$\mathcal{N} = 2$ AdS higher spins from the harmonic approach

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Supersymmetry and higher spins

- Supersymmetric higher-spin theories provide a bridge between superstring theory and low-energy (super)gauge theories.
- The component approach to $4D$, $\mathcal{N}=1$ supersymmetric free massless higher spin models: Courtright, 1979; Vasiliev, 1980.
- The complete off-shell $\mathcal{N}=1$ superfield Lagrangian formulation of $\mathcal{N}=1, 4D$ free higher spins: Kuzenko et al, 1993, 1994.
- An off-shell superfield Lagrangian formulation for higher-spin **extended** supersymmetric theories, with all supersymmetries manifest, was unknown for long even for free theories.
- This gap was filled in I. Buchbinder, E. Ivanov, N. Zaigraev, 2021, 2022, 2023. An off-shell manifestly $\mathcal{N}=2$ supersymmetric unconstrained formulation of $4D, \mathcal{N}=2$ integer higher spins and their couplings to hypermultiplets was constructed in the harmonic superspace approach.
- Quite recently, we generalized the HSS non-conformal construction to $\mathcal{N}=2$ superconformal multiplets and their hypermultiplet coupling (arXiv:2404.19016 [hep-th], JHEP 08 (2024) 120).

Harmonic superspace

- In $4D$, the only self-consistent off-shell superfield formalism for $\mathcal{N}=2$ (and $\mathcal{N}=3$) theories is the harmonic superspace approach (Galperin, Ivanov, Kalitzin, Ogievetsky, Sokatchev, 1984, 1985).

- Harmonic $\mathcal{N}=2$ superspace:

$$Z=(x^m\,,\ \theta_i^\alpha\,,\ \bar\theta^{\dot\alpha j}, u^{\pm i}),\quad u^{\pm i}\in SU(2)/U(1),\ u^{+i}u_i^-=1$$

- Analytic harmonic $\mathcal{N}=2$ superspace:

$$\zeta_A=(x_A^m,\theta^{+\alpha},\bar\theta^{+\dot\alpha},u^{\pm i}),\ \theta^{+\alpha,\dot\alpha}:=\theta^{\alpha,\dot\alpha i}u_i^+,\ x_A^m:=x^m-2i\theta^{(i}\sigma^m\bar\theta^{j)}u_i^+u_j^+$$

- All basic $\mathcal{N}=2$ superfields are analytic:

$$\underline{\text{SYM}}:\qquad V^{++}(\zeta_A)\,,\ \ \underline{\text{matter hypermultiplets}}:\ q^+(\zeta_A)\,,\ \bar q^+(\zeta_A)$$

$$\underline{\text{supergravity}}:\qquad H^{++m}(\zeta_A)\,,\ H^{++\alpha+}(\zeta_A)\,,\ H^{++5}(\zeta_A)\,,\hat\alpha=(\alpha,\dot\alpha)$$

- The general case of $\mathcal{N}=2$ gauge multiplet with the maximal spin **s** is spanned by the analytic gauge potentials

$$h^{++\alpha(s-1)\dot{\alpha}(s-1)}(\zeta),\, h^{++\alpha(s-2)\dot{\alpha}(s-2)}(\zeta),\, h^{++\alpha(s-1)\dot{\alpha}(s-2)+}(\zeta),\, h^{++\dot{\alpha}(s-1)\alpha(s-2)+}(\zeta),$$

where $\alpha(s) := (\alpha_1 \ldots \alpha_s), \dot{\alpha}(s) := (\dot{\alpha}_1 \ldots \dot{\alpha}_s)$

- The relevant gauge transformations can be defined and shown to leave, in the WZ-like gauge, the physical field multiplet $(\mathbf{s}, \mathbf{s}-\mathbf{1/2}, \mathbf{s}-\mathbf{1/2}, \mathbf{s}-\mathbf{1})$.

- The on-shell spin contents of $\mathcal{N}=2$ higher-spin multiplets:

$$\underline{spin\ 1}:\ 1,\ (1/2)^2,\ (0)^2$$

$$\underline{spin\ 2}:\ 2,\ (3/2)^2,\ 1$$

$$\underline{spin\ 3}:\ 3,\ (5/2)^2,\ 2$$

....

$$\underline{spin\ s}:\ s,\ (s-1/2)^2,\ s-1$$

- The off-shell contents of the spin **s** multiplet: $8[\mathbf{s}^2 + (\mathbf{s}-\mathbf{1})^2]_B + 8[\mathbf{s}^2 + (\mathbf{s}-\mathbf{1})^2]_F$.

Hypermultiplet couplings

- Supersymmetric $\mathcal{N} = 1$ generalizations of the bosonic cubic vertices with matter were explored in terms of $\mathcal{N} = 1$ superfields by Gates, Koutrolikos, Kuzenko, I. Buchbinder, E. Buchbinder and many others.
- In JHEP 05 (2022) 104 we have constructed the off-shell manifestly $\mathcal{N} = 2$ supersymmetric cubic couplings $(\frac{1}{2}, \frac{1}{2}, \mathbf{s})$ of an arbitrary gauge $\mathcal{N} = 2$ multiplet with higher integer spin **s** to the hypermultiplet matter in $4D, \mathcal{N} = 2$ harmonic superspace.
- In our approach $\mathcal{N} = 2$ supersymmetry of cubic vertices is always manifest and off-shell, in contrast, e.g., to the non-manifest light-cone formulations.

- The starting point is the $\mathcal{N}=2$ hypermultiplet off-shell free action:

$$S = \int d\zeta^{(-4)}\, \mathcal{L}^{+4}_{free} = -\int d\zeta^{(-4)}\, \frac{1}{2} q^{+a} \mathcal{D}^{++} q^{+}_{a}\,, a = 1, 2$$

- Analytic gauge potentials for any spin **s** with the correct transformation rules are recovered by proper gauge-covariantization of the harmonic derivative $\mathcal{D}^{++}$. The simplest option is gauging of $U(1)$,

$$\delta q^{+a} = -\lambda_0 J q^{+a}, \quad J q^{+a} = i(\tau_3)^a{}_b q^{+b}\,,$$
$$\mathcal{D}^{++} \Rightarrow \mathcal{D}^{++} + \hat{\mathcal{H}}^{++}_{(1)}\,, \quad \hat{\mathcal{H}}^{++}_{(1)} = h^{++} J\,,$$
$$\delta_\lambda \hat{\mathcal{H}}^{++}_{(1)} = [\mathcal{D}^{++}, \hat{\Lambda}]\,, \quad \hat{\Lambda} = \lambda J \Rightarrow \delta_\lambda h^{++} = \mathcal{D}^{++}\lambda$$

- In $\mathcal{N}=2$ supergravity, that is for $\mathbf{s}=2$,

$$S_{(2)} = -\int d\zeta^{(-4)}\, \frac{1}{2} q^{+a}\left(\mathcal{D}^{++} + \mathcal{H}_{(2)}\right) q^{+}_{a}\,, \quad \delta\mathcal{H}_{(2)} = [\mathcal{D}^{++}, \hat{\Lambda}_{(2)}],$$
$$\mathcal{H}_{(2)} = h^{++M}(\zeta)\partial_M,\ \hat{\Lambda}_{(2)} = \lambda^M(\zeta)\partial_M,\ M := (\alpha\dot{\beta}, 5, \hat{\mu}+)$$

- For higher **s** everything goes analogously. For $\mathbf{s}=3$

$$S_{(3)} = -\int d\zeta^{(-4)}\, \frac{1}{2} q^{+a}\left(\mathcal{D}^{++} + \mathcal{H}_{(3)} J\right) q^{+}_{a}\,,$$
$$\delta\mathcal{H}_{(3)} = [\mathcal{D}^{++}, \hat{\Lambda}_{(3)}], \quad \mathcal{H}_{(3)} = h^{++\alpha\dot{\alpha}M}(\zeta)\partial_M\partial_{\alpha\dot{\alpha}}, \quad \hat{\Lambda}_{(3)} = \lambda^{\alpha\dot{\alpha}M}(\zeta)\partial_M\partial_{\alpha\dot{\alpha}}$$

Superconformal couplings

- Free conformal higher-spin actions in $4D$ Minkowski space were pioneered by Fradkin & Tseytlin, 1985; Fradkin & Linetsky, 1989, 1991. Since then, a lot of works on (super)conformal higher spins appeared.

- (Super)conformal higher-spin theories are considered as a basis for all other types of higher-spin models. Non-conformal ones follow from the superconformal ones through couplings to the superfield compensators.

- In (Buchbinder, Ivanov, Zaigraev, arXiv:2404.19016 [hep-th]), we extended the off-shell $\mathcal{N}=2, 4D$ higher spins and their hypermultiplet couplings to the superconformal case. Rigid $\mathcal{N}=2, 4D$ superconformal symmetry plays a crucial role in fixing the structure of the theory.

- $\mathcal{N}=2, 4D$ SCA preserves harmonic analyticity and is a closure of the rigid $\mathcal{N}=2$ supersymmetry and special conformal symmetry

$$\delta_\epsilon \theta^{+\hat{\alpha}} = \epsilon^{\hat{\alpha}i} u^+_i\,, \;\; \delta_\epsilon x^{\alpha\dot{\alpha}} = -4i\left(\epsilon^{\alpha i}\bar{\theta}^{+\dot{\alpha}} + \theta^{+\alpha}\bar{\epsilon}^{\dot{\alpha}i}\right) u^-_i\,, \hat{\alpha} = (\alpha, \dot{\alpha})\,,$$

$$\delta_k \theta^{+\alpha} = x^{\alpha\dot{\beta}} k_{\beta\dot{\beta}} \theta^{\hat{\beta}}\,, \;\; \delta_k x^{\alpha\dot{\alpha}} = x^{\rho\dot{\alpha}} k_{\rho\dot{\rho}} x^{\dot{\rho}\alpha}\,, \;\; \delta_k u^{+i} = (4i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}} k_{\alpha\dot{\alpha}}) u^{-i}$$

- What about the conformal properties of various analytic higher-spin potentials? No problems with the spin **1** potential V^{++}:

$$\delta_{sc} V^{++} = -\hat{\Lambda}_{sc} V^{++}\,, \quad \hat{\Lambda}_{sc} := \lambda^{\alpha\dot{\alpha}}_{sc}\partial_{\alpha\dot{\alpha}} + \lambda^{\hat{\alpha}+}_{sc}\partial_{\hat{\alpha}+} + \lambda^{++}_{sc}\partial^{--}$$

- The cubic vertex $\sim q^{+a}V^{++}Jq^{+}_{a}$ is invariant up to total derivative if

$$\delta_{sk} q^{+a} = -\hat{\Lambda}_{sc} q^{+a} - \frac{1}{2}\Omega q^{+a}\,, \quad \Omega := (-1)^{P(M)}\partial_M \lambda^M$$

- Situation gets more complicated for $\mathbf{s} \geq 2$. Requiring $\mathcal{N} = 2$ gauge potentials for $\mathbf{s} = 2$ to be closed under $\mathcal{N} = 2$ SCA necessarily leads to

$$\mathcal{D}^{++} \to \mathcal{D}^{++} + \kappa_2 \hat{\mathcal{H}}^{++}_{(s=2)}\,,$$
$$\hat{\mathcal{H}}^{++}_{(s=2)} := h^{++M}\partial_M = h^{++\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}} + h^{++\alpha+}\partial^{-}_{\alpha} + h^{++\dot{\alpha}+}\partial^{-}_{\dot{\alpha}} + h^{(+4)}\partial^{--}$$
$$\delta_{k_{\alpha\dot{\alpha}}} h^{(+4)} = -\hat{\Lambda} h^{(+4)} + 4i h^{++\alpha+}\bar{\theta}^{+\dot{\alpha}} k_{\alpha\dot{\alpha}} + 4i\theta^{+\alpha} h^{++\dot{\alpha}+} k_{\alpha\dot{\alpha}}$$

For ensuring conformal covariance, it is imperative to introduce the extra potential $h^{(+4)}$. The extended set of potentials embodies $\mathcal{N} = 2$ **Weyl multiplet** ($\mathcal{N} = 2$ conformal SG gauge multiplet).

- For $\mathbf{s} \geq 3$ the gauge-covariantization of the free q^{+a} action requires adding the gauge superfield differential operators of rank $\mathbf{s-1}$ in ∂_M,

$$\mathcal{D}^{++} \to \mathcal{D}^{++} + \kappa_s \hat{\mathcal{H}}^{++}_{(s)} (J)^{P(s)}, \quad P(s) = \frac{1 + (-1)^{s-1}}{2}$$

- For $\mathbf{s} = 3$:

$$\hat{\mathcal{H}}_{(s=3)} = h^{++MN}\partial_N\partial_M + h^{++}, \quad h^{++MN} = (-1)^{P(M)P(N)} h^{++NM}$$

- $\mathcal{N} = 2$ SCA mixes different entries of h^{++MN}, so we need to take into account all these entries, as distinct from non-conformal case where it was enough to consider, e.g., $h^{++\alpha\dot{\alpha}M}$.
- All the potentials except $h^{++\alpha\dot{\alpha}M}$ can be put equal to zero using the original extensive gauge freedom:

$$S^{(s=3)}_{int|fixed} = -\frac{\kappa_3}{2} \int d\zeta^{(-4)}\, q^{+a} h^{++\alpha\dot{\alpha}M} \partial_M \partial_{\alpha\dot{\alpha}} J q^+_a \qquad (1)$$

- In such a gauge the superconformal transformations are accompanied by the proper compensating gauge transformations in order to preserve the gauge, so the final SC transformations are nonlinear in $h^{++M\alpha\dot{\alpha}}$.
- The whole consideration can be generalized to the general integer higher-spin s case: $8(2s-1)_B + 8(2s-1)_F$ d.o.f. off shell.

On AdS background

- It is most interesting to explicitly construct $\mathcal{N}=2$ higher spins in the AdS background, with the superconformal symmetry $SU(2,2|2)$ being broken to the AdS supersymmetry $OSp(2|4;R)$. The latter involves the spinor generators $\Psi^i_\alpha, \Psi^j_{\dot\alpha}$, the Lorentz $SO(1,3)$ generators $L_{(\alpha\beta)}, \bar{L}_{(\dot\alpha\dot\beta)}$, nonlinear $SO(2,3)/SO(1,3)$ translation generators $R_{\alpha\dot\beta}$ and the internal $SO(2)$ symmetry generator T

$$\{\Psi^i_\alpha, \Psi^j_\beta\} \sim (L, T) \qquad \{\Psi^i_\alpha, \bar\Psi^j_{\dot\beta}\} \sim \varepsilon^{ij} R_{\alpha\dot\beta}$$

- The embedding of the $\mathcal{N}=2$ AdS superalgebra into $SU(2,2|2)$ is realized through the identification (Bandos, Ivanov, Lukierski, Sorokin, 2002)

$$\Psi^i_\alpha = Q^i_\alpha + c^{ik} S_{k\alpha}, \quad \bar\Psi^i_{\dot\alpha} = \overline{\Psi^i_\alpha} = \bar{Q}_{\dot\alpha i} + c_{ik}\bar{S}^k_{\dot\alpha},$$
$$c^{ik} = c^{ki} \qquad \overline{c^{ik}} = c_{ik} = \varepsilon_{il}\varepsilon_{kj} c^{lj}$$

- The $SU(2,2|2)$ commutation relations imply for super AdS generators

$$\{\Psi^i_\alpha, \Psi^k_\beta\} = c^{ik} L_{(\alpha\beta)} + 4i\varepsilon_{\alpha\beta}\varepsilon^{ik} T, \quad T := c_{lm} T^{lm}, \quad [J, \Psi^i_\alpha] \sim c^{ik}\Psi_{k\alpha},$$
$$\{\Psi^i_\alpha, \bar\Psi_{\dot\beta k}\} = 2\delta^i_k R_{\alpha\dot\beta}, \quad R_{\alpha\dot\beta} = P_{\alpha\dot\beta} + \frac{1}{2}c^2 K_{\alpha\dot\beta}, \quad c^2 \sim \frac{1}{R^2_{AdS}},$$
$$[R_{\alpha\dot\alpha}, R_{\gamma\dot\gamma}] \sim c^2\left(\varepsilon_{\alpha\gamma} L_{\dot\alpha\dot\gamma} + \varepsilon_{\dot\alpha\dot\gamma} L_{\alpha\gamma}\right), \quad [R_{\alpha\dot\beta}, \Psi^i_\beta] \sim \varepsilon_{\alpha\beta}\bar\Psi^i_{\dot\beta} \text{ (and c.c.)}$$

- The super AdS transformation properties of analytic coordinates:

$$\delta u^{+i} = u^{-i}\big[u^+_k c^{kl}(\epsilon_{\alpha l}\theta^{+\alpha} + \bar\epsilon_{\dot\alpha l}\bar\theta^{+\dot\alpha})\big],$$
$$\delta x^{\alpha\dot\alpha} = -4i\, u^-_i\left[\epsilon^{\alpha i}\bar\theta^{+\dot\alpha} + \theta^{+\alpha}\bar\epsilon^{\dot\alpha i} - c^{ik}\left(x^{\alpha\dot\beta}\bar\epsilon_{\dot\beta k}\bar\theta^{+\dot\alpha} + x^{\beta\dot\alpha}\epsilon_{\beta k}\theta^{+\alpha}\right)\right],$$
$$\delta\theta^{+\alpha} = \left(\epsilon^{\alpha i} - x^{\alpha\dot\alpha}c^{ik}\bar\epsilon_{\dot\alpha k}\right)u^+_i - 2i(\theta^+)^2 c^{ki}\epsilon^\alpha_k u^-_i\,,$$
$$\delta\bar\theta^{+\dot\alpha} = \left(\bar\epsilon^{\dot\alpha i} + x^{\alpha\dot\alpha}c^{ik}\epsilon_{\alpha k}\right)u^+_i + 2i(\bar\theta^+)^2 c^{ik}\bar\epsilon^{\dot\alpha}_k u^-_i\,.$$

- The nonlinear AdS translations:

$$\delta x^{\alpha\dot\alpha} = a^{\alpha\dot\alpha} + \frac{1}{2}c^2 a_{\beta\dot\beta}x^{\alpha\dot\beta}x^{\beta\dot\alpha} = a^{\alpha\dot\alpha}\left(1 - \frac{1}{4}c^2x^2\right) + \frac{1}{2}c^2(ax)x^{\alpha\dot\alpha}\,,$$
$$\delta\theta^{+\alpha} = \frac{1}{2}c^2 a_{\beta\dot\alpha}\theta^{+\beta}x^{\alpha\dot\alpha}\,,\quad \delta\bar\theta^{+\dot\alpha} = \frac{1}{2}c^2 a_{\beta\dot\beta}\bar\theta^{+\dot\beta}x^{\beta\dot\alpha}\,,$$
$$\delta u^{+i} = \frac{1}{2}c^2 u^{-i}a_{\alpha\dot\alpha}\theta^{+\alpha}\bar\theta^{+\dot\alpha}\,.$$

- The T transformations (with parameter ω) can also be easily found

$$\delta u^{+i} = u^{-i}c^{++}\omega,\quad \delta x^{\alpha\dot\alpha} = 4ic^{--}\bar\theta^{+\dot\alpha}\theta^{+\alpha}\omega,\quad \delta\theta^{+\alpha} = c^{+-}\theta^{+\alpha}\omega\,,$$
$$c^{\pm\pm} = c^{ik}u^\pm_i u^\pm_k\,,\quad c^{+-} = c^{ik}u^+_i u^-_k\,.$$

- The first step toward constructing an off-shell $\mathcal{N}=2$ AdS higher spin theory (Ivanov & Zaigraev, in progress) is to define the super AdS invariant Lagrangian of hypermultiplet, such that it respect no full superconformal invariance, but only the super AdS one. To this end, one needs to define the AdS covariant version of the analyticity-preserving harmonic derivative $\mathcal{D}^{++}$. One way to find it is to require its commutativity with the super AdS generators acting on the analytic harmonic coordinates. Without enering into details, the appropriate $\mathcal{D}^{++}_{AdS}$ acting on $q^{+a}=(q^+,\tilde{q}^+)$ has the structure

$$\mathcal{D}^{++}_{AdS} = \partial^{++} - 4i\hat{\theta}^{+\alpha}\hat{\bar{\theta}}^{+\dot{\alpha}}\nabla_{\alpha\dot{\alpha}} + h^{++}\hat{T} + \mathcal{O}(c)$$

$$\nabla_{\alpha\dot{\alpha}} = (1+c^2x^2)\partial_{\alpha\dot{\alpha}}, \quad h^{++} = i\big[(\hat{\theta}^+)^2 - (\hat{\bar{\theta}}^+)^2\big] + \mathcal{O}(c),$$

$$\hat{T}\,(q^+,\,\tilde{q}^+) = (q^+,\,-\tilde{q}^+)\,,$$

  where $\hat{\theta}^+_\alpha, \hat{\bar{\theta}}^+_{\dot{\alpha}}$ are some redefinitions of the original Grassmann coordinates and $\mathcal{O}(c)$ stand for terms vanishing in the limit $c^{ik}\to 0$.
- An extra term $\sim \hat{T}$ in $\mathcal{D}^{++}_{AdS}$ is necessary for breaking superconformal invariance and it produces a mass of q^+ proportional to $1/R^2_{AdS}$. In the properly defined flat limit this term becomes the central charge extension of flat D^{++} and $\hat{T}$ goes just into the derivative ∂_5.
- More details on the AdS invariant q^+ Lagrangians, including interactions with superspins $\mathbf{s}\geq 3$, will be given in my work with Nikita Zaigraev.

Summary and outlook

The theory of $\mathcal{N} = 2$ supersymmetric higher spins $s \geq 3$ opens a new direction of applications of the harmonic superspace approach which earlier proved to be indispensable for description of more conventional $\mathcal{N} = 2$ theories with maximal spins $s \leq 2$. Once again, the basic property underlying these new higher-spin theories is the preservation of harmonic Grassmann analyticity (all basic gauge potentials are unconstrained analytic superfields involving an infinite number of degrees of freedom off shell, before fixing WZ-type gauges). The theory of conformal higher spins already embodies that of AdS higher spins because the supergroup underlying the latter is a subgroup of $\mathcal{N} = 2$ superconformal group.

Under way:

- The linearized actions of conformal higher-spin $\mathcal{N}=2$ multiplets ($\mathcal{N}=2$ analogs of the square of Weyl tensor) and their AdS descendants.

- Quantization, induced actions,...

- $\mathcal{N}=2$ supersymmetric half-integer spins? (talk by N. Zaigraev at this Session).

- From the linearized theory to its full nonlinear version? At present, the latter is known only for $s \leq 2$ ($\mathcal{N}=2$ super Yang - Mills and $\mathcal{N}=2$ supergravities). This problem will seemingly require accounting for **ALL** higher $\mathcal{N}=2$ superspins simultaneously. New supergeometries?

THANK YOU FOR YOUR ATTENTION!