Космологические модели без начальной сингулярности в теории Хорндески и их устойчивость

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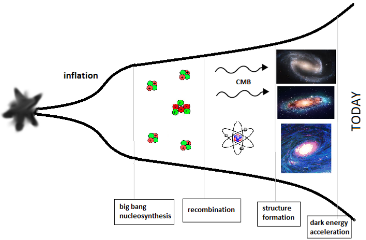
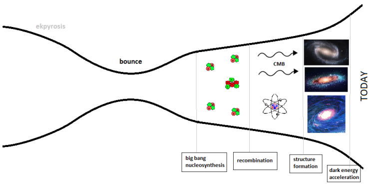
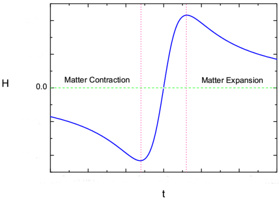
Motivation



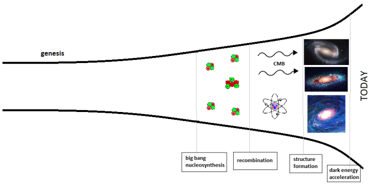
- Standard Big Bang cosmology has its characteristic set of problems at early times including an initial singularity problem.
- Inflation can successfully solve the majority of issues, but it **is geodesically incomplete in the past**.
- Alternative scenarios propose non-singular solutions:
    - Bouncing models (contraction → bounce → expansion)
    - Genesis models (expansion from Minkowski space)

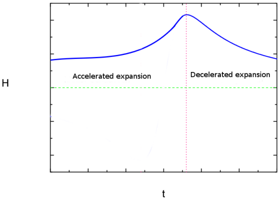
- Both scenarios require the Hubble parameter H to grow:
    - a Universe with a bounce ($\dot{H} > 0$ during the bouncing stage)





- a Universe starting off with Genesis ($\dot{H} > 0$ at the onset of expansion)





Non-singular cosmologies and NEC violation

- Both scenarios require **violation of the Null Energy Condition** (NEC):

$$T_{\mu\nu} k^{\mu} k^{\nu} > 0 \quad (g_{\mu\nu} k^{\mu} k^{\nu} = 0)$$

NEC for a homogeneous stationary fluid: $p + \rho > 0$

- NEC ensures that the Hubble parameter never grows

$$\dot{H} = -4\pi G(p + \rho) + \frac{\kappa}{a^2} < 0$$

and the energy density in standard cosmologies always decreases

$$\frac{d\rho}{dt} = -3H(\rho + p) < 0.$$

- NEC is satisfied in GR + conventional matter
- Ways to violate NEC:
  - add exotic matter
  - modify GR $\longrightarrow$ scalar-tensor theories like Horndeski theories

Horndeski theory

Horndeski (1974), Nicolis, Rattazzi, Trincherini (2009), Deffayet, Gao, Steer, Zahariade (2011)

$$S = \int \mathrm{d}^4x \sqrt{-g} \left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 \right),$$

$$\mathcal{L}_2 = F(\pi, X),$$
$$\mathcal{L}_3 = K(\pi, X)\Box\pi,$$
$$\mathcal{L}_4 = -G_4(\pi, X)R + 2G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right],$$
$$\mathcal{L}_5 = G_5(\pi, X)G^{\mu\nu}\pi_{;\mu\nu} + \frac{1}{3}G_{5X}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu\nu}\pi^{;\mu\rho}\pi_{;\rho}{}^{\nu}\right].$$

π is a scalar field, $X = \pi_{,\mu}\pi^{,\mu}$, $\pi_{,\mu} = \partial_\mu\pi$, $\pi_{;\mu\nu} = \nabla_\nu\nabla_\mu\pi$, $\Box\pi = g^{\mu\nu}\nabla_\nu\nabla_\mu\pi$, $G_{iX} = \partial G_i/\partial X$.

- Equations of motion are 2nd order (2+1 DOFs and no Ostrograsky ghost)
- Generality: any STT with 2nd order EOMs, belong to the Horndeski group.
- Special cases: Brans-Dicke theory, $f(R)$-gravity, k-essence, kinetic gravity braiding, Fab Four, any inflation on a scalar and many more...
- Healthy NEC violation $\longrightarrow$ suitable framework for non-singular cosmologies

Stability issues

- Another subtlety with non-singular cosmologies: stability at the perturbative level.

$$ds^2 = (1+2\alpha)dt^2 - \partial_i\beta \, dtdx^i - a^2(1+2\zeta\delta_{ij} + h^T_{ij})dx^i dx^j$$

Quadratic action for tensor h^T_{ij} and scalar ζ DOFs

$$S^{(2)}_{h+\zeta} = \int \mathrm{d}t\mathrm{d}^3x \; a^3\left[\frac{\mathcal{G}_\mathcal{T}}{8}\left(\dot{h}^T_{ij}\right)^2 - \frac{\mathcal{F}_\mathcal{T}}{8a^2}\left(\partial_k h^T_{ij}\right)^2 + \mathcal{G}_\mathcal{S}\dot{\zeta}^2 - \mathcal{F}_\mathcal{S}\frac{(\nabla\zeta)^2}{a^2}\right]$$

$$\mathcal{G}_\mathcal{S} = \frac{\Sigma{\mathcal{G}_\mathcal{T}}^2}{\Theta^2} + 3\mathcal{G}_\mathcal{T}, \qquad \mathcal{F}_\mathcal{S} = \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F}_\mathcal{T}, \qquad \xi = \frac{a{\mathcal{G}_\mathcal{T}}^2}{\Theta}$$

Stability and (sub)luminality conditions

$$\mathcal{G}_\mathcal{T}, \mathcal{F}_\mathcal{T} > \epsilon > 0, \quad \mathcal{G}_\mathcal{S}, \mathcal{F}_\mathcal{S} > \epsilon > 0, \quad \mathcal{F}_\mathcal{T} \le \mathcal{G}_\mathcal{T}, \quad \mathcal{F}_\mathcal{S} \le \mathcal{G}_\mathcal{S}$$

- ***Complete*** stability for $\forall t$? $\longrightarrow$ *No-go theorem*

No-go theorem

Libanov, Mironov, Rubakov (2016)

Kobayshi (2016)

No-go theorem: there are no cosmological solutions with $a > 0$ in Horndeski theory which are free from gradient instabilities for $t \in (-\infty, +\infty)$.

$$\mathcal{F}_\mathcal{S} = \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F}_\mathcal{T} \qquad \longrightarrow \qquad \xi(t_2) - \xi(t_1) = \int_{t_1}^{t_2} a(t)\left(\mathcal{F}_\mathcal{T} + \mathcal{F}_\mathcal{S}\right)\mathrm{d}t$$

- To avoid gradient instabilities there must be a moment t_0 when $\xi(t)$ changes sign, i.e, $\xi(t_0) = 0$.
- With $a \neq 0$ making $\xi = \dfrac{a\mathcal{G}_\mathcal{T}{}^2}{\Theta}$ cross zero requires either
  - $\mathcal{G}_\mathcal{T} \to 0$, which corresponds to a strong coupling regime, or
  - $\Theta \to \infty$, which corresponds to a singularity in the Lagrangian

Hence, ξ cannot cross zero in a healthy way $\longrightarrow$ *non-singular cosmologies in Horndeski theory are always plagued with gradient instabilities* .

Ways to Evade the No-Go Theorem

Basic strategy: *resolve the contradiction between stability and healthy $\xi(t_0) = 0$.*

- Protect ξ from crossing zero with rapid decay of $\mathcal{G}_{\mathcal{S},\mathcal{T}}, \mathcal{F}_{\mathcal{S},\mathcal{T}} \to 0$ in

$$\xi(t_2) - \xi(t_1) = \int_{t_1}^{t_2} a(t)\,(\mathcal{F}_\mathcal{T} + \mathcal{F}_\mathcal{S})\,\mathrm{d}t \quad \to \quad \text{converges}$$

  Solution: stability does not require ξ to cross zero.
  Note: Strong coupling problem? Not necessarily (see Yulia's talk).

  Ageeva, Evseev, Melichev, Rubakov (2018)
  Ageeva, Petrov, Rubakov (2021)

- Make $\xi = \dfrac{a\mathcal{G}_\mathcal{T}{}^2}{\Theta}$ cross zero by adjusting Θ:
  - Vanishing $\mathcal{G}_\mathcal{T}(t_*) = \Theta(t_*) = 0$: fine-tuning and strong coupling at t_*
  - $\Theta \equiv 0$ for $\forall t$ $\to$ constraints on the Lagrangian $\to$ 2 DOFs like in GR

  Mironov, Shtennikova (2023)

- Cuscuton theory: $\mathcal{G}_\mathcal{S} \equiv 0 \to$ non-dynamical scalar mode
- Go beyond Horndeski

Beyond Horndeski theory (GLPV)

Zumalacárregui, García-Bellido (2014) *Gleyzes, Langlois, Piazza, Vernizzi (2015)*

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_{\mathcal{BH}}\right),$$

$$\mathcal{L}_2 = F(\pi, X),$$

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$$\mathcal{L}_{\mathcal{BH}} = F_4(\pi, X)\epsilon^{\mu\nu\rho}{}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'}$$
$$+F_5(\pi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'}\pi_{;\sigma\sigma'}.$$

- 2+1 DOFs if: $F_4\, G_{5X}X = -3F_5\left[G_4 - 2XG_{4X} + \frac{1}{2}G_{5\pi}X\right]$
- Further generalization: DHOST theories. *Langlois, Noui (2015)*

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- Effect on the No-go theorem: new definition $\xi = \dfrac{a\mathcal{G}_{\mathcal{T}}\,(\mathcal{G}_{\mathcal{T}} + \mathcal{D}\dot{\pi})}{\Theta}$ (no contradiction between stability and $\xi(t_0) = 0$).
- Completely stable bouncing and Genesis solutions were constructed in GLPV theory

Kolevatov, Mironov, Sukhov, VV (2017)

Mironov, Rubakov, VV (2018, 2019)

Matter Coupling: Subtleties & Problems

- **Matter Coupling Can Spoil Degeneracy**
  - Additional matter fields can break degeneracy conditions, resurrecting the Ostrogradsky ghost. Examples:
    - another Horndeski-like scalar field
    - vector field coupled through derivative
  - (Beyond) Horndeski and DHOST Ia theories allow safe couplings to a conventional scalar field and to the classical fermions.

Deffayet, Garcia-Saenz (2020, 2021)

- **Emergent Superluminal Modes**
  - Even stable theories may develop **superluminal propagation** when coupled to matter. *Easson, Sawicki, Vikman (2013)*
  - The effect arises due to **kinetic mixing (braiding)**
  - General case of "(beyond) Horndeski + perfect fluid":
    - Horndeski theory: the speed of Horndeski scalar mode is modified (no superluminality)
    - beyond Horndeski theory: fluid with $\omega \sim 1$ induces superluminal scalar mode *Mironov, Rubakov, VV (2020)*
  - Exceptional DHOST Ia subclass where the superluminality is not induced *Mironov, Rubakov, VV (2021)*

New developments and outlook

- Viable non-singular cosmological models: primordial spectrum and non-Gaussianities.

  Yulia Ageeva's talk

- Non-singular cosmological solutions in Horndeski theory with torsion.

  Sergey Mironov's talk

- No-go theorem in Horndeski theory over a time-dependent spherically-symmetric background.

  Mikhail Sharov's talk

- Kaluza-Klein reduction in Horndeski theory.

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Thank you for your attention!