# Scalar oscillons and the mechanisms of their longevity

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Based on:

Dmitry Levkov, VM, Emin Nugaev, Alexander Panin, JHEP12(2022)079
Dmitry Levkov, VM, Phys. Rev. D 108, 063514 (2023)
Dmitry Levkov, VM, Emin Nugaev, Alexander Panin, *work in progress*

Rubakov–70 memorial RAS session-conference,

Moscow, February 19, 2025

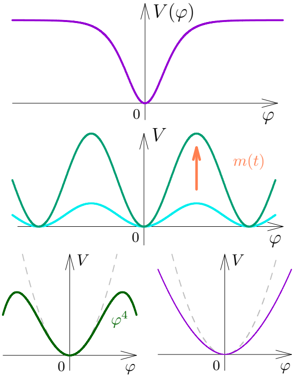
**Scalar field theory**

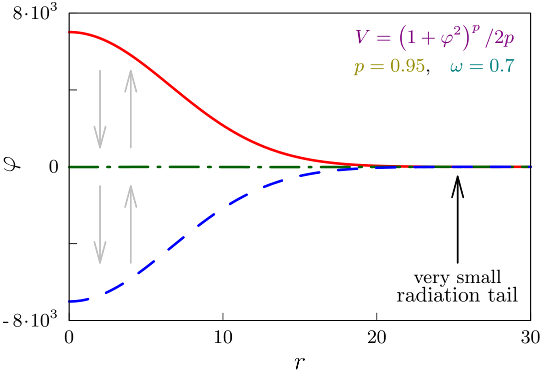
$$\partial_t^2\varphi - \Delta\varphi = -V'(\varphi)$$

**Generic lifetimes:**

$\gtrsim 10^5$ periods

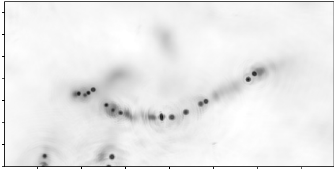
Plethora of theories:



# Oscillons in cosmology

- nucleate during generation of axion or ultra-light DM



*Kolb, Tkachev '94*

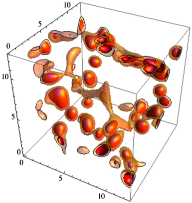
*Vaquero, Redondo, Stadler '19*

*Buschmann, Foster, Safdi '20*

- accompany cosmological phase transitions

*Dymnikova, Kozel, Khlopov, Rubin '00*
*Gleiser, Graham, Stamatopoulos '10*

- formed by inflaton field during preheating

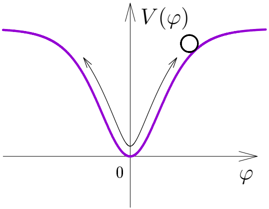


*Amin, Easther, Finkel, Flauger, Herzberg' 12*

*Hong, Kawasaki, Yamazaki '18*

Why are oscillons so long-lived? How to describe them?

- **Consider large-sized oscillons** $\Longrightarrow$ pursue gradient expansion
- Zero order approx.: $\partial_t^2 \varphi - \cancel{\Delta\varphi} = -V'(\varphi) \Longrightarrow$ Nonlinear oscillator




- Action-angle variables in full nonlinearity

$$\varphi = \Phi(I,\theta), \quad \dot{\varphi} = \Pi(I,\theta)$$

- Hamiltonian: $h = \dot{\varphi}^2/2 + V(\varphi) \equiv h(I)$
- Classical solution: $I = \text{const}, \quad \theta = \Omega t + \text{const}, \qquad \boxed{\Omega = \frac{\partial h}{\partial I}}$

- Leading order: restore $\Delta\varphi$

  $I(t,\boldsymbol{x})$, $\theta(t,\boldsymbol{x})$ now depend on $\boldsymbol{x}$ but **slowly**.

$$\mathcal{S} = \int dt\, d^d\boldsymbol{x} \left( \underbrace{\frac{1}{2}\dot{\varphi}^2 - V(\varphi)}_{I\partial_t\theta - h} - \underbrace{\frac{1}{2}(\partial_i\varphi)^2}_{\text{subleading}} \right)$$

- Averaging over period

$$(\partial_i\varphi)^2 \longrightarrow \langle(\partial_i\varphi)^2\rangle = \frac{1}{2\pi}\int_0^{2\pi} (\partial_i\Phi(I,\theta))^2\, d\theta$$

# Effective Field Theory (EFT) for oscillons

- Slow-varying $\partial_i I$, $\partial_i \theta$ are moved *out* of the average

$$\langle (\partial_i \varphi)^2 \rangle \approx \frac{(\partial_i I)^2}{\mu_I(I)} + \frac{(\partial_i \theta)^2}{\mu_\theta(I)} + \langle \partial_I \Phi \, \partial_\theta \Phi \rangle \partial_i I \partial_i \theta$$

$$\mu_I \equiv \left\langle (\partial_I \Phi)^2 \right\rangle^{-1} , \quad \mu_\theta \equiv \left\langle (\partial_\theta \Phi)^2 \right\rangle^{-1}$$

## Leading-order effective action for large oscillons

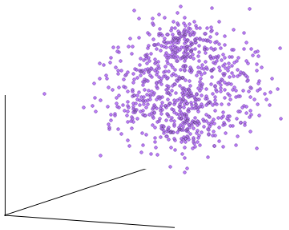
$$\mathcal{S}_{\text{eff}} = \int dt\, d^d \boldsymbol{x} \left( I \partial_t \theta - h(I) - \frac{(\partial_i I)^2}{2\mu_I(I)} - \frac{(\partial_i \theta)^2}{2\mu_\theta(I)} \right)$$

- Global symmetry: $\theta \to \theta + \alpha$

$$\Downarrow$$

- Conserved charge: $N = \int d^3\boldsymbol{x}\; I(t, \boldsymbol{x})$

\+ attraction ⟹ solitons!



# Oscillons in EFT as nontopological solitons

- Stationary ansatz: $I(t, \boldsymbol{x}) = \psi^2(\boldsymbol{x}), \ \theta(t, \boldsymbol{x}) = \omega t$

  or **minimize** energy $E$ at fixed charge $N$.

$$\frac{dE}{dN} = \omega$$

- Oscillon profile equation (gives longevity criterion)

$$-\frac{2\psi^2}{\mu_I}\Delta\psi - (\partial_i\psi)^2\frac{d}{d\psi}\left(\psi^2/\mu_I\right) + \Omega\psi = \omega\psi$$

$$\Downarrow$$

$$\Omega = \partial h/\partial I$$

$$\omega - \Omega \sim (mR)^{-2} \Longrightarrow \left|\frac{d^2h}{dI^2}\right| = \left|\frac{d\Omega}{dI}\right| \ll \frac{\Omega}{I}$$

potential is close to quadratic!

$N = \text{const}$

$\Downarrow$

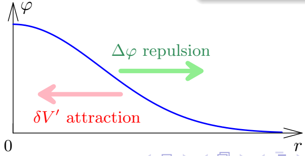
oscillons — eternal in EFT

$\Downarrow$

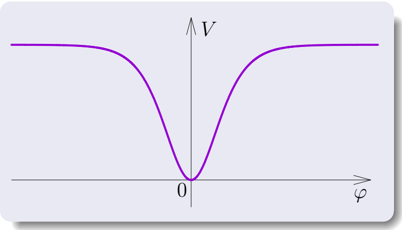
evaporation — beyond all orders

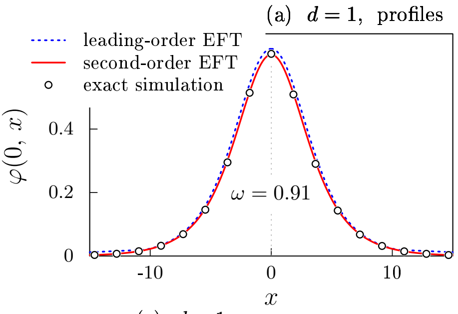
Restoring field values:

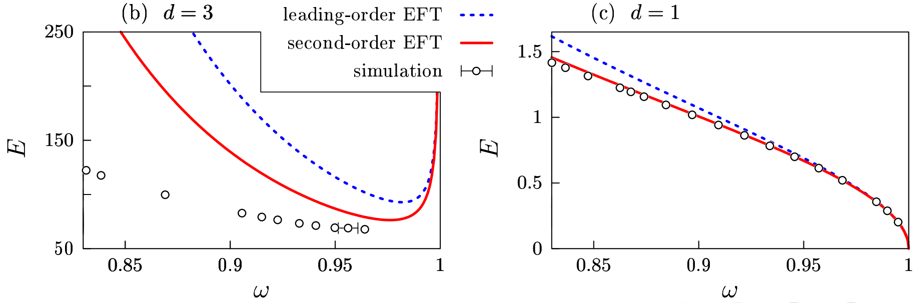
$$\varphi(t, \boldsymbol{x}) = \Phi(\psi^2(\boldsymbol{x}), \omega t)$$



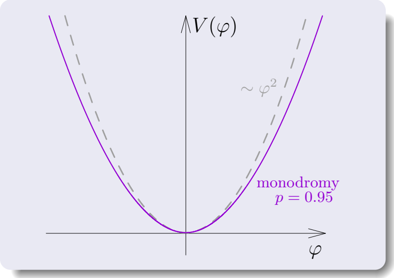
# Example: $V(\varphi) = \frac{1}{2}\tanh^2\varphi$

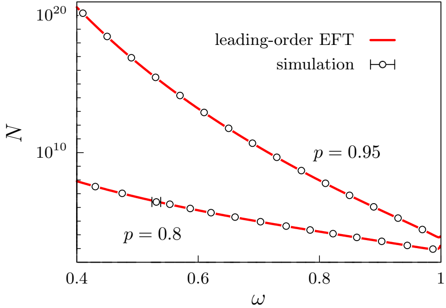


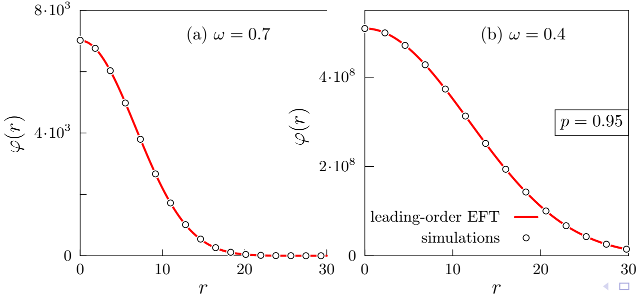



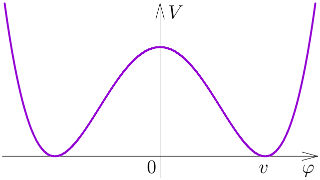
# Monodromy potentials: $V(\varphi) = \frac{1}{2p}\left(1+\varphi^2\right)^p$, $p \lesssim 1$, 3D



# 1D oscillons in a double well?




$$V(\varphi) = \frac{\lambda}{4}\left(\varphi^2 - v^2\right)^2$$

$$\alpha = v\sqrt{\lambda/2}$$

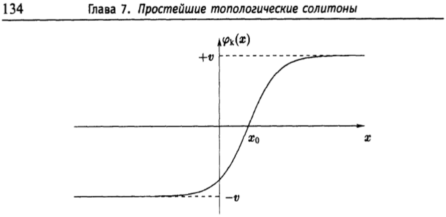



Рис. 7.1

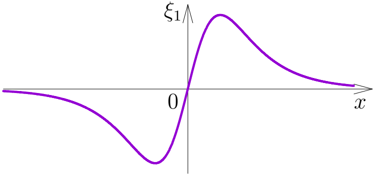
*V. A. Rubakov, "Classical Gauge Fields"*

Kink: $\varphi_k = v \tanh \alpha x$

- Wobbling mode:

$$\varphi_k(x) + \xi_1(x)\ \mathrm{Re}\, e^{i\omega_1 t}$$

$$\xi_1(x) \propto \frac{\tanh \alpha x}{\cosh \alpha x}, \qquad \omega_1^2 = 3\alpha^2$$



# Wobbling modes over kink–antikink pair

- Kink & antikink attract: $\varphi_R = \varphi_k(x+R) + \varphi_k(R-x) - v$

$$\Downarrow$$

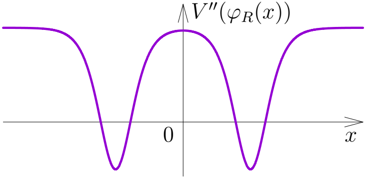
Effective potential: $U_0(R) \approx -16\alpha v^2 e^{-4\alpha R}$

Can wobbling save them? Goal: $\dot{R} = 0$.

- Excitations over kink-antikink pair:

$$\left[-\partial_x^2 + V''(\varphi_R(x))\right]\xi = \omega^2\xi$$

Double well $\Longrightarrow$ $\omega^2$ levels split!




$$\xi_{a,s} = \frac{1}{\sqrt{2}}\left[\xi_1(x+R) \pm \xi_1(x-R)\right], \qquad \omega_{a,s}^2 = 3\alpha^2 \cdot \left(1 \pm 4e^{-2\alpha R}\right)$$

# Wobbling kink–antikink pair — a type of oscillon!

$$\text{Ansatz: } \varphi(t,x) = \varphi_R(x) + \sum_{n=a,s} K_n(t)\cdot\xi_n(x)$$

- Collective coordinates: $K_a(t)$, $K_s(t)$; $R(t)$ — considered as slow–varying: $|\dot{R}/R| \ll \omega_n$
- Plug into action & $\int dx$:

$$S_{\text{eff}} = \int dt \left[ \frac{M(R,K_i)}{2}\dot{R}^2 - U_0(R) + \sum_{n=a,s}\left[\frac{1}{2}\dot{K}_n^2 - \frac{1}{2}\omega_n^2(R)K_n^2\right] + \dots \right]$$

- Adiabatic invariants: $2\pi I_n = \oint dKn\sqrt{2\varepsilon_n - \omega_n^2(R)K_n^2} \approx \text{const.}$
- Effective potential for $R(t)$: $E = M\dot{R}^2/2 + U_{\text{eff}}(R)$,

$$U_{\text{eff}}(R) = U_0(R) + I_a\omega_a(R) + I_s\omega_s(R)$$

- $\omega_a(R)$ decreases with $R \Longrightarrow$ pulls kinks apart!

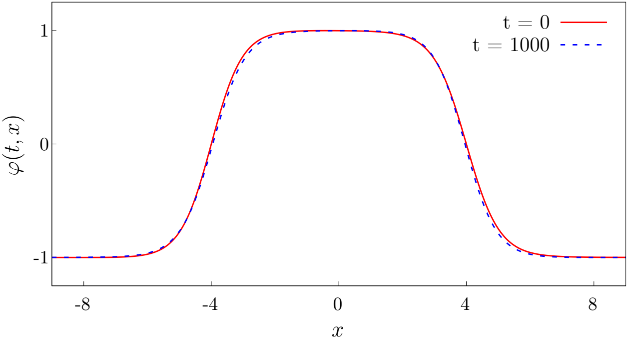
# Wobbling kink–antikink pair — a type of oscillon!

- For antisymmetric mode: $\dfrac{\partial U_{\text{eff}}(R)}{\partial R} = \dfrac{\partial U_0(R)}{\partial R} + I_a \dfrac{\partial \omega_a}{\partial R} = 0.$

$$\Downarrow$$

$$\dot{R} = 0 \quad \Longleftrightarrow \quad v^2 e^{-2\alpha R} = \frac{\sqrt{3}}{16} I_a$$

- Prediction tested numerically:




- Slowly evaporates, then collapses after $10^2 - 10^3$ periods, lifetime grows with $R$.

## EFT.

- Large oscillons — held together by weak nonlinearity
- Parameter of the expansion: $(mR)^{-2}$
- Global $U(1)$ symmetry $\Longrightarrow$ oscillons
- Conditions for existence of long-lived oscillons:

$$V(\varphi)\ \begin{cases} \text{attractive} \\ \text{nearly quadratic potential} \end{cases}$$

## Beyond EFT.

- Conserved Noether charge is not the only way to achieve oscillons.
- Instead, in double well potential:
  close to a topological soliton + adiabatic invariant

## Perspective.

- Decay of oscillons — nonperturbative in EFT?
  May be perturbative for wobbling kinks?

# Thank you for your attention!

# Large oscillons, weak nonlinearities

**Large oscillons**: $R \gg m^{-1}$

- Small repulsion from $\Delta\varphi$

  $\Downarrow$

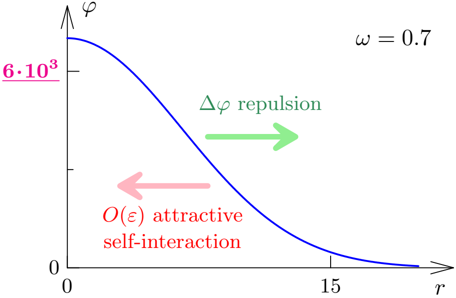
  Weak attraction expected

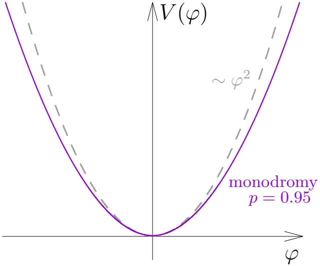
**Example: monodromy potentials**

$$V(\varphi) = \frac{1}{2p}\left(1+\varphi^2\right)^p, \qquad p \lesssim 1$$

- Attractive nonlinearity $\quad \varepsilon \equiv 1 - p$
- Large radius: $R^{-2} \sim O(\varepsilon)$.
- Lifetime: up to $10^{14}$ periods!

  *Ollé, Pujolàs, Rompineve '20*
- Very strong fields: how to account for small nonlinearities?



# Isolating small nonlinearity at strong fields

$$\partial_t^2\varphi - \Delta\varphi = -V'(\varphi)$$

- Zero-order approximation: still a **parabola**, but **not** expansion around the vacuum

$$-V'(\varphi) = -\mu^2\varphi - \delta V'(\varphi)$$

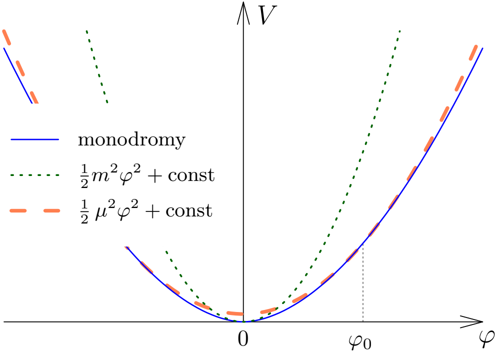
$$\delta V \equiv V - \mu^2\varphi^2/2$$

- Wise choice of $\boldsymbol{\mu} \neq m$ to make $\delta V'$ small:

$$\mu^2 = V'(\varphi_0)/\varphi_0$$

for some scale $\varphi_0 \sim \varphi$

- In the end: scale $\varphi_0$ — tuned to the oscillon amplitude.




**Example: monodromy potential**

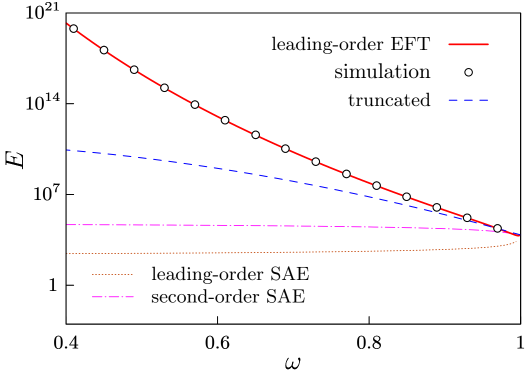
$$V'(\varphi) = \left(1+\varphi^2\right)^{-\varepsilon}\cdot\varphi$$

$$= \underbrace{\left(1+\varphi_0^2\right)^{-\varepsilon}}_{\mu^2}\cdot\varphi + \delta V'$$

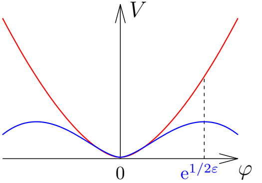
- Small-amplitude expansion: $|\varphi| \ll 1, \quad R \gg m^{-1}$
- Monodromy potential: expansion in $\varepsilon$ at $|\varphi| \gg 1$

$$V = \underbrace{\frac{\varphi^2}{2}\left[1 + \varepsilon - \varepsilon \ln \varphi^2\right.}_{\text{admits exactly periodic solutions}} \left. + O(\varphi^{-2}) + O(\varepsilon^2 \ln^2 |\varphi|)\right] .$$

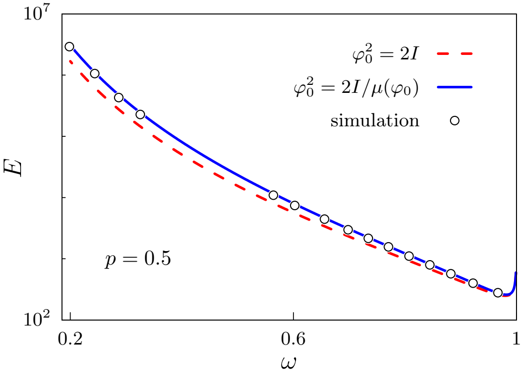
$d = 3$; $p = 0.95$

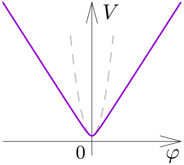



$\varepsilon$-expansion **breaks down** at $\varepsilon \ln |\varphi| \gtrsim 1$.



- Significantly nonlinear: $p = 0.5$.
- How does that affect the EFT precisi?




$\delta N/N \lesssim 0.4$

$\Downarrow$

$\delta N/N \lesssim 0.1$

- Proper choice of $\varphi_0$ scale cures the method!
- Does not mean the EFT series converge well: $\varepsilon = 0.5$.

# Higher-order corrections

- Goal: Develop asymptotic expansion in $R^{-2}$:

$$\mathcal{S}_{\text{eff}} = \underbrace{\mathcal{S}^{(1)}_{\text{eff}}}_{R^0+R^{-2}} + \overbrace{\underbrace{\mathcal{S}^{(2)}_{\text{eff}}}_{R^{-4}} + \underbrace{\mathcal{S}^{(3)}_{\text{eff}}}_{R^{-6}} + \dots}^{\text{corrections}}$$

- Field corrections:

$$I = \underbrace{\bar{I}}_{\text{slow}} + \underbrace{\delta I}_{\text{fast}}, \qquad \theta = \underbrace{\bar{\theta}}_{\text{slow}} + \underbrace{\delta\theta}_{\text{fast}}$$

$$\langle \delta I \rangle = \langle \delta\theta \rangle = 0, \quad \delta I \ll I, \quad \delta\theta \ll \theta$$

- Solve eqs. for $\delta I$, $\delta\theta$ $\Rightarrow$ plug the result into action + $\boxed{\bar{\theta} = \omega t}$

$$\mathcal{S}_{\text{eff}} = \mathcal{S}^{(1)}_{\text{eff}} + \mathcal{S}^{(2)}_{\text{eff}}$$

$$\mathcal{S}^{(2)}_{\text{eff}} = \int dt\, d^d\mathbf{x} \left[ d_1 (\partial_i\psi)^4 + d_2\, \psi\Delta\psi(\partial_i\psi)^2 + d_3 (\Delta\psi)^2 \right]$$

Note. Four spatial derivatives $\qquad d_i(\psi^2)$ — form factors