.**

- **Galaxy clusters scales: ways to explain dark energy & comparing with  $\Lambda$ CDM.**
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Thank you  
for your  
attention!

