Non-singular Horndeski cosmologies in the light of latest experimental data

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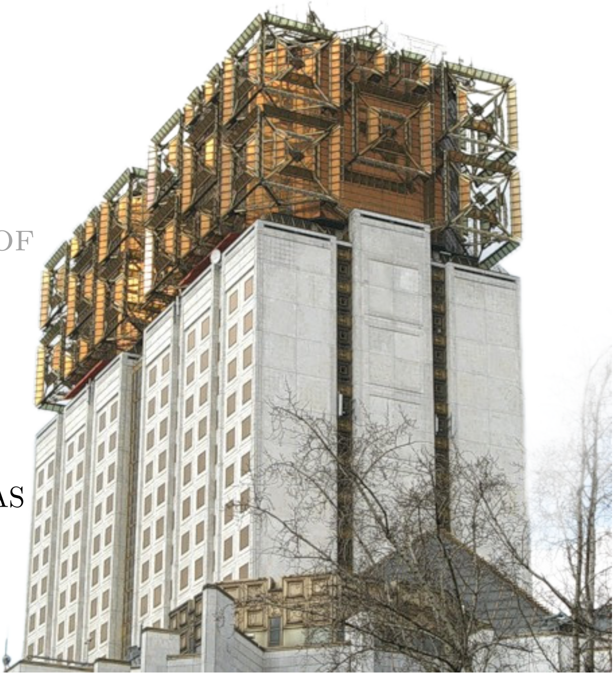

These works:

- arXiv:2104.13412 (PRD),
- arXiv:2207.04071 (JHEP),
- work in progress...,

are done in the collaboration with V. Rubakov, P. Petrov, and M. Kotenko.

- The search of non-singular alternatives/completions to **inflation** [*Starobinsky'1980, Guth'1981, Sato'1981, Linde'1982, Albrecht, Steinhardt'1982*] seems as an important problem.

Motivation

Why non-singular cosmologies?

- We study contracting Universe with subsequent **bounce**
  [*Veneziano'2004;*
  Aref'eva, Joukovskaya, Vernov'2007;
  Qui et al'2011,2013;
  Easson, Sawicki, Vikman'2011;
  Cai, Easson, Brandenberger'2012;
  Osipov, Rubakov'2013;
  Koehn et al'2013; Battarra et al'2014; Ijjas, Steinhardt'2016...]
  epoch as such alternative/completion to inflation;
- This model (as any viable cosmological model) should obey the set of different **theoretical and experimental constraints**...

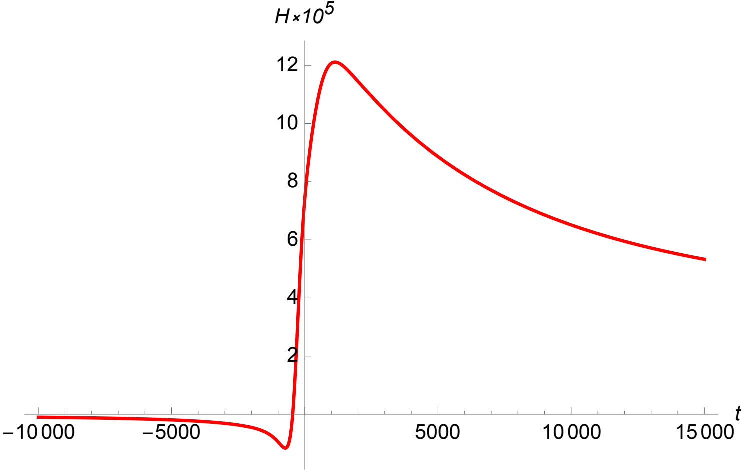

Figure: Hubble parameter for the early Universe with bounce

Violation of NEC/NCC without obvious pathologies is possible in the class of Horndeski theories [*Horndeski'74*]:

$$
\begin{aligned}
\mathcal{L}_H = & \, G_2(\phi, X) - G_3(\phi, X)\Box\phi + \\
& G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\
& + G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi \\
& - \frac{1}{6}G_{5,X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right],
\end{aligned}
$$

where $X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ and $\Box\phi = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi$.
In the framework of this theory one can (quite straightforwardly) obtain healthy bounce epoch.

No-Go theorem

Stability during the whole evolution

- One way is to go beyond Horndeski and DHOST [*Cai et.al.' 2016, Creminelli et.al.'2016, Kolevatov et.al.'2017, Cai, Piao'2017, Mironov, Volkova, Rubakov'2018...*]
- Another way to avoid No-Go theorem for Horndeski is to obtain such a model/solution that $\mathcal{F}_{S,T}$ coefficients have asymptotics

$$\mathcal{F}_{S,T} \to 0 \text{ as } t \to -\infty, \text{ where } \mathcal{F}_T = 2G_4.$$

- This means that

$$G_4 \to 0 \text{ as } t \to -\infty.$$

- Effective Planck mass goes to zero and it signalizes that we may have strong coupling at $t \to -\infty$.

Solution: no SC regime at $t \to -\infty$ in some region of Lagrangian parameters.

Concrete bounce model

Theoretical constraints

With the appropriate choice of Lagrangian functions, the bounce solution reads

$$a = d(-t)^{\chi} \ ,$$

where $\chi > 0$ is a constant and t is cosmic time, so that $H = \chi/t$. Coefficients from quadratic actions are

$$\mathcal{G}_T = \mathcal{F}_T = \frac{g}{(-t)^{2\mu}}, \quad \mathcal{G}_S = g\frac{g_S}{2(-t)^{2\mu}}, \quad \mathcal{F}_S = g\frac{f_S}{2(-t)^{2\mu}},$$

and

$$u_T^2 = \frac{\mathcal{F}_T}{\mathcal{G}_T} = 1, \quad u_S^2 = \frac{\mathcal{F}_S}{\mathcal{G}_S} = \frac{f_S}{g_S} \neq 1.$$

To avoid No-Go:

$$1 > \chi > 0, \ 2\mu > \chi + 1.$$

To avoid SC regime ($t \to -\infty$):

$$\mu < 1.$$

Spectra are given by

$$\mathcal{P}_\zeta \equiv \mathcal{A}_\zeta \left(\frac{k}{k_*}\right)^{n_s-1} , \qquad \mathcal{P}_T \equiv \mathcal{A}_T \left(\frac{k}{k_*}\right)^{n_T} ,$$

where k_* is pivot scale, the spectral tilts are

$$n_S - 1 = n_T = 2 \cdot \left(\frac{1-\mu}{1-\chi}\right),$$

$$n_S = 0.9649 \pm 0.0042.$$

The amplitudes in our model are

$$\mathcal{A}_\zeta = \frac{C}{g}\frac{1}{g_S u_S^{2\nu}} , \; \mathcal{A}_T = \frac{8C}{g},$$

where

$$\nu = \frac{1+2\mu-3\chi}{2(1-\chi)} = \frac{3}{2} + \frac{1-n_S}{2} \approx \frac{3}{2}.$$

Power spectrum

Observational constraints

- Approximate flatness is ensured in our set of models by choosing $\mu \approx 1$...
- ...while the slightly red spectrum is found for $\mu > 1$.

Power spectrum

Observational constraints

The problem №1: red-tilted spectrum requires $\mu > 1$, while absence of strong coupling $\mu < 1$!

Solution: consider time-dependent μ: changes from $\mu < 1$ to $\mu > 1$ (time runs as $-\infty < t < \infty$).

Try to escape from SC and generate spectrum, consistent with experiment.

Horizon exit must occur in weak coupling regime!

The problem №2: r-ratio is small:

$$r = \frac{\mathcal{A}_T}{\mathcal{A}_\zeta} \approx 8 g_S u_S^3 < 0.032.$$ *Tristram et al'2022*

Solution: choose $u_S \ll 1 \to$ similar to **k-inflation** *Mukhanov et al'1999, 2000*

Cubic action for tensors

$$\mathcal{S}^{(3)}_{TTT} = \int dt\ a^3 d^3x \Big[\frac{\mathcal{F}_T}{4a^2}\Big(h_{ik}h_{jl} - \frac{1}{2}h_{ij}h_{kl}\Big)h_{ij,kl}\Big] \, .$$

Corresponding SC and classical scales are

$$E^{TTT}_{strong} \sim \frac{\mathcal{G}_T^{3/2}}{\mathcal{F}_T} = \frac{g^{1/2}}{|t|^{\mu}} \, , \quad E_{cl} \sim H \sim |t|^{-1},$$

$$E^{TTT}_{strong} > E_{cl} \quad \rightarrow \quad |t|^{2\mu-2} < g \, .$$

Tensors exit (effective) horizon:

$$t_f^{(T)}(k) \sim \left(\frac{d}{k}\right)^{\frac{1}{1-\chi}}$$

so the absence of SC at $t = t_f$: $\frac{1}{g}\left(\frac{d}{k}\right)^{2\frac{\mu-1}{1-\chi}} \sim \mathcal{A}_T \ll 1.$

Scalars exit (effective) horizon:

$$t_f^{2(\mu-1)} \sim g\mathcal{A}_\zeta u_S^3 \ .$$

$$\left(\frac{g u_S^{11}}{|t_f(k_{min})|^{2(\mu-1)}}\right)^{1/6} \sim \left(\frac{u_S^8}{\mathcal{A}_\zeta}\right)^{1/6} \sim \left(\frac{r^{8/3}}{\mathcal{A}_\zeta}\right)^{1/6},$$

$$\left(\frac{r^{8/3}}{\mathcal{A}_\zeta}\right)^{1/6} > 1 \ .$$

Strong coupling and r-ratio

Region of healthy parameters

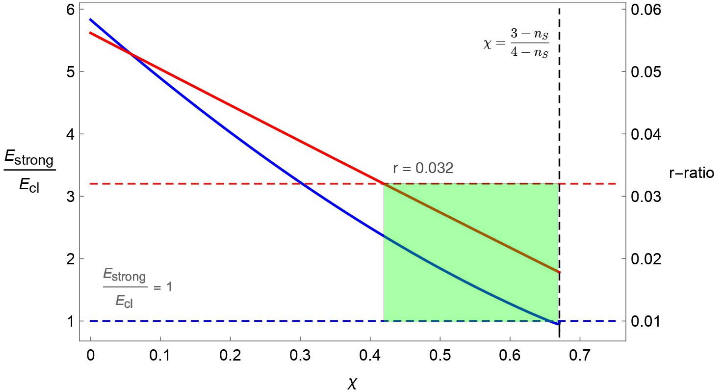

Figure: The r-ratio (red line) and ratio $E_{strong}(k_*)/E_{cl}(k_*)$ (blue line) as functions of χ for the central value $n_S = 0.9649$.

- Deviation from the Gaussian features for primordial perturbation field:

$$\zeta = \zeta_{\rm g} + \frac{3}{5} f_{\rm NL} \zeta_{\rm g}^2,$$

[*Komatsu, Spergel'2001; 2009*] where

$$f_{\rm NL} = \frac{10}{3} \frac{\mathcal{A}_\zeta}{\sum_{i=1}^{3} k_i^3},$$

and

$$B_\zeta\left(k_1, k_2, k_3\right) = \frac{(2\pi)^4 \left(\mathcal{P}_\zeta\right)^2}{\prod_{i=1}^{3} k_i^3} \mathcal{A}_\zeta\left(k_1, k_2, k_3\right),$$

$$\begin{aligned}
&\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3)\rangle \\
&= -i \int_{\tau_i}^{\tau_f} d\tau \; a \; \langle 0| \left[\zeta(\tau_f, \vec{k}_1)\zeta(\tau_f, \vec{k}_2)\zeta(\tau_f, \vec{k}_3), \mathcal{H}_{\rm int}(\tau)\right] |0\rangle \,.
\end{aligned}$$

The following observational constraints were acquired by *Planck collaboration'2018*:

- $f_{\mathrm{NL}}^{\mathrm{local}} = -0.9 \pm 5.1$,
- $f_{\mathrm{NL}}^{\mathrm{equil}} = -26 \pm 47$,
- $f_{\mathrm{NL}}^{\mathrm{enfold}} = 6 \pm 30.5$.

Our model $\rightarrow$ small f_{NL} $\rightarrow$ weak constraints on model parameters...

Conclusion

...and outlook

- We construct the model of bounce, within one can generate nearly flat (red-tilted) power spectrum of scalar perturbations. But it is not so automatic as in inflation!
- In such models the requirement of strong coupling absence leads to the fact that the r-ratio cannot be arbitrarily small and, moreover, it is close to the boundary $r < 0.032$ suggested by the observational data.
- It seems that observational constraints from non-Gaussianities provide very weak conditions for model parameters $\rightarrow$ in progress...

Thank you for attention!