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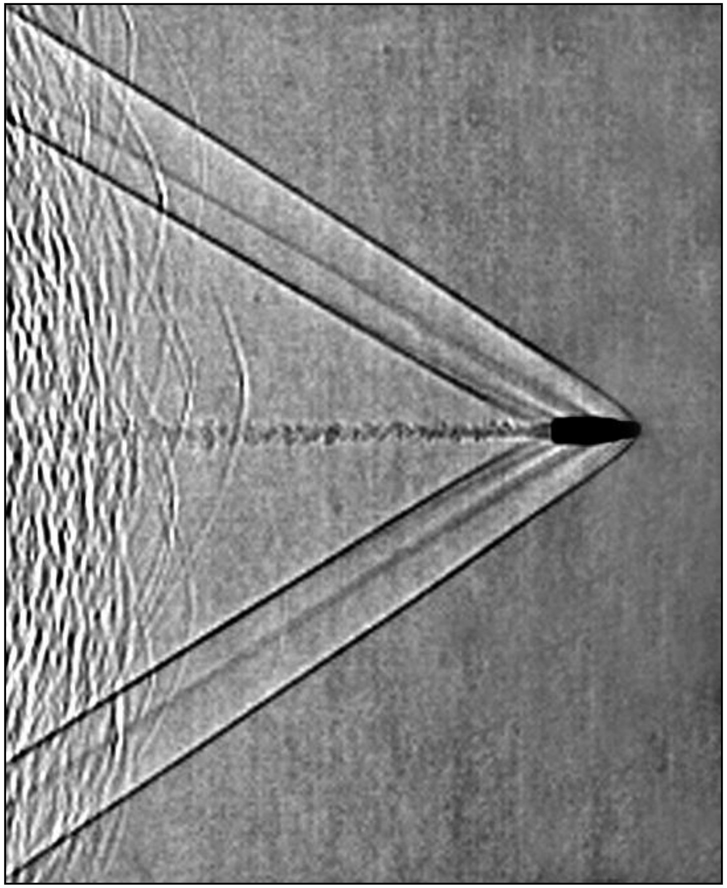
Shock waves: geometry, symmetries and new physics

Basic facts about shockwaves (SW)

SW in *nonrelativistic physics* – waves with rapid changes of pressure, density etc. on their fronts;

SW perturbation travels faster than a local speed of sound

SW are produced by supersonic sources



A sound shockwave from a supersonic bullet

Gravitational shockwaves are solutions to the Einstein equations with rapid changes of curvature tensor on null hypersurfaces (which are wave fronts).

Definitions

A general class of plane-fronted shockwave geometries

$$ds^2 = -dvdu - H(v,u,y)du^2 + dy_i^2 \quad ,$$
$$H(v,u,y) = \chi(u) f(v,y) \quad ,$$

$f(v,y)$ is a profile function.

Symmetries: metric holds its form under null rotations

$$u' = u \quad , \quad y_i' = y_i + \omega^i u \quad , \quad v' = v + 2\omega^i y_i + (\omega^i)^2 u \quad , \quad f'(v + 2\omega y, y) = f(v,y)$$

Nonvanishing components of the Riemann, Ricci, the Einstein tensors are

$$R_{uaub} = \frac{1}{2}\partial_a \partial_b H \quad ,$$

$$R_{uu} = \frac{1}{2}\partial_\perp^2 H + 2H\partial_v^2 H \quad , \quad R_{ua} = \partial_v \partial_a H \quad .$$

$$\boxed{G_{uu} = \frac{1}{2}\partial_\perp^2 H \quad , \quad G_{uv} = \partial_v \partial_i H \quad , \quad G_{ij} = 2\delta_{ij}\partial_v^2 H}$$

Change of velocities

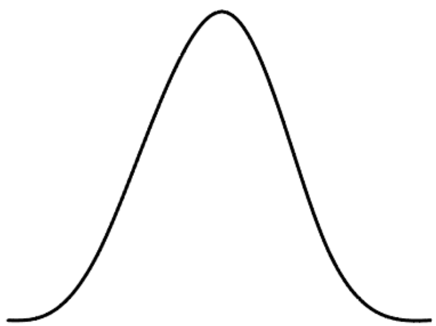
Let $\bar{u}^\mu$ be velocity of a particle before gravitational perturbation,

Let u^μ be velocity after the perturbation. Geodesic equations yield:

$$u^\mu = \bar{u}^\mu - \frac{1}{2} H l^\mu + \bar{\chi}(u) \mathcal{L}_\zeta \bar{u}^\mu \quad ,$$

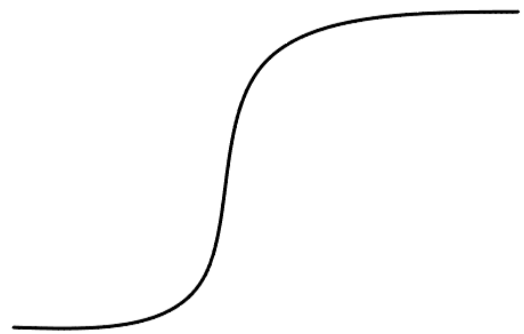
$$\zeta^\mu = \frac{1}{2}(f l^\mu + u f^{,\mu}) \quad , \quad \bar{\chi}(u) = \int_{-\infty}^{u} du' \chi(u')$$

$\chi(u)$



$0 \le u \le \delta$

$\bar{\chi}(u)$

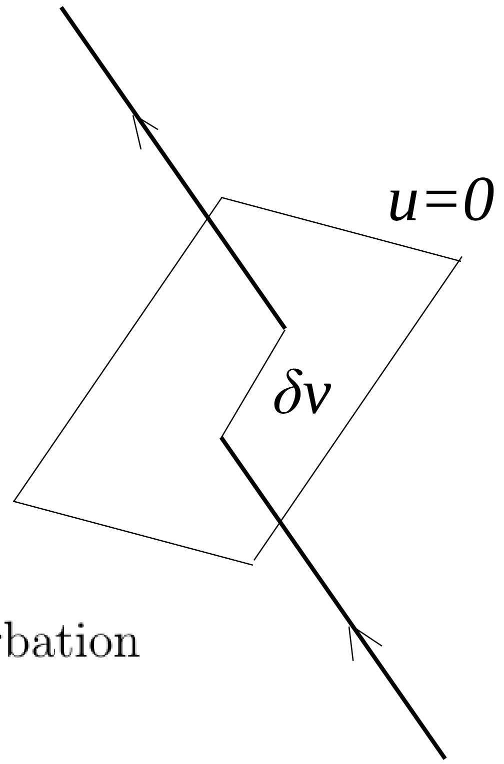


$0 \le u \le \delta$

particle "does not remember" form of the shock !

$$u^\mu = \bar{u}^\mu + \mathcal{L}_\zeta \bar{u}^\mu \quad , \quad u \simeq \delta$$

Change of coordinates



$$\delta v = \int_0^\delta du' u^v = -f \quad ,$$

change of coordinates and velocities caused by the perturbation look as coordinate transformations

$$\delta x^\mu = -\zeta^\mu \quad , \quad u = 0 \quad ,$$
$$\delta u^\mu = \mathcal{L}_\zeta \bar{u}^\mu$$

name of transformations - supertranslations

Since the form of the shock does not matter, one can work in impulsive limit

Shockwaves (SW) with wave front localized at $u = 0$ are obtained when

$$\boxed{\chi(u) \to \delta(u)}$$

Define a tetrad at $u = 0$

$$l = 2\partial_v \quad , \quad e^i = \partial_i \quad , \quad n = \partial_u \quad , \quad (n \cdot l) = -1 \quad ,$$

l, e^i are tangent to $u = 0$.

In the Einstein theory SW are produced by sources with the the stress-energy tensor

$$T_{\mu\nu} = \delta(u) \left(\sigma l_\mu l_\nu + j_i (l_\mu e^i_\nu + l_\nu e^i_\mu) + p \delta_{ij} e^i_\mu e^j_\nu\right) \quad ,$$

where surface energy σ, current j_i and pressure p of the wave are

$$\sigma = \frac{f_{,ii}}{16\pi G} \quad , \quad j_i = -\frac{f_{,vi}}{8\pi G} \quad , \quad p = \frac{f_{,vv}}{4\pi G} \quad ,$$

WEC or NEC conditions:

$$T_{\mu\nu} V^\mu V^\nu \geq 0 \quad \to \quad \sigma p - j_i^2 \geq 0 \quad , \quad \sigma \geq 0 \quad .$$

for any time-like or null V

4-momentum of SW

Null rotations of the tetrade

$$l' = l \quad , \quad e_i' = e_i - \omega^i l \quad , \quad n' = n - \omega^i l + \frac{1}{2}\omega^2 l \quad ,$$

induce transformations

$$p' = p \quad , \quad j_i' = j_i + \omega^i p \quad , \quad \sigma' = \sigma + 2\omega^i j_i + \omega^2 p$$

One can define 4-momentum of SW

$$P^v = \sigma \quad , \quad P^u = p \quad , \quad P^i = j_i$$

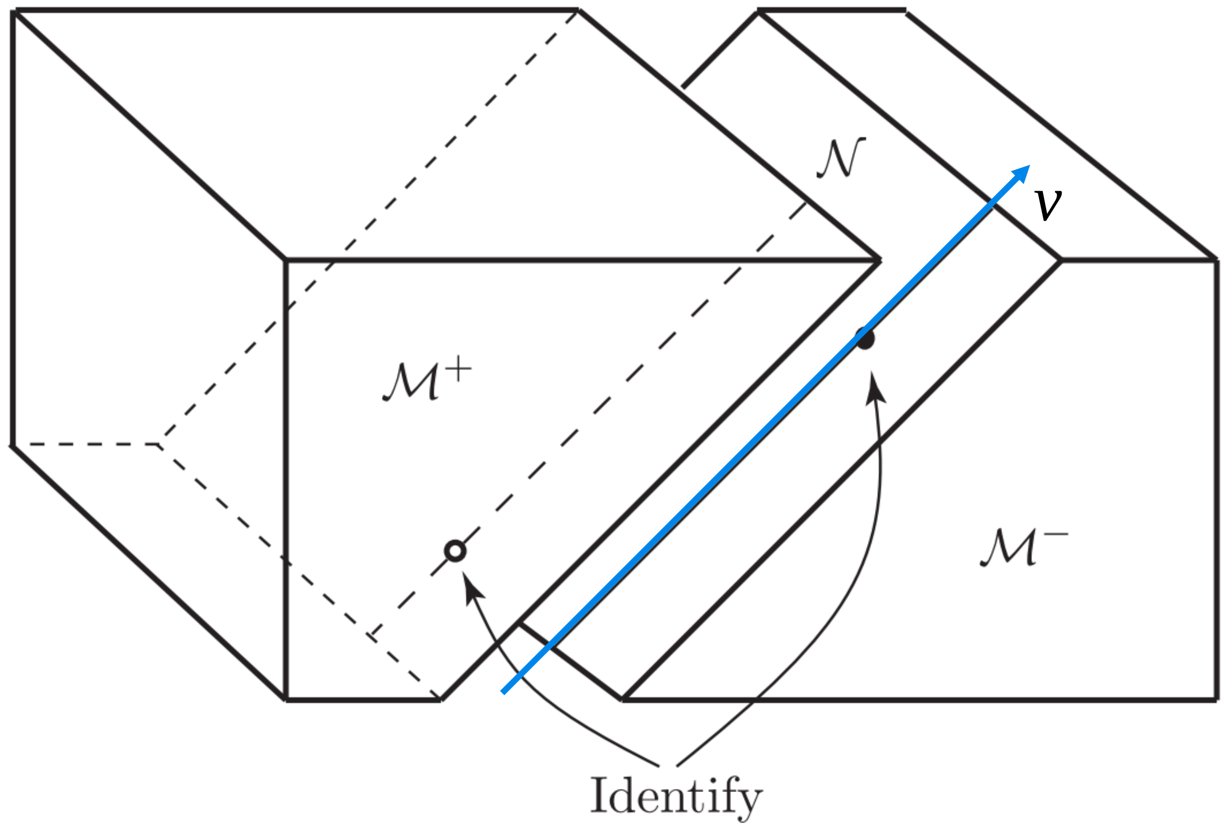
which transforms as 4-vector under null rotations.

P is time-like ,

$$P^2 = \eta_{MN} P^M P^N = -\sigma p + j_i^2 \leq 0 \quad ,$$

if WEC or SEC hold.

A 'cut-and-glue' method by R.Penrose (soldering spacetimes)



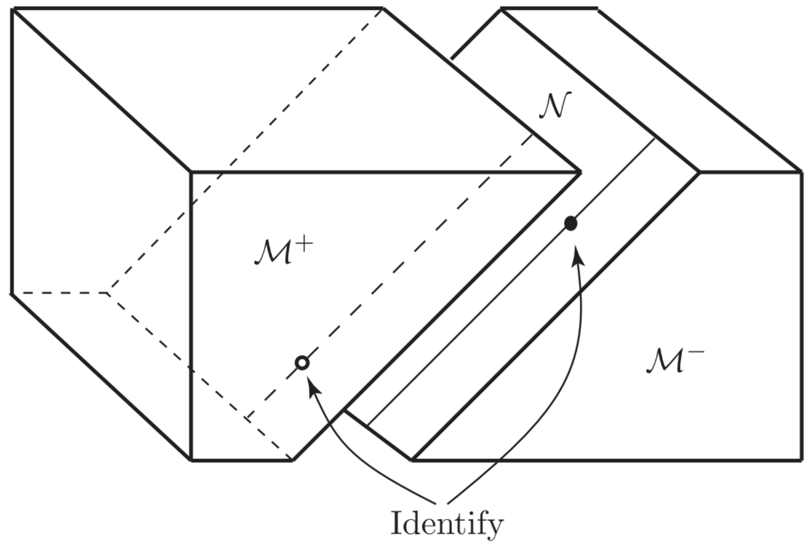
$\mathcal{M}^+$ and $\mathcal{M}^-$ are glued along the wave front

with the shift

Penrose supertranslation

$$v_+ = v_- + f + O(f^2)$$

Scalar fields



Soldering conditions for the metrics of $\mathcal{M}^{\pm}$

$$x^+ = x^- - \zeta \quad , \quad g^+_{\mu\nu} = g^-_{\mu\nu} + \mathcal{L}_\zeta g^-_{\mu\nu} = g^-_{\mu\nu} \quad u = 0 \quad ,$$

By studying impulsive limit in the Klein-Gordon equation

$$(-\Box + m^2)\phi = j \quad ,$$

one comes to condition

$$\boxed{\phi^+ = \phi^- + \mathcal{L}_\zeta \phi^- \quad , \quad u = 0}\,.$$

An observer does not see discontinuities in ϕ on SW front.
(but not its normal deriavtives)

Characteristic Cauchy problem for perturbations

Let ϕ be a field in the absences of the shock wave

$$P\phi = j$$

Perturbation caused by the shock wave in $\mathcal{M}^+$ is defined as

$$\phi_{sw} \equiv \phi^+ - \phi \quad , \quad j^+(x) \equiv j(x) \quad , \quad u > 0$$

$$\boxed{\begin{array}{c} P\phi_{sw} = 0 \quad , \quad u > 0 \quad , \\ \phi_{sw} = \phi^+ - \phi = \mathcal{L}_\varsigma \phi^- - (\phi - \phi^-) \quad , \quad u = 0 \end{array}}$$

characteristic initial-value problem

In $\mathcal{M}^-$ fields ϕ and ϕ^- are solutions to the same problem but with different sources j and j^-, thus

$$\phi - \phi^- = \delta_j \phi \quad , \quad u = 0$$

Hints for particle creation by SW (in QFT)

$$\phi_-(u,v,y^i) = e^{ik_\mu x^\mu} \quad , \quad u < 0$$

$$k^2 = -m^2 \quad , \quad k_u = \frac{k_i^2 + m^2}{4k_v}$$

Problem for perturbation

$$(-\Box + m^2)\phi_{sw} = 0 \quad , \quad \phi_{sw} = f\partial_v \phi_- \quad , \quad u = 0 \quad .$$

For SW with time-dependent profile

$$f(v) = \int dp e^{ipv} \tilde{f}(p)$$

$$\phi_{sw}(x) = i \int dp e^{ik_\mu(p)x^\mu} p_v \tilde{f}(p)$$

$$k_\mu(p) = (k_u(p), k_v + p, k_i) \quad , \quad k_u(p) = \frac{k_i^2 + m^2}{4(k_v + p)}$$

Conversion of positive energy modes to negative energy modes happens when

$$k_v(k_v + p) < 0$$

Transition Electro-Magnetic radiation on SW

$$\partial_\mu F^{\mu\nu} = j^\nu \quad , \quad F_+^{\mu\nu} = F_-^{\mu\nu} + F_{sw}^{\mu\nu}$$

perturbation caused by gravity pulse is a transition radiation (V.Ginzburg)

$$F_{sw}^{\mu\nu} = \chi(u) F_1^{\mu\nu} + \bar{\chi}(u) F_2^{\mu\nu}$$

One can show that

$$F_1^{\mu\nu} = \sigma_i (l^\mu e_i^\nu - l^\nu e_i^\mu) \quad , \quad F_2^{\mu\nu} = e_i^\mu \sigma_i{}^{,\nu} - e_i^\nu \sigma_i{}^{,\mu} + \rho_i (l^\mu e_i^\nu - l^\nu e_i^\mu)$$

Outside the source of EM field

$$l_\mu F_2^{\mu\nu} = l_\mu \mathcal{L}_\zeta F_-^{\mu\nu}$$

Cauchy problem for EM perturbation

$$\boxed{\begin{array}{c} \partial_\mu F_{sw}^{\mu\nu} = 0 \quad , \quad u > 0 \\[2em] l_\mu F_{sw}^{\mu\nu} = l_\mu \mathcal{L}_\zeta F_-^{\mu\nu} \quad , \quad u = 0 \end{array}}$$

It is enough to set Cauchy data for $l_\mu F_{sw}^{\mu\nu}$ to determine other components

Main new physical effects

- SW produce non-stationary perturbations of classical fields, the perturbations are transformed (at late times) into (scalar, electromagnetic or gravitational) waves with a specific energy flux to null infinity;

- SW produce secondary shock waves in a field system as well$

- one secondary shock wave is a plane SW accompanying the initial gravitational SW;

- there may other secondary shock wave: for example when SW hits the source of field it produces spherical shocks;

- discontinuities of secondary shocks are theta-function-like (not delta-function-like)

Shockwaves passing charged bodies or bodies with magnetic moments (more in talk by E. Davydov)

SW create perturbations of EM fields of an electric charge or a magnetic dipole

The perturbations are non-stationary and at late-times convert to spherical EM outgoing from the charge (magnetic dipole)

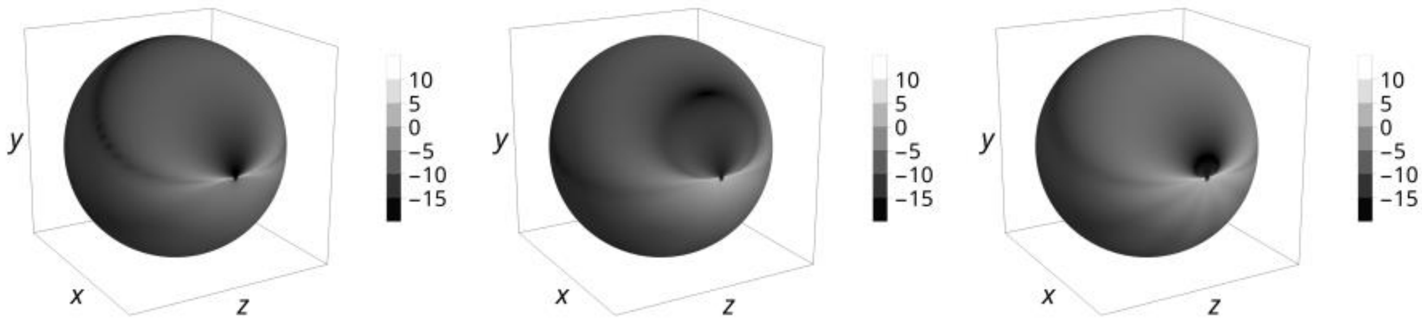
The flux of the radiation is finite at the future null infinity and it is mainly directed toward the motion of the charge (magnetic dipole)

This may result in observable effects for shockwaves passing near pulsars (possible explanation of Fast Radio Bursts)

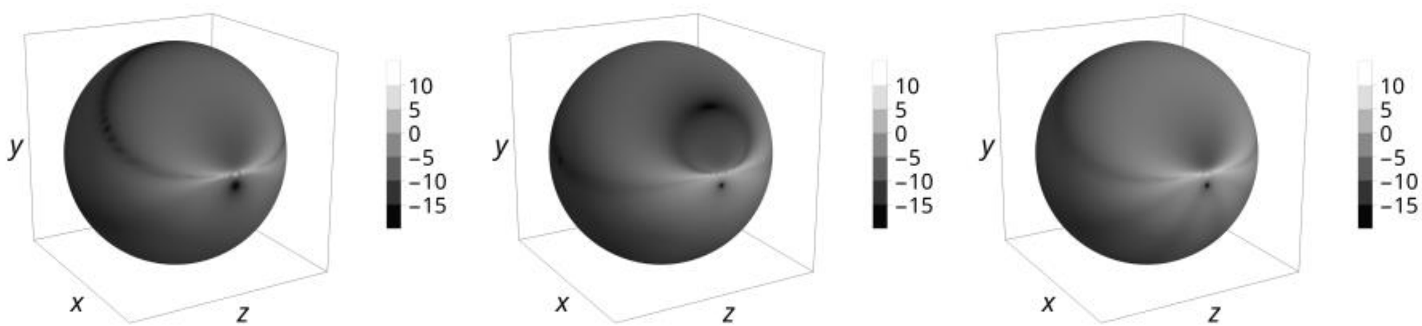
Angular distribution of EM radiation for the case of a magnetic dipole (for shockwave produced by massless object)

$M_x = 1$ $\qquad$ $M_y = 1$ $\qquad$ $M_z = 1$

1. $U/a = 0.01$



2. $U/a = 0.1$



thank you for attention

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