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**Influence of axions and axion-
like particles on momentum and
spin dynamics of Standard
Model particles – New results**

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OUTLINE

- **Dark matter axions: preliminary remarks**
- **Relativistic particle spin dynamics caused by generally accepted interactions of axions and axion-like particles**
- **New manifestations of axion-photon coupling**
- **Summary**



Dark matter axions: preliminary remarks

CP -noninvariant interactions caused by dark matter axions are time-dependent. Like photons, moving axions form a wave which pseudoscalar field reads

$$a(\mathbf{r}, t) = a_0 \cos(E_a t - \mathbf{p}_a \cdot \mathbf{r} + \phi_a).$$

Here $E_a = \sqrt{m_a^2 + \mathbf{p}_a^2}$, \mathbf{p}_a , and m_a are the energy, momentum, and mass of axions. The Earth motion through our galactic define its velocity relative to dark matter, $V \sim 10^{-3}c$. Therefore, $|\mathbf{p}_a| \approx m_a V$ and axions and axion-like particles have momenta of the order of $|\nabla a| \sim 10^{-3} \dot{a} c$.

We suppose that axion-like dark matter interacts like the axion. The Peccei-Quinn theory introduces a new anomalous U(1) symmetry to the Standard Model along with a new pseudoscalar field which spontaneously breaks the symmetry at low energies, giving rise to an axion that suppresses the problematic CP violation.

Strong CP problem and dark matter

QCD Lagrangian:

contains CP violating term:

$$\mathcal{L}_{CP} = -\frac{g^2}{32\pi^2} \Theta \text{Tr} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

Neutron electric dipole moment

$$d_n \approx \Theta 10^{-16} e \cdot \text{cm} < 10^{-25} e \cdot \text{cm}$$

Problem: why so small?

$$\Theta < 10^{-9}$$

Peccei&Quinn'77, Wiczeck'78, Weinberg'78

The tilde denotes a dual tensor

Postulate new global U(1) symmetry - Peccei-Quinn symmetry

Re-interpret Θ as a scalar field a - axion - Nambu-Goldstone boson

$$\mathcal{L}_{CP} = -\frac{g^2}{32\pi^2} \Theta \text{Tr} G_{\mu\nu} \tilde{G}^{\mu\nu} \implies \mathcal{L}_{CP} = -\frac{g^2}{32\pi^2} \frac{a(x)}{f_a} \text{Tr} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

General relativity effects in precision spin experimental tests of fundamental symmetries

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Regular Article - Theoretical Physics

Relativistic spin dynamics conditioned by dark matter axions

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First, we consider spin dynamics caused by generally accepted axion interactions and effects. Second, we analyze new manifestations of axion-photon coupling.

A. J. Silenko, Effective oscillatory magnetic charges and electric dipole moments induced by axion-photon coupling, arXiv:2305.19703

The result of axion-gluon interactions is an oscillating EDM of a strongly interacting particle like a nucleon:

$$\mathcal{L}_{aEDM} = -\frac{i}{2}g_d a \sigma^{\mu\nu} \gamma^5 F_{\mu\nu}$$

where the EDM is equal to $d_a = g_d a$ and g_d is proportional to $g_{a\gamma\gamma}$

The axion-photon interaction leads to mixing of electric and magnetic fields and results in the Lagrangian density

$$\mathcal{L}_\gamma = -\frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu} = g_{a\gamma\gamma} a \mathbf{E} \cdot \mathbf{B}$$

Another contribution to the total Lagrangian density (Pospelov et al.) is defined by the gradient interaction (axion wind effect):

$$\mathcal{L}_N = g_{aNN} \gamma^\mu \gamma^5 \partial_\mu a$$

M. Pospelov, A. Ritz, and M. Voloshin, Phys. Rev. D 78, 115012 (2008); V. A. Dzuba, V. V. Flambaum, and M. Pospelov, Phys. Rev. D 81, 103520 (2010).


$$m_a = (110 \pm 2) \mu\text{eV}$$

C. Beck, Axion mass estimates from resonant Josephson junctions, Phys. Dark Universe 7–8, 6 (2015).

$$(40 \geq m_a \leq 180) \mu\text{eV}$$

M. Buschmann, J. W. Foster, A. Hook, A. Peterson, D. E. Willcox, W. Zhang and B. R. Safdi, Dark matter from axion strings with adaptive mesh refinement, Nature Comm. 13, 1049 (2022).



**Relativistic particle spin
dynamics caused by generally
accepted interactions of axions
and axion-like particles**

The Lagrangian $L = \bar{\psi}\mathcal{L}\psi$ describing electromagnetic interactions of a Dirac particle with allowance for a pseudoscalar axion field is defined by

$$\mathcal{L} = \gamma^\mu (i\hbar\partial_\mu - eA_\mu) - m + \frac{\mu'}{2}\sigma^{\mu\nu}F_{\mu\nu} - i\frac{d}{2}\sigma^{\mu\nu}\gamma^5 F_{\mu\nu} + g_{aNN}\gamma^\mu\gamma^5\Lambda_\mu,$$

$$\Lambda_\mu = \partial_\mu a, \quad \gamma^5 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix},$$

where μ' and d are the anomalous magnetic and electric dipole moments. In the last term, $a = a_0 \cos(m_a t - \mathbf{p}_a \cdot \mathbf{r})$ is the axion field.

The corresponding Hamiltonian in the Dirac representation reads

$$\mathcal{H} = \beta m + \boldsymbol{\alpha} \cdot (\mathbf{p} - e\mathbf{A}) + e\Phi + \mu'(i\boldsymbol{\gamma} \cdot \mathbf{E} - \boldsymbol{\Pi} \cdot \mathbf{B})$$

$$- d(\boldsymbol{\Pi} \cdot \mathbf{E} + i\boldsymbol{\gamma} \cdot \mathbf{B}) - g_{aNN}(\gamma^5\Lambda_0 + \boldsymbol{\Sigma} \cdot \boldsymbol{\Lambda}).$$

The relativistic Foldy-Wouthuysen Hamiltonian has the form

$$\begin{aligned} \mathcal{H}_{FW} &= \mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_3, \\ \mathcal{H}_1 &= \beta\epsilon' + e\Phi - \frac{1}{2} \left\{ \left(\frac{\mu_0 m}{\epsilon'} + \mu' \right), \boldsymbol{\Pi} \cdot \mathbf{B} \right\} \\ &+ \frac{1}{4} \left\{ \left(\frac{\mu_0 m}{\epsilon' + m} + \mu' \right) \frac{1}{\epsilon'}, \left(\boldsymbol{\Sigma} \cdot [\boldsymbol{\pi} \times \mathbf{E}] - \boldsymbol{\Sigma} \cdot [\mathbf{E} \times \boldsymbol{\pi}] - \nabla \cdot \mathbf{E} \right) \right\} \\ &+ \frac{\mu'}{4} \left\{ \frac{1}{\epsilon'(\epsilon' + m)}, \left[(\mathbf{B} \cdot \boldsymbol{\pi})(\boldsymbol{\Pi} \cdot \boldsymbol{\pi}) + (\boldsymbol{\Pi} \cdot \boldsymbol{\pi})(\boldsymbol{\pi} \cdot \mathbf{B}) + 2\boldsymbol{\pi}(\boldsymbol{\pi} \cdot \mathbf{j} + \mathbf{j} \cdot \boldsymbol{\pi}) \right] \right\}, \end{aligned}$$

where \mathcal{H}_1 defines the CP -conserving part of the total Hamiltonian \mathcal{H}_{FW} , $\mu_0 = e\hbar/(2m)$ is the Dirac magnetic moment, $\epsilon' = \sqrt{m^2 + \boldsymbol{\pi}^2}$, and $\mathbf{j} = \frac{1}{4\pi} \left(c \nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} \right)$ is the density of external electric current.

$$\mathcal{H}_2 = -d\mathbf{\Pi} \cdot \mathbf{E} + \frac{d}{4} \left\{ \frac{1}{\epsilon'(\epsilon' + m)}, \left[(\mathbf{E} \cdot \boldsymbol{\pi})(\mathbf{\Pi} \cdot \boldsymbol{\pi}) + (\mathbf{\Pi} \cdot \boldsymbol{\pi})(\boldsymbol{\pi} \cdot \mathbf{E}) \right] \right\} \\ - \frac{d}{4} \left\{ \frac{1}{\epsilon'}, \left(\boldsymbol{\Sigma} \cdot [\boldsymbol{\pi} \times \mathbf{B}] - \boldsymbol{\Sigma} \cdot [\mathbf{B} \times \boldsymbol{\pi}] \right) \right\},$$

The terms describing the direct interaction with the axion field are given by

$$\mathcal{H}_3 = \frac{g_{aNN}}{2} \left\{ \frac{\mathbf{\Pi} \cdot \mathbf{p}}{\epsilon'}, \Lambda_0 \right\} \\ - \frac{g_{aNN}}{2} \left[\left\{ \frac{m}{\epsilon'}, \boldsymbol{\Sigma} \cdot \boldsymbol{\Lambda} \right\} + \frac{(\boldsymbol{\Sigma} \cdot \mathbf{p})}{\epsilon'(\epsilon' + m)} (\mathbf{p} \cdot \boldsymbol{\Lambda}) + (\boldsymbol{\Lambda} \cdot \mathbf{p}) \frac{(\boldsymbol{\Sigma} \cdot \mathbf{p})}{\epsilon'(\epsilon' + m)} \right].$$

In the semiclassical approximation, the angular velocity of the spin rotation has the form

$$\boldsymbol{\Omega} = \boldsymbol{\Omega}_{TBMT} + \boldsymbol{\Omega}_{EDM} + \boldsymbol{\Omega}_{axion},$$

$$\boldsymbol{\Omega}_{TBMT} = -\frac{e}{2m} \left\{ \left(g - 2 + \frac{2}{\gamma} \right) \mathbf{B} - \frac{(g-2)\gamma}{\gamma+1} \boldsymbol{\beta}(\boldsymbol{\beta} \cdot \mathbf{B}) - \left(g - 2 + \frac{2}{\gamma+1} \right) (\boldsymbol{\beta} \times \mathbf{E}) \right\},$$

$$\boldsymbol{\Omega}_{EDM} = -\frac{e\eta}{2m} \left[\mathbf{E} - \frac{\gamma}{\gamma+1} \boldsymbol{\beta}(\boldsymbol{\beta} \cdot \mathbf{E}) + \boldsymbol{\beta} \times \mathbf{H} \right],$$

$$\boldsymbol{\Omega}_{axion} = 2g_{aNN} \left(\Lambda_0 \boldsymbol{\beta} - \frac{\boldsymbol{\Lambda}}{\gamma} - \frac{\gamma}{\gamma+1} (\boldsymbol{\beta} \cdot \boldsymbol{\Lambda}) \boldsymbol{\beta} \right),$$

where $\boldsymbol{\Omega}_{TBMT}$ is determined by the Thomas-Bargmann-Michel-Telegdi equation and the factors $g = 4(\mu_0 + \mu')m/e$ and $\eta = 4dm/e$ are introduced.

The newly added first term in Ω_{axion} is three orders of magnitude larger than the second term. This fact significantly increases an importance of a search for a possible manifestation of the axion field in storage ring experiments.

A. J. Silenko, Relativistic spin dynamics conditioned by dark matter axions, Eur. Phys. J. C 82, 856 (2022).



New manifestations of axion-photon coupling

DYONS OF CHARGE $e\theta/2\pi$ E. WITTEN ¹*CERN, Geneva, Switzerland*

If a non-zero vacuum angle θ is the only mechanism for CP violation, the electric charge of the monopole is exactly calculable and is $-\theta e/2\pi$, plus an integer:

$$q = ne - \theta e/2\pi$$

It has been found much later

ChunJun Cao and A. Zhitnitsky, Axion detection via topological Casimir effect, *Phys. Rev. D* **96**, 015013 (2017);

A. Zhitnitsky, A few thoughts on θ and the electric dipole moments, *Phys. Rev. D* **108**, 076021 (2023).

that magnetic dipole moment μ of any microscopical

configuration in the background of θ_{QED} generates the electric dipole moment $\langle d_{\text{ind}} \rangle$ proportional to θ_{QED} ,

i.e., $\langle d_{\text{ind}} \rangle = -\frac{\theta_{\text{QED}} \alpha}{\pi} \mu$. We also argue that many CP odd correlations such as $\langle \vec{B}_{\text{ext}} \cdot \vec{E} \rangle = -\frac{\alpha \theta_{\text{QED}}}{\pi} \vec{B}_{\text{ext}}^2$ will

be generated in the background of an external magnetic field \vec{B}_{ext} as a result of the same physics.

There is also the new idea to use electric-magnetic duality to motivate the possible existence of non-standard axion couplings, which can both violate the usual quantization rule and exchange the roles of electric and magnetic fields in axion electrodynamics.

In this case, an electrically charged particle acquires also a magnetic charge and becomes a dyon.

We can use this idea and find equations of motion of a particle with electric and magnetic charges and dipole moments (dyon) in electromagnetic fields.

A. J. Silenko, Equation of spin motion for a particle with electric and magnetic charges and dipole moments, Phys. Scr. 99, 085306 (2024).

A. J. Silenko, Effective oscillatory magnetic charges and electric dipole moments induced by axion-photon coupling, arXiv: 2305.19703 [hep-ph] (2023).

■ The distorted Lagrangian density is given by

$$\mathcal{L}' = -\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} = \mathcal{L} + \mathcal{L}_\gamma, \quad \mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$

$$\mathcal{L}_\gamma = -\frac{g_{a\gamma\gamma}}{\Lambda}a(x)F_{\mu\nu}\tilde{F}^{\mu\nu} = g_{a\gamma\gamma}a(x)\mathbf{E} \cdot \mathbf{B},$$

where \mathcal{L} , $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and A_μ are the Lagrangian density, the electromagnetic field tensor, and the four-potential without the axion field. The tilde denotes a dual tensor.

Lagrangians should describe not only the light field but also electric and magnetic fields of other origin.

$$F'_{\mu\nu} = F_{\mu\nu} + \frac{g_{a\gamma\gamma}}{2}a(x)\tilde{F}_{\mu\nu}, \quad F_{\mu\nu} = (\mathbf{E}, \mathbf{B}), \quad \tilde{F}_{\mu\nu} = (\mathbf{B}, -\mathbf{E})$$

$$\mathbf{E}' = \mathbf{E} + \frac{g_{a\gamma\gamma}}{2}a(x)\mathbf{B}, \quad \mathbf{B}' = \mathbf{B} - \frac{g_{a\gamma\gamma}}{2}a(x)\mathbf{E}.$$

Total Lagrangian density without axions:

$$\mathcal{L}_{tot} = \mathcal{L} - A_{\mu}j^{\mu}, \quad j^{\mu} = (\rho, \mathbf{j})$$

Total Lagrangian density with axions:

$$\mathcal{L}'_{tot} = \mathcal{L}' - A'_{\mu}j^{\mu}.$$

Quantum-mechanical equations of motion:

$$\frac{d\boldsymbol{\pi}}{dt} = e\mathbf{E}' + \frac{e}{4} \left\{ \frac{1}{\epsilon}, \left(\boldsymbol{\pi} \times \mathbf{B}' - \mathbf{B}' \times \boldsymbol{\pi} \right) \right\}, \quad \epsilon = \sqrt{m^2 + \boldsymbol{\pi}^2}.$$

$$\frac{d\boldsymbol{\Pi}}{dt} = \frac{1}{2} \left\{ \left(\frac{\mu_0 m}{\epsilon + m} + \mu' \right) \frac{1}{\epsilon}, \left[\boldsymbol{\Pi} \times (\mathbf{E}' \times \boldsymbol{\pi} - \boldsymbol{\pi} \times \mathbf{E}') \right] \right\} \\ + \left\{ \left(\frac{\mu_0 m}{\epsilon} + \mu' \right), [\boldsymbol{\Sigma} \times \mathbf{B}'] \right\}$$

$$- \frac{\mu'}{2} \left\{ \frac{1}{\epsilon(\epsilon + m)}, \left([\boldsymbol{\Sigma} \times \boldsymbol{\pi}](\boldsymbol{\pi} \cdot \mathbf{B}') + (\mathbf{B}' \cdot \boldsymbol{\pi})[\boldsymbol{\Sigma} \times \boldsymbol{\pi}] \right) \right\},$$

where $\mu_0 + \mu' = \mu = eg\hbar s / (2mc)$, μ_0 and μ' are the normal (Dirac) and anomalous magnetic moments, s is the spin number, and $\mathbf{\Pi}$ is the polarization operator.

Maxwell-like equations

$$\partial_\mu F'^{\mu\nu} = 4\pi j^\nu, \quad \partial_\lambda F'_{\mu\nu} + \partial_\mu F'_{\nu\lambda} + \partial_\nu F'_{\lambda\mu} = 0.$$

Maxwell-like equations in vector form read

$$\nabla \cdot \mathbf{E}' = 4\pi\rho, \quad \nabla \cdot \mathbf{B}' = 0.$$

$$\nabla \times \mathbf{E}' = -\frac{\partial \mathbf{B}'}{\partial t}, \quad \nabla \times \mathbf{B}' = 4\pi\mathbf{j} + \frac{\partial \mathbf{E}'}{\partial t}.$$

With the use of unperturbed fields, we obtain

$$\nabla \cdot \mathbf{E} = 4\pi\rho, \quad \nabla \cdot \mathbf{B} = 2\pi g_{a\gamma\gamma} a(x)\rho.$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} - 2\pi g_{a\gamma\gamma} a(x) \mathbf{j} + \frac{g_{a\gamma\gamma}}{2} \dot{a}(x) \mathbf{E},$$

$$\nabla \times \mathbf{B} = 4\pi \mathbf{j} + \frac{\partial \mathbf{E}}{\partial t} + \frac{g_{a\gamma\gamma}}{2} \dot{a}(x) \mathbf{B}.$$

Dual axion electrodynamics

The Lorentz force \mathbf{F} acting on the electric charge e and the Lorentz-like force \mathbf{F}^* acting on the magnetic charge e^* are given by

$$\frac{d\mathbf{p}}{dt} = \mathbf{F} + \mathbf{F}^* = e(\mathbf{E} + \boldsymbol{\beta} \times \mathbf{B}) + e^*(\mathbf{B} - \boldsymbol{\beta} \times \mathbf{E}).$$

Equation of motion:

$$m \frac{du^\mu}{d\tau} = e F^{\mu\nu} u_\nu + e^* \tilde{F}^{\mu\nu} u_\nu$$

$$e^* = \frac{g_{a\gamma\gamma}}{2} a(x) e.$$

Equation for the angular velocity of spin motion:

A. J. Silenko, Relativistic spin dynamics conditioned by dark matter axions, *Eur. Phys. J. C* **82**, 856 (2022).

$$\begin{aligned} \boldsymbol{\Omega} = & -\frac{e}{m} \left[\left(G + \frac{1}{\gamma} \right) \mathbf{B} - \frac{G\gamma}{\gamma+1} (\boldsymbol{\beta} \cdot \mathbf{B}) \boldsymbol{\beta} \right. \\ & \left. - \left(G + \frac{1}{\gamma+1} \right) \boldsymbol{\beta} \times \mathbf{E} \right] + \frac{e^*}{m} \left[\left(G^* + \frac{1}{\gamma} \right) \mathbf{E} \right. \\ & \left. - \frac{G^*\gamma}{\gamma+1} (\boldsymbol{\beta} \cdot \mathbf{E}) \boldsymbol{\beta} + \left(G^* + \frac{1}{\gamma+1} \right) \boldsymbol{\beta} \times \mathbf{B} \right], \end{aligned}$$

where $\boldsymbol{\beta} = \boldsymbol{\pi}/\epsilon$, $\gamma = \epsilon/m$, $G = (g-2)/2$, $g = 2mc\mu/(es)$, $G^* = (g^*-2)/2$, and $g^* = -2mcd/(e^*s)$.

We can propose to study a passage of strongly decelerated electrons or positrons through a solenoid.

Summary

- **The relativistic spin dynamics caused by generally accepted axion interactions has been rigorously described**
- **The direct axion-particle coupling (axion wind effect) results in the spin rotation about the radial axis**
- **The distortion of any electromagnetic field by the axion field takes place. As a result, electric and magnetic fields acquire oscillating magnetic and electric components, respectively. One can also use the equivalent approach based on introducing effective (fictitious) oscillating magnetic charges and EDMs in undistorted electromagnetic fields**

Thank you for your attention

