Dirac Singleton as a $4d$ Field Beyond SM

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Singleton as a Candidate for Dark Matter?!

Singleton $S = Di + Rac$ was discovered as a specific branch of the solutions of certain wave equations that survive at infinity of AdS_4.

P.A.M. Dirac, A Remarkable representation of the 3 + 2 de Sitter group

J. Math. Phys. 4 (1963), 901-909

Later it was realized that S is a free conformal field at the boundary of AdS_4.

Two new issues on the physics of singletons:

- Lorentz covariant field equations for singleton in $(A)dS_4$
- Speculation on its interpretation as dark matter supported by dark energy

Unusual: belongs to an infinite-dimensional IRREP of the Lorentz group.

Related phenomenon: S cannot be localised at a point in the $3d$ space.

From the $4d$ perspective it is nowhere (everywhere).

Unfolded Dynamics

First-order form of differential equations

$$\dot{q}^i(t) = \varphi^i(q(t)) \qquad \text{initial values:} \quad q^i(t_0)$$

Unfolded dynamics: multidimensional generalization

$$\frac{\partial}{\partial t} \to \mathrm{d}\,, \qquad q^i(t) \to W^\Omega(x) = \theta^{\underline{n}_1} \dots \theta^{\underline{n}_p} W^\Omega_{\underline{n}_1 \dots \underline{n}_p}(x)$$

$$\mathrm{d}\mathbf{W}^\Omega(\mathbf{x}) = \mathbf{G}^\Omega(\mathbf{W}(\mathbf{x}))\,, \qquad \mathrm{d} = \theta^{\underline{n}} \partial_{\underline{n}} \qquad \mathbf{MV} \quad \mathbf{1988}$$

$G^\Omega(W)$: **function of "supercoordinates"** W^Ω

$$G^\Omega(W) = \sum_{n=1}^{\infty} f^\Omega{}_{\Phi_1 \dots \Phi_{\underline{n}}} W^{\Phi_1} \dots W^{\Phi_{\underline{n}}}$$

Covariant first-order differential equations

$d > 1$: **Compatibility conditions**

$$G^\Phi(W) \frac{\partial G^\Omega(W)}{\partial W^\Phi} = 0$$

Properties

- General applicability
- Coordinate independence
- Clear group-theoretical interpretation of fields and equations in terms of representations of the symmetry algebra $\mathfrak{s}$
- Local degrees of freedom are in zero-forms $C^i(x_0)$ at any $x = x_0$ (as $q(t_0)$) infinite-dimensional module dual to the space of single-particle states: $C^i(x_0)$ moduli of solutions
- Space-Time Metamorphoses

Key observation: unfolded equation makes sense in any space-time

$$\mathrm{d}W^\Omega(x) = G^\Omega(W(x))\,, \quad x \to X = (x, z)\,, \quad \mathrm{d}_x \to \mathrm{d}_X = \mathrm{d}_x + \mathrm{d}_z\,, \quad \mathrm{d}_z = dz^u \frac{\partial}{\partial z^u}$$

X-dependence is reconstructed in terms of fields $W^\Omega(X_0) = W^\Omega(x_0, z_0)$ at any X_0. To take $W^\Omega(x_0, z_0)$ in space M_X with coordinates X_0 is the same as to take $W^\Omega(x_0)$ in the space $M_x \in M_X$ with coordinates x

Holography

Boundary conformal currents are dual to fields in the bulk AdS_d

Holography is based on the isomorphism of the boundary conformal group $O(d, 2)$ with the symmetry of AdS_{d+1}.

The correspondence is based on the representation theory

A version of this phenomenon was presented in the paper
Gelfond, MV JETP 120 (2015) dedicated to Valery Rubakov at 60

Today I will do the opposite: start from the boundary conformal field to see what is its bulk dual. The result will be interesting both formallyr and, hopefully, from physics point of view.

Singleton Field Equations

Singleton is described as a field in AdS_4 with auxiliary variables y^+_α, y^-_β

$$|\phi(y^+|x)\rangle = \phi(y^+|x)|0\rangle$$

$$[y_\alpha\,, y^+_\beta] = \varepsilon_{\alpha\beta}\,, \qquad y_-|0\rangle = 0\,.$$

Field equations are

$$D|\phi\rangle = 0\,, \qquad \phi(y^+|x) = \sum_{n=0}^{\infty} \frac{1}{n!}\phi_{\alpha_1\ldots\alpha_n}(x) y^{+\alpha_1}\ldots y^{+\alpha_n}$$

where

$$D = \mathsf{d}_x + \frac{i}{z} d\mathbf{x}^{\alpha\beta} y^-_\alpha y^-_\beta - \frac{dz}{2z} y^-_\alpha y^{+\alpha}\,, \qquad \mathsf{d}_x := dx^{\alpha\dot\beta}\frac{\partial}{\partial x^{\alpha\dot\beta}}$$

$$x^{\alpha\dot\alpha} = (\mathbf{x}^{\alpha\dot\alpha}, -\frac{i}{2}\epsilon^{\alpha\dot\alpha} z^{-1})\,,$$

AdS_4 connection in Poincaré coordinates

$$e^{\alpha\dot\alpha} = \frac{1}{2z} dx^{\alpha\dot\alpha}\,, \qquad \omega^{\alpha\beta} = -\frac{i}{4z} d\mathbf{x}^{\alpha\beta}\,, \qquad \bar\omega^{\dot\alpha\dot\beta} = \frac{i}{4z} d\mathbf{x}^{\dot\alpha\dot\beta}\,.$$

How to observe?

The field equations are three-dimensional rather than four-dimensional

$$\frac{\partial^2}{\partial x^{\alpha\beta}\partial x_{\alpha\beta}}\phi(x) = 0.$$

Direct scattering is unlikely observable. But

Flato-Fronsdal Thm:

$$S \bigotimes S = \sum_{s=0}^{\infty} \phi_{s,m=0}(x) = graviton + \ldots$$

implying in particular, that bilinears of singletons contain graviton, that may have direct observable consequences via additional induced gravitational field.

To evaluate the effect one has to introduce interactions. That S admits a Lorentz covariant formulation allows one to introduce interactions with gravity via usual covariantization of derivatives

Important comment: To be dynamically active singleton should live in the $(A)dS$ space. In other words all this may only work in presence of dark energy.

Conclusion

New type of relativistic matter in presence of dark energy

Interesting to explore in the context of long-standing problems including dark matter and even baryon asymmetry induced by the appropriately charged singletons.