Entanglement islands in regular black holes

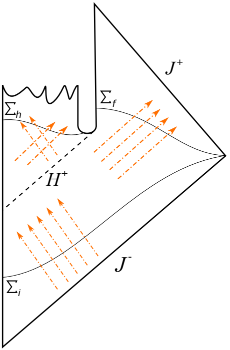
Maxim Fitkevich

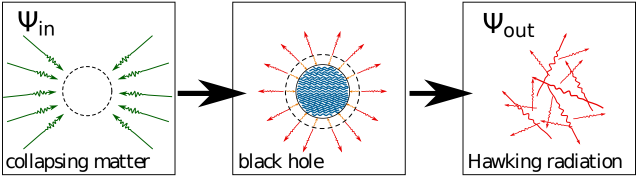
Institute for Nuclear Research of RAS
Moscow Institute of Physics and Technology

Rubakov 70th Anniversary

2025 February 17, Moscow

Quantum Theory vs General Relativity





Disconnected Cauchy surfaces Σ_h and Σ_f contain information about Σ_i

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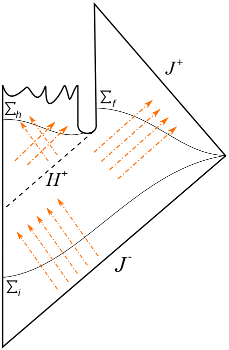

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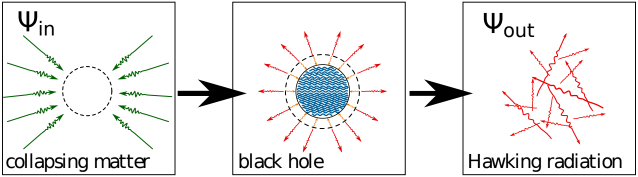

Apparent violation of unitarity:

$$\hat{\rho}_{in} = |\Psi_{in}\rangle\langle\Psi_{in}| \mapsto$$

$$\hat{\rho}_{out} = \mathrm{Tr}_{BH}\left(|\Psi_{ext}\rangle|\Psi_{BH}\rangle\langle\Psi_{BH}|\langle\Psi_{ext}|\right)$$

$$\mathrm{Tr}(\hat{\rho}^2_{out}) < 1 \qquad \text{Danger to quantum laws!}$$

This is *information paradox*.

S.W. Hawking, 1976

Quantum Theory vs General Relativity

- Pro-unitary arguments:
  - Black hole complementarity

    Susskind et al.
  - Remnants/baby universes
  - Holography: gauge/string duality (AdS/CFT)

    Maldacena et al.
  - Islands: unitary Page curve - MAIN FOCUS HERE

    1911.12333 [hep-th] Almheiri, 1905.08255 [hep-th] Pennington ...

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- Problems?
  - AMPS-firewall: unitarity vs equivalence principle.

    Almheiri et al.
  - Dynamics: S-matrix derivation.

    ArXiv:gr-qc/9607022 't Hooft
  - Microscopic: fundamental dofs in Quantum Gravity.

Toy models

CGHS model

$$S = \int d^2x\sqrt{-g}\left[e^{-2\phi}(R+4(\nabla\phi)^2+4\lambda^2) - \frac{1}{2}(\nabla f)^2\right]$$

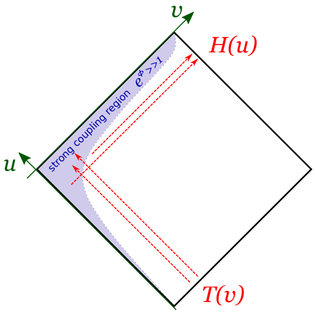
ArXiv:9111056 [hep-th] C. Callan, S. Giddings, J. Harvey, A. Strominger, 1991

In the bulk: $ds^2 = -e^{2\phi}dvdu$,
$f(v,u) = f_{out}(u) + f_{in}(v)$
$e^{-2\phi} = -\lambda^2 vu - \mathcal{T}(v) - \mathcal{H}(u)$
$\partial_v^2\mathcal{T} = (\partial_v f_{in})^2/2$, $\partial_u^2\mathcal{H} = (\partial_u f_{out})^2/2$

Eternal black hole metric: linear $\phi = -\lambda r$,

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 \ ,$$

$$f(r) = 1 - \frac{M}{2\lambda\exp(2\lambda r)} \ .$$



Models

Sinh-CGHS model

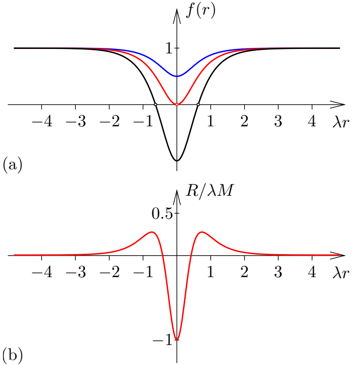
$$S_{\rm sinh} = -\frac{M_{\rm ext}}{2\lambda}\int d^2x\,\sqrt{-g}\,\sinh(2\phi)\,\left(R + 4(\nabla\phi)^2 + 4\lambda^2\right)$$

ArXiv:2202.00023 [gr-qc] M.F.

Regular black holes: linear $\phi = -\lambda r$,

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2\ ,$$

$$f(r) = 1 - \frac{M}{M_{\rm ext}\cosh(2\lambda r)}\ .$$



Motivation:

- Limiting curvature $R_{\mu\nu\rho\sigma}{}^2 < \Lambda^2$.

  Markov, 2111.14318 [gr-qc] Frolov ...

- Other models: Bardeen's black hole, black bounces, planck stars...

  1812.07114 Visser, 1802.04264 Rovelli...

Thermodynamic properties

Euclidean solution

$$ds_E{}^2 = f(r)dt_E{}^2 + \frac{dr^2}{f(r)}\,, \qquad 0 \le t_E < \beta_H\,,$$

has imaginary time period

$$\beta_H = T_H{}^{-1} = 4\pi/f'(r_\mathrm{h})$$

$\Leftarrow$ no conifold singularity at $r = r_\mathrm{h}$.

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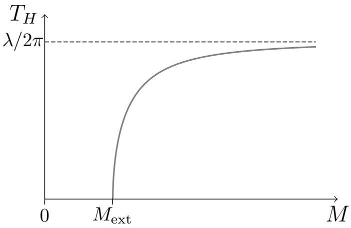
Regular black hole temperature and entropy

$$S_{BH} = \frac{2\pi}{\lambda} M \sqrt{1 - \frac{M_{\rm ext}^2}{M^2}}$$

$$T_H = \frac{\lambda}{2\pi} \sqrt{1 - \frac{M_{\rm ext}^2}{M^2}}$$



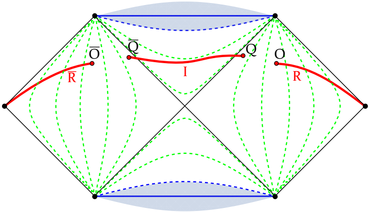
Sinh-CGHS reduces to CGHS in $M_{\rm ext} \to 0$.

Entropy from entanglement island

Hawking's semiclassical answer (for $R \cup R^*$):

$$\dot{S}_{\text{ent}} \simeq 2\pi NT/3\,, \qquad \text{at } r_O \to +\infty\,, \quad \Rightarrow \quad S_{\text{ent}} \leq 2S_{BH}$$

- in violation of the Bekenstein bound.



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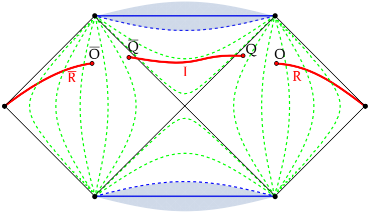
Island formula for black hole entropy

$$\boxed{S_{\rm gen}[R] = \min_{I} \operatorname*{ext}_{\partial I} \left(S_{\rm grav}[\partial I] + S_{\rm ent}[R \cup I]\right)}$$



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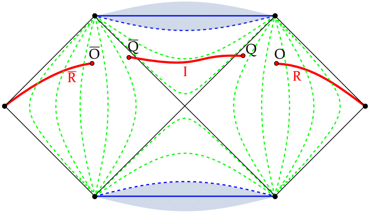


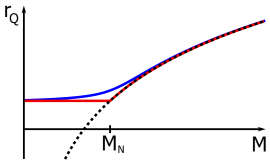
For linear dilaton CGHS $W(\phi) = e^{-2\phi}$ and sinh-CHGS $W(\phi) = -\frac{M_{\rm ext}}{2\lambda}\sinh(2\phi)$

$$S_{\rm gen} = 8\pi W(-\lambda r_Q) + \frac{N}{3}\log(\epsilon^{-2}(v_O - v_Q)(u_Q - u_O)) + \frac{N}{3}(\rho_O + \rho_Q)$$

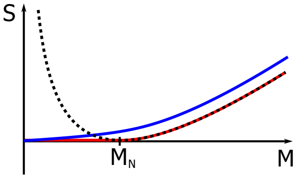
where ρ is metric factor: $ds^2 = -e^{2\rho}dvdu$. Vary $S_{\rm gen}$ with respect to t_Q and r_Q.

Quantum Extremal Point and $S_{\rm ent}$ in CGHS model

Position of QEP



Entanglement entropy



Horizon at $r_{\rm hor}(M) = \frac{1}{2\lambda}\ln(M/2\lambda)$ (black dashed)
Position of QEP at finite r_O (blue line)
Position of QEP at $r_O \to +\infty$ (red line)

$$1. \qquad r_Q \simeq r_{\rm hor}(M)\,, \qquad M > M_N\,,$$

$$\text{where} \quad M_N = \frac{N\lambda}{24\pi}$$

$$2. \qquad r_Q \simeq r_{\rm hor}(M_N)\,, \qquad M < M_N$$

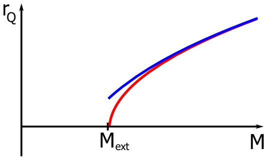
Zero entropy for lightest black hole: $S^0_{\rm ent}(0) = 0$
Analytic answer at $r_O \to +\infty$

$$1. \quad S_{\rm ent} = \frac{2\pi}{\lambda}M - \frac{N}{12}\left(\ln(M/M_N)+1\right)\,, \quad M > M_N$$

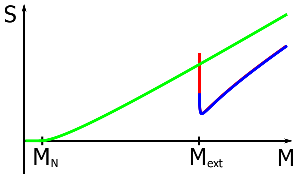
$$2. \qquad S^0_{\rm ent}(M) = 0\,, \qquad M < M_N$$

Quantum Extremal Point and $S_{\rm ent}$ in sh-CGHS model

Position of QEP



Entanglement entropy



Position of QEP at finite r_O (blue line)

QEP at $r_O \to +\infty$ and horizon at (red line) $r_{\rm hor}(M) = \frac{1}{2\lambda} \ln \left(M/M_{\rm ext} + \sqrt{(M/M_{\rm ext})^2 - 1} \right)$

Extremal hole is heavy $M_{\rm ext} > M_N$. At finite r_A its entropy diverges!

$$S_{\rm ent} \simeq \frac{4\pi}{\lambda} \frac{M_{\rm ext}}{\lambda r_O} , \quad M \to M_{\rm ext}$$

What if far observer can not distinguish CGHS and sh-CGHS, $S_{\rm ent}(M)/S^0_{\rm ent}(M) \simeq 1$ as $M \to +\infty$,

$$S_{\rm ent} \simeq \frac{c}{6} \ln \left(\frac{M_N}{M - M_{\rm ext}} \right) + \ldots ,$$

Minimal entropy at $M_{\rm qb} \simeq M_{\rm ext} + 2M_N^2/M_{\rm ext}$,

$$S_{\rm min} \simeq \frac{N}{6} (1 + \ln(M_{\rm ext}/M_N) - 2M_N/M_{\rm ext}))$$

Law of evaporation

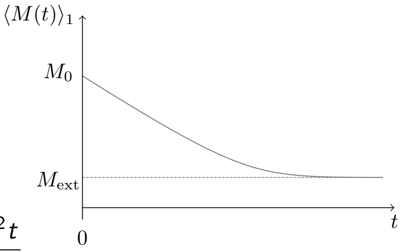
2D Stefan–Boltzmann

$$\frac{dM}{dt} = -\frac{\pi}{12} T_H^2(M)$$

$\Rightarrow$ averaged mass function

$$M(t) + \frac{M_{\text{ext}}}{2} \log\left(\frac{M(t) - M_{\text{ext}}}{M(t) + M_{\text{ext}}}\right) = M_0 - \frac{\lambda^2 t}{48\pi}$$

with initial value $M_0 \gg M_{\text{ext}}$.



Remnants formation

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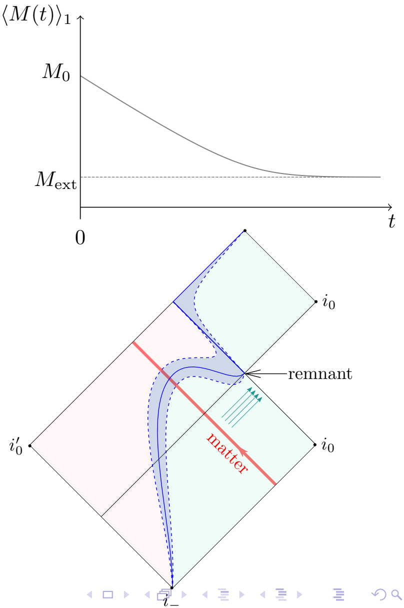
$$M \simeq M_{\rm ext}\left(1 + \exp\left(-\frac{\lambda^2 t}{24\pi M_{\rm ext}}\right)\right)$$

i.e. remnant is formed.



Remnants ~~formation~~ decay

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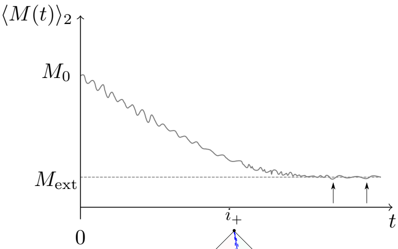
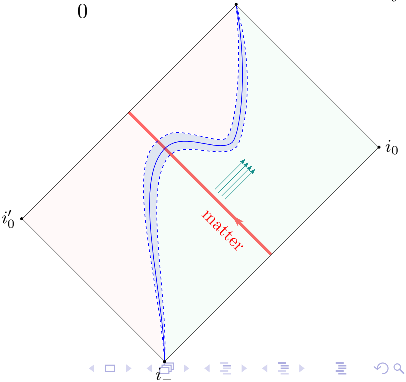
Fluctuations of Hawking flux

$$\langle : \Delta \hat{T}_{tr} : \rangle = O(1) \langle : \hat{T}_{tr} : \rangle$$

on timescale $O(M)$

gr-qc/9905012 Wu, Ford





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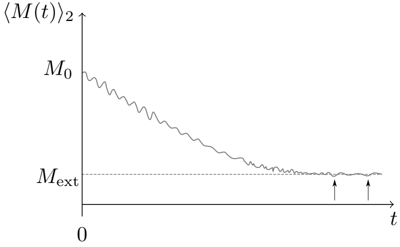

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From fluctuations theory

$$\langle(\Delta M)^2\rangle = -\frac{\partial\langle E\rangle}{\partial\beta} \simeq \frac{\lambda^2}{M_{\text{ext}}} O(1)$$

assuming $\Delta M \ll M_{\text{ext}}$.

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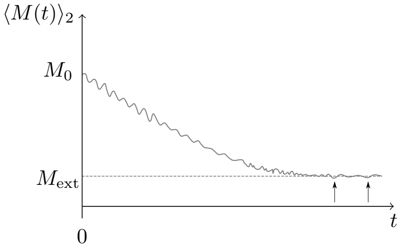

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$$t_{\text{dec}} \simeq 48\pi \frac{M_{\text{ext}}}{\lambda^2} \log\left(\frac{M_{\text{ext}}}{\lambda}\right)$$

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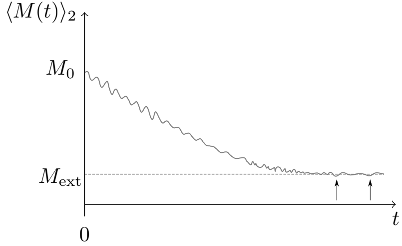

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Adiabaticity condition: change in mass is negligible

$$T\frac{\partial T}{\partial M} \ll T \;\Rightarrow\; T\frac{\partial S}{\partial T} \gg 1 \;\Rightarrow\; T \gg \frac{\lambda^2}{4\pi^2 M_{ext}}$$

We need quantum treatment of remnant decay.

What try to do

Either regularize classical solutions or modify field equations to make black hole decay.

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- Complex saddle points:

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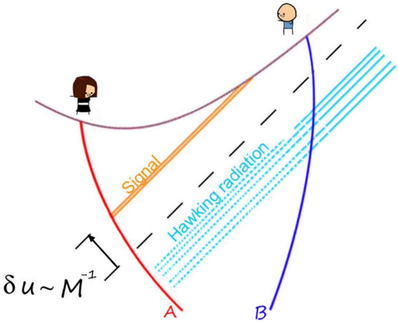
- Real EFT solutions:

$$S \;\mapsto\; S + \frac{N}{96\pi} \int R \frac{1}{\Box} R\,, \qquad\qquad (1\text{-loop effective action})$$

Example: Bardeen black hole, arXiv: 2405.13373

Looming problem

Alice and Bob perform quantum cloning...



...but singularity prevents their attempts

Quantum xerox/cloner paradox:

$|\psi\rangle \mapsto |\psi\rangle|\psi\rangle$ - forbidden

Wooters, Zurek (1982)

but black holes seems to do exactly this

Solution (?): complementarity

with scrambling time $t_{\rm scr} \sim T_H^{-1} \ln S_{BH}$

Page, Preskill et al.

Looming problem

But it literally can be infinite for regular solutions!

Vaydia : $ds^2 = -\left(1 - \frac{M(v)}{M_{\text{ext}}\cosh(2\lambda r)}\right) dv^2 + 2dvdr$, $M(v)$ - Bondi mass

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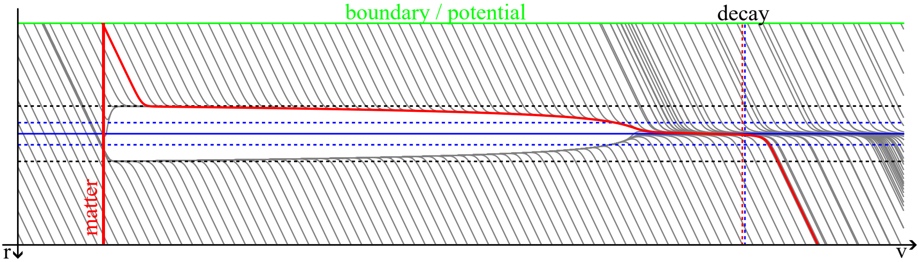

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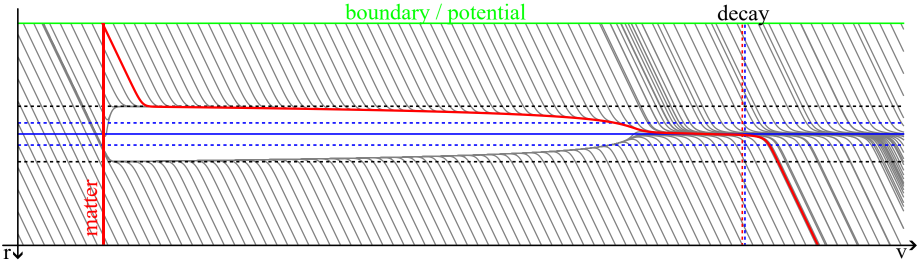

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Obstacle to perturbative/numerical analysis is mass inflation.

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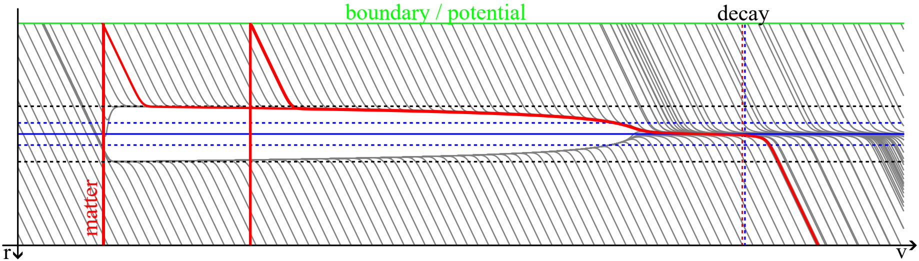

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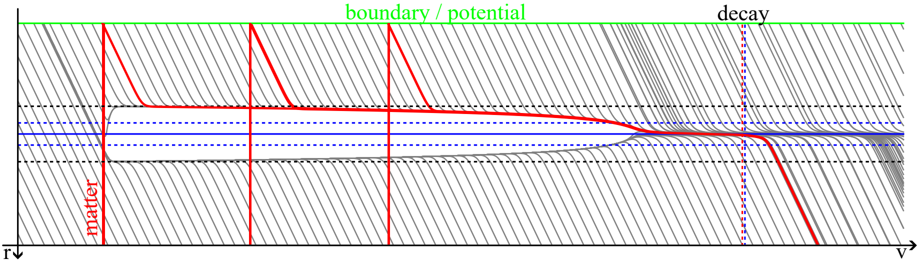

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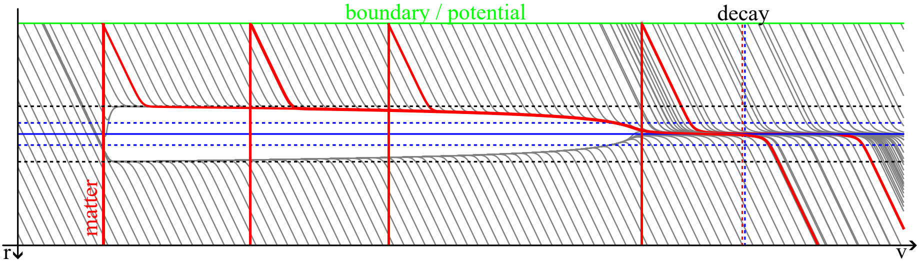

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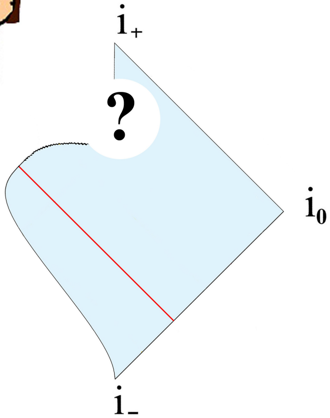
- Toy model with regular black holes was investigated:
  - Islands do not reproduce unitary Page curve in quasi–stationary situations.
  - Singularity removal does not automatically save unitarity.
- What next:
  - Model with one–loop corrections: numerical solutions for black hole decay.
  - Regularization: tunneling solutions.
  - Use them both to test unitarity

$$\int \mathcal{D}c_k^* \mathcal{D}c_k \; e^{-\int dk \; c_k^* c_k} \langle b|\hat{S}^\dagger|c\rangle\langle c|\hat{S}|a\rangle = \langle b|a\rangle \; ,$$

where path integrals for the S–matrix are evaluated using saddle points method.



Thank you!