Смешанная неоднородная фаза во вращающейся кварк-глюонной плазме
(Mixed inhomogeneous phase in rotating quark-gluon plasma)

Artem Roenko[1],

in collaboration with

V. V. Braguta, M. N. Chernodub, Ya. A. Gershtein A. Yu. Kotov, I. E. Kudrov, D. A. Sychev

[1]Joint Institute for Nuclear Research (JINR), Dubna
roenko@theor.jinr.ru

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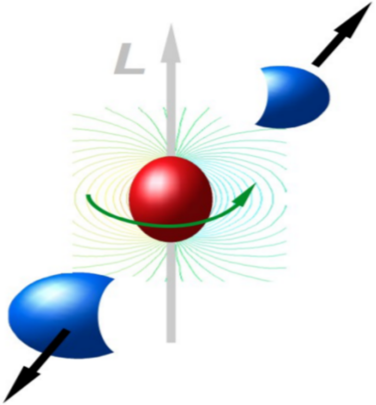
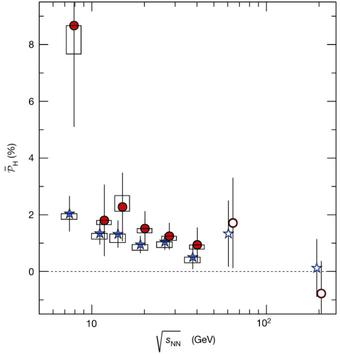
- The rotation occurs with relativistic velocities.



$\omega = 10$ MeV ~ 0.05 fm^{-1} $\quad v \sim c$ at $r \sim 20$ fm



[L. Adamczyk et al. (STAR), Nature **548**, 62–65 (2017), arXiv:1701.06657 [nucl-ex]]

$\langle\omega\rangle \sim 7$ MeV $\quad (\sqrt{s_{NN}}$-averaged)

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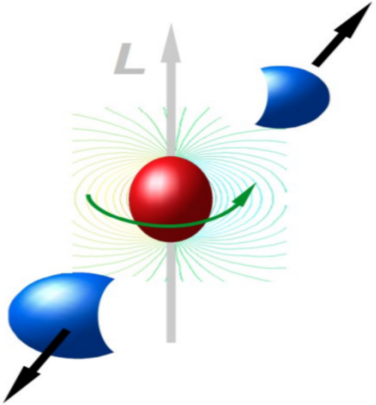

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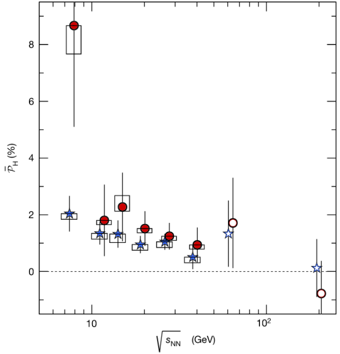

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- How does the rotation affect QCD properties?

Lattice study of rotating QCD properties

Formulation of rotating QCD on the lattice

- A. Yamamoto and Y. Hirono, Phys. Rev. Lett. **111**, 081601 (2013), arXiv:1303.6292 [hep-lat]

Bulk-averaged critical temperature in rotating gluodynamics:

- V. Braguta, A. Kotov, D. Kuznedelev, and A. Roenko, JETP Lett. **112**, 6–12 (2020)
- V. Braguta, A. Kotov, D. Kuznedelev, and A. Roenko, Phys. Rev. D **103**, 094515 (2021), arXiv:2102.05084 [hep-lat]

Rotating gluodynamics in laboratory frame ($\Omega_I = \pi/(2T)$):

- M. N. Chernodub, V. A. Goy, and A. V. Molochkov, Phys. Rev. D **107**, 114502 (2023), arXiv:2209.15534 [hep-lat]

Bulk-averaged critical temperature in rotating QCD:

- V. Braguta, A. Kotov, A. Roenko, and D. Sychev, PoS **LATTICE2022**, 190 (2023), arXiv:2212.03224 [hep-lat]
- J.-C. Yang and X.-G. Huang, (2023), arXiv:2307.05755 [hep-lat]

Thermodynamical properties and moment of inertia of rotating gluon plasma:

- V. V. Braguta, M. N. Chernodub, A. A. Roenko, and D. A. Sychev, Phys. Lett. B **852**, 138604 (2024), arXiv:2303.03147 [hep-lat]
- V. V. Braguta et al., JETP Lett. **117**, 639–644 (2023)
- V. V. Braguta et al., Phys. Rev. D **110**, 014511 (2024), arXiv:2310.16036 [hep-ph]

Mixed inhomogeneous phase in rotating gluon plasma:

- V. V. Braguta, M. N. Chernodub, and A. A. Roenko, Phys. Lett. B **855**, 138783 (2024), arXiv:2312.13994 [hep-lat]
- V. V. Braguta, M. N. Chernodub, Y. A. Gershtein, and A. A. Roenko, (2024), arXiv:2411.15085 [hep-lat]

It is convenient to describe the system in the co-rotating reference frame, $x^\mu = (t, x, y, z)$,

$$\varphi = [\varphi_{\text{lab}} - \Omega t]_{2\pi}\,, \quad t = t_{\text{lab}}, \quad z = z_{\text{lab}}, \quad r = r_{\text{lab}}, \tag{1}$$

with the metric

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{2}$$

The Dirac Lagrangian in curved space is given by

$$\mathcal{L}_\psi = \bar{\psi}\left(i\gamma^\mu(D_\mu + \Gamma_\mu) - m\right)\psi = \mathcal{L}_\psi^{(0)} + \mathcal{L}_\psi^{(1)}\,, \tag{3}$$

and the Lagrangian of Yang-Mills theory in the Minkowski curved spacetime is

$$\mathcal{L}_G = -\frac{1}{4g_{YM}^2} g^{\mu\nu} g^{\alpha\beta} F^a_{\mu\alpha} F^a_{\nu\beta} = \mathcal{L}_G^{(0)} + \mathcal{L}_G^{(1)} + \mathcal{L}_G^{(2)}\,, \tag{4}$$

where $\mathcal{L}^{(n)} \propto \Omega^n$, and $\Omega = \partial_t \varphi_{\text{lab}}$.

Rotating QCD in Minkowksi space

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where $\mathcal{L}^{(n)} \propto \Omega^n$, and $\Omega = \partial_t \varphi_{\text{lab}}$. The causality restriction: $\boxed{\Omega r < 1}$.

Rotating QCD in Euclidean space

The rotating system at thermal equilibrium is studied on the lattice. The partition function is

$$\mathcal{Z} = \mathrm{Tr}\left[e^{-\hat{H}/T_0}\right] = \int \mathcal{D}[U]\,\mathcal{D}[\psi,\bar{\psi}]\; e^{-S_G[U,\Omega]-S_F[U,\psi,\bar{\psi},\Omega]}, \tag{5}$$

where the Euclidean action, $S_G + S_F$, is formulated in curved space $(t \to -i\tau)$, $x^\mu = (x, y, z, \tau)$,

$$g^E_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & -y\Omega_I \\ 0 & 1 & 0 & x\Omega_I \\ 0 & 0 & 1 & 0 \\ -y\Omega_I & x\Omega_I & 0 & 1 + r^2\Omega_I^2 \end{pmatrix}, \tag{6}$$

and the angular velocity is imaginary, $\Omega_I = \partial_\tau \varphi_{\mathrm{lab}} = -i\partial_t \varphi_{\mathrm{lab}} = -i\Omega$, to avoid the sign problem.

There is no causality restriction in Euclidean space.

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There is no causality restriction in Euclidean space.

- The inverse temperature $1/T_0$ sets the system length in τ-direction.
- Ehrenfest–Tolman (TE) law: the local temperature *depends on the coordinates*

$$T(r)\sqrt{g_{00}} = T(r)\sqrt{1 - r^2\Omega^2} = T(r)\sqrt{1 + r^2\Omega_I^2} = T_0\,.$$

- We denote by $T \equiv T_0$ the temperature at the rotation axis ($r = 0$).

Rotating QCD in Euclidean space

The quark action is a linear function in angular velocity:

$$S_F = \int d^4x \sqrt{g_E}\, \bar{\psi} \left(\gamma^\mu (\partial_\mu + \Gamma_\mu) + m\right) \psi =$$

$$= \int d^4x\, \bar{\psi} \Bigg(\left(\gamma^1 + y\Omega_I \gamma^4\right) D_x + \left(\gamma^2 - x\Omega_I \gamma^4\right) D_y + \gamma^3 D_z + \gamma^4 \left(D_\tau + i\Omega_I \frac{\sigma^{12}}{2} \right) + m \Bigg) \psi\,, \quad (7)$$

The gluon action is a quadratic function in angular velocity:

$$S_G = \frac{1}{4g_{YM}^2} \int d^4x \sqrt{g_E}\, g_E^{\mu\nu} g_E^{\alpha\beta} F^a_{\mu\alpha} F^a_{\nu\beta} \equiv S_0 + S_1 \Omega_I + S_2 \frac{\Omega_I^2}{2} =$$

$$= \frac{1}{g_{YM}^2} \int d^4x \Bigg(\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \Omega_I \left[-y F^a_{xy} F^a_{y\tau} - y F^a_{xz} F^a_{z\tau} + x F^a_{yx} F^a_{x\tau} + x F^a_{yz} F^a_{z\tau} \right] +$$

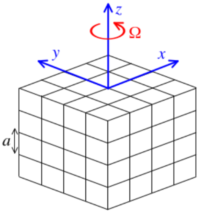
$$+ \Omega_I^2 \left[r^2 (F^a_{xy})^2 + y^2 (F^a_{xz})^2 + x^2 (F^a_{yz})^2 + 2xy F^a_{xz} F^a_{zy} \right] \Bigg) \quad (8)$$

So, for quarks $\mathcal{L}^{(1)}_\psi = \bar{\psi}(\boldsymbol{\Omega} \cdot \hat{\boldsymbol{J}})\psi$ (note $\hat{\boldsymbol{J}} = \hat{\boldsymbol{L}} + \hat{\boldsymbol{S}}$), whereas for gluons $\mathcal{L}^{(1)}_G = \boldsymbol{\Omega} \cdot \boldsymbol{J}_G$ and $\mathcal{L}^{(2)}_G \propto B^2$.

▷ sign problem ▷ inhomogeneous action ▷ asymmetry between E^2 and B^2

Causality restriction

- Analytic continuation is allowed only for bounded system with $\Omega r < 1$
- Boundary conditions are important! (they influence the result in all approaches)



[A. Yamamoto and Y. Hirono, Phys. Rev. Lett. **111**, 081601 (2013), arXiv:1303.6292 [hep-lat]]

- Euclidean action $S_G + S_F$ is discretized
- Lattice size: $N_t \times N_z \times N_s^2$ $(N_x = N_y = N_s)$
- "Radius" of the square cylinder: $R = a(N_s - 1)/2$
- Boundary velocity: $v_I^2 = (\Omega_I R)^2 < 1/2$
- periodic b.c. in directions τ, z.
- Infinite volume limit: $N_z \to \infty$
- different types of b.c. in directions x,y:
  open / periodic / Dirichlet / ...

Observables:

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}[\bar{\psi}, \psi] \, \mathcal{O}(U, \psi, \bar{\psi}) \, e^{-S_G[U] - S_F[U,\psi,\bar{\psi}]} \tag{9}$$

We start from rotating gluons.

Observables

The Polyakov loop is an order parameter, in gluodynamics ($\mathbb{Z}_3$ symmetry).

$$L(x,y) = \frac{1}{N_z} \sum_z \mathrm{Tr}\left[\prod_{\tau=0}^{N_t-1} U_4(\vec{r},\tau)\right], \qquad L = \frac{1}{N_s^2} \sum_{x,y} L(x,y)\,. \tag{10}$$

In confinement $\langle L \rangle = 0$; in deconfinement $\langle L \rangle \neq 0$. $\langle L \rangle = e^{-F_Q/T}$

The local critical temperature is associated with the peak of the local Polyakov loop susceptibility

$$\chi_L(r) = \langle |L(r)|^2 \rangle - \langle |L(r)| \rangle^2\,. \tag{11}$$

We use tree-level improved (Symanzik) lattice action for S_0 and chair/plaquette discretization for S_1, S_2.[1]

The temperature is $T = 1/N_t a$. It coincides with the temperature on the rotation axis T_0.

[1] A. Yamamoto and Y. Hirono, Phys. Rev. Lett. **111**, 081601 (2013), arXiv:1303.6292 [hep-lat].

Inhomogeneous phases for imaginary rotation

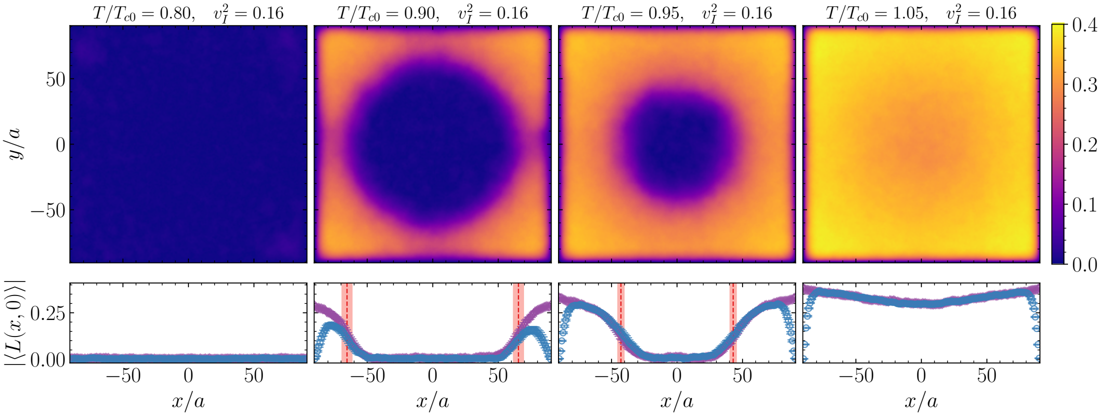


Figure: The distribution of the local Polyakov loop in x, y-plane for the lattice of size $5 \times 30 \times 181^2$ at the fixed imaginary velocity at the boundary $v_I^2 \equiv (\Omega_I R)^2 = 0.16$ and different on-axis temperatures, $T = 1/N_t a$.

- As the (on-axis) temperature increases, the radius of the inner confining region shrinks.
- Boundary is screened; Rotating symmetry is restored.
- Local thermalization takes place; Phase transition occurs as a vortex evolution.

Inhomogeneous phases for imaginary rotation

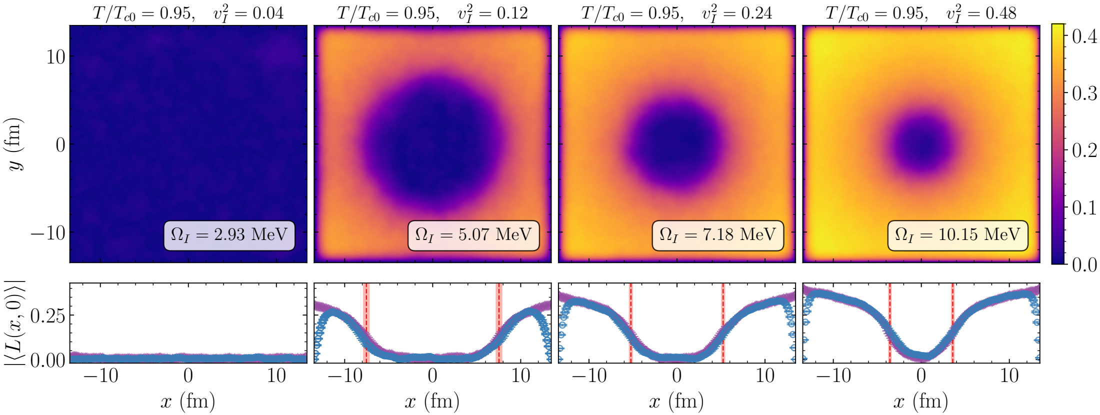
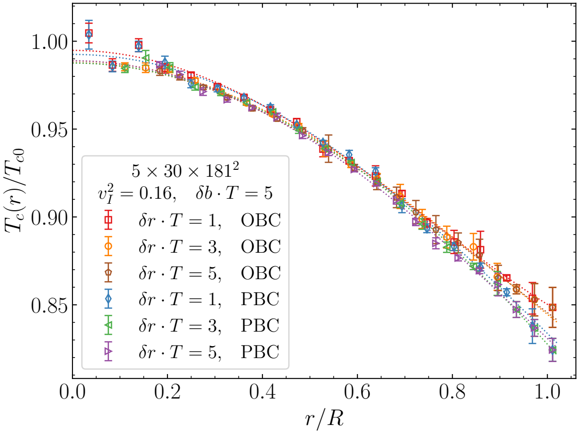


Figure: The distribution of the local Polyakov loop in x, y-plane for the lattice of size $5 \times 30 \times 181^2$ at the fixed temperature $T = 0.95\,T_{c0}$ and different Ω_I; System size $R = 13.5$ fm.

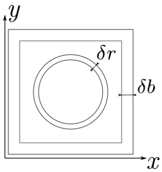
- Mixed inhomogeneous phase may be observed for $T \lesssim T_{c0}$. For imaginary rotation, deconfinement appears at the periphery; confinement is in the central regions.
- The confinement region shrinks with the increase in Ω_I;

The local critical temperature $T_c(r)$ is the temperature at the rotation axis when the phase transition occurs at radius r.

- Technical details: We split the system into thin cylinders of width δr and measure $T_c(r)$.

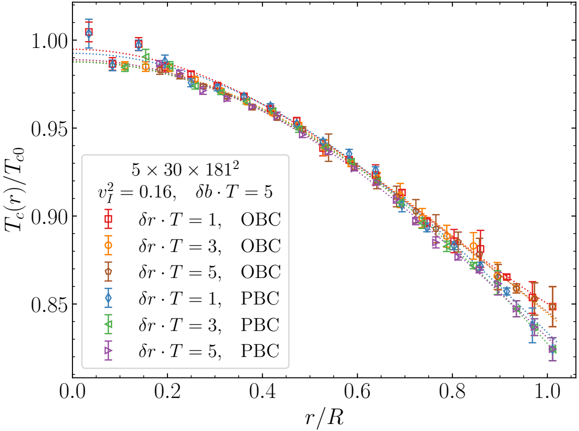


- Results for different $\delta r \cdot T = 1, \ldots, 5$ are in agreement.
- δb is a width of ignored boundary layer
- Minor difference on b.c. appears at $r/R \sim 1$



The local critical temperature $T_c(r)$ is the temperature at the rotation axis when the phase transition occurs at radius r.

- Lattice parameters: $4 \times 24 \times 145^2$, $5 \times 30 \times 181^2$, $6 \times 36 \times 216^2$, with $v_I^2 = 0.04, \ldots, 0.48$.



- Results: The local critical temperature decreases with imaginary angular velocity.

$$\frac{T_c(r,\Omega_I)}{T_{c0}} = 1 - \kappa_2(\Omega_I r)^2 - \kappa_4\left(\frac{r}{R}\right)^2(\Omega_I r)^2 \quad (12)$$

- The continuum limit result of the vortical curvature in the bulk (from quadratic fit) is

$$\kappa_2 = 0.902(33)\,, \quad (13)$$

(next terms are affected by b.c.)

- How analyticaly continue the inhomogeneous phase?

Decomposition of rotating action for gluons

The action of rotating gluons is a quadratic function in Ω_I,

$$S_G = S_0 + \lambda_1 S_1 \Omega_I + \lambda_2 S_2 \Omega_I^2 \,, \tag{14}$$

where we introduce switching factors λ_1, λ_2.

- The first operator S_1 is an angular momentum of gluons (in laboratory frame).
- The second operator S_2 is related to the chromomagnetic fields F_{ij}^2 .

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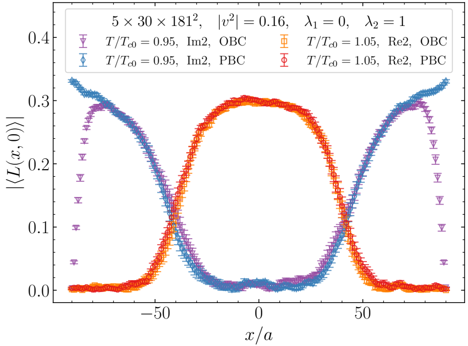

- $\lambda_1 = 0,\ \lambda_2 \neq 0$: no sign problem

  Re2: $T = T_{c0} + \Delta T$ vs Im2: $T = T_{c0} - \Delta T$

  $$\frac{T_c(r,\Omega)}{T_{c0}} = 1+\kappa(\Omega r)^2 \quad \text{vs} \quad \frac{T_c(r,\Omega_I)}{T_{c0}} = 1-\kappa(\Omega_I r)^2$$

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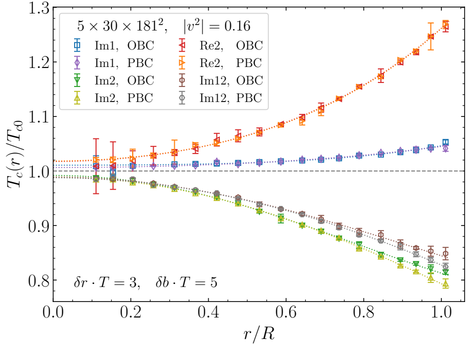

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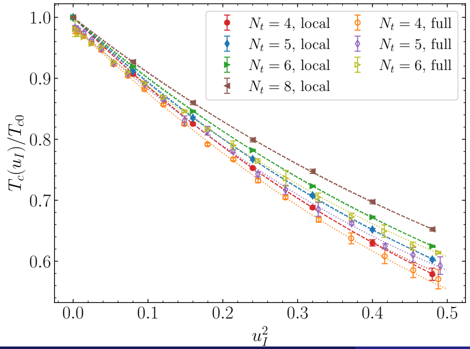
  Re2: $T = T_{c0} + \Delta T$ vs Im2: $T = T_{c0} - \Delta T$
  $\dfrac{T_c(r,\Omega)}{T_{c0}} = 1 + \kappa(\Omega r)^2$ vs $\dfrac{T_c(r,\Omega_I)}{T_{c0}} = 1 - \kappa(\Omega_I r)^2$
- S_1 and S_2 have opposite influence on T_c.
- Effect of asymmetry (S_2) dominates.
- The results resemble the decomposition of I (see below)

The homogeneous local action (in the vicinity of the point $x = r_0$, $y = 0$) is

$$S_G = \frac{1}{2g_0^2}\int d^4x \Big[F^a_{x\tau}F^a_{x\tau} + F^a_{y\tau}F^a_{y\tau} + F^a_{z\tau}F^a_{z\tau} + F^a_{xz}F^a_{xz} + \\ + \left(1+u_I^2\right)F^a_{yz}F^a_{yz} + \left(1+u_I^2\right)F^a_{xy}F^a_{xy} - 2u_I\left(F^a_{yx}F^a_{x\tau} + F^a_{yz}F^a_{z\tau}\right)\Big], \quad (15)$$

where $u_I = \Omega_I r_0$ is a local velocity. The local critical temperature increases with real velocity $u = \Omega r$.



- Data are fitted by two different functions:

$$\frac{T_c(u_I)}{T_{c0}} = 1 + k_2u^2 + k_4u^4\,, \quad (16)$$

$$\frac{T_c(u)}{T_{c0}} = \frac{1 + c_2u^2}{1 - b_2u^2}\,. \quad (17)$$

- In continuum limit the coefficients are

$$k_2 = 0.869(31)\,, \qquad k_4 = 0.388(53)\,. \quad (18)$$

$$c_2 = 0.206(66)\,, \qquad b_2 = 0.694(101)\,. \quad (19)$$

A mechanical response of a thermodynamic ensemble to rigid rotation $\boldsymbol{\Omega} = \Omega \boldsymbol{e}$ is described in terms of the total angular momentum $\boldsymbol{J}$. The energy in co-rotating reference frame is

$$E = E^{(lab)} - \boldsymbol{J} \cdot \boldsymbol{\Omega} , \qquad F = E - TS , \qquad dF = -SdT - \boldsymbol{J} \cdot d\boldsymbol{\Omega} + \ldots ,$$

The moment of inertia is a scalar quantity, $\boldsymbol{J} = I(T,\Omega)\boldsymbol{\Omega}$,

$$I(T,\Omega) = \frac{J(T,\Omega)}{\Omega} = -\frac{1}{\Omega}\left(\frac{\partial F}{\partial \Omega}\right)_T ,$$

For a classical system with characteristic radius R the moment of inertia is given by

$$I(T,\Omega) = \int_V d^3x\, x_\perp^2 \rho(T, x_\perp, \Omega) \simeq \alpha\, \rho_0(T) V R^2 ,$$

The free energy may be represented as a series in angular velocity (or linear velocity $v_R = \Omega R$)

$$F(T,V,\Omega) = F_0(T,V) - \frac{F_2(T,V)}{2}\Omega^2 + \mathcal{O}(\Omega^4) \equiv F_0(T,V) - \frac{\mathsf{i}_2(T)}{2} V v_R^2 + \mathcal{O}(v_R^4) ,$$

where $F_2(T,V) = I(T,V,\Omega = 0) \equiv \mathsf{i}_2(T) V R^2$, and $\mathsf{i}_2(T)$ is a *specific* moment of inertia.

Taking the derivative at $\Omega = 0$, we obtain:

$$I = F_2 = T \frac{\partial^2 \log Z}{\partial \Omega^2}\bigg|_{\Omega=0} = T\left(\langle\!\langle S_1^2 \rangle\!\rangle_T + \langle\!\langle S_2 \rangle\!\rangle_T\right),$$

where $\langle\!\langle \mathcal{O} \rangle\!\rangle_T = \langle \mathcal{O} \rangle_T - \langle \mathcal{O} \rangle_{T=0}$.
Using the exact forms of S_1, S_2, we get

$$I = I_{\rm mech} + I_{\rm magn}$$

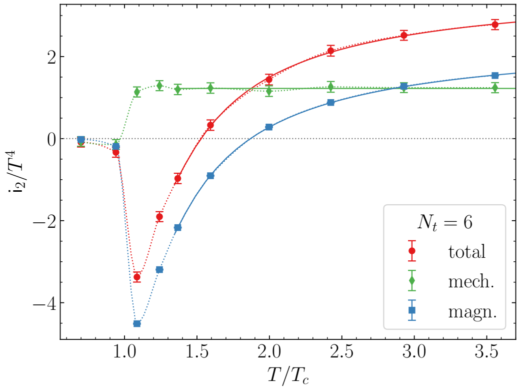
where ($\langle J \rangle = 0$ for any T) and

$$I_{\rm mech} = \frac{1}{T}\left(\langle\!\langle J^2 \rangle\!\rangle_T - \langle\!\langle J \rangle\!\rangle_T^2\right) \geq 0,$$

$$I_{\rm magn} = \frac{1}{3}\int_V d^3x\, x_\perp^2 \langle\!\langle (F_{ij}^a)^2 \rangle\!\rangle_T = \frac{\alpha}{3} V R^2 \langle\!\langle (G_{\rm magn})^2 \rangle\!\rangle_T\,.$$

J is the total angular momentum of gluon field.

- $I < 0$ for $T < T_s \simeq 1.5 T_c$ and $I > 0$ for $T > T_s$.
- Mass density $\rho_0(T) \leftrightarrow \langle\!\langle (G_{\rm magn})^2 \rangle\!\rangle_T / 3$.



[V. V. Braguta et al., Phys. Rev. D **110**, 014511 (2024), arXiv:2310.16036 [hep-ph]]

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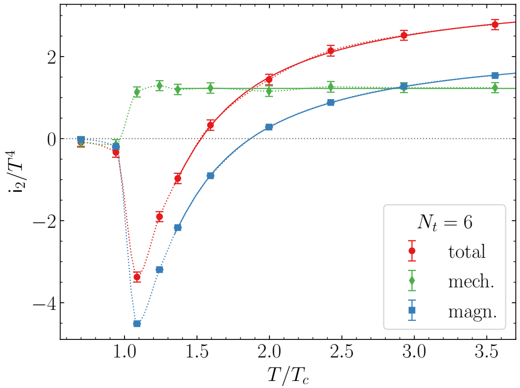
$$I_{\text{magn}} = \frac{1}{3}\int_V d^3x\, x_\perp^2 \langle\!\langle (F_{ij}^a)^2 \rangle\!\rangle_T = \frac{\alpha}{3} V R^2 \langle\!\langle (G_{\text{magn}})^2 \rangle\!\rangle_T .$$

J is the total angular momentum of gluon field.

- $I < 0$ for $T < T_s \simeq 1.5 T_c$ and $I > 0$ for $T > T_s$.
- Mass density $\rho_0(T) \leftrightarrow \langle\!\langle (G_{\text{magn}})^2 \rangle\!\rangle_T / 3$.



[V. V. Braguta et al., Phys. Rev. D **110**, 014511 (2024), arXiv:2310.16036 [hep-ph]]

Negative moment of inertia and magnetic gluon condensate

Taking the derivative at $\Omega = 0$, we obtain:

$$I = F_2 = T \frac{\partial^2 \log Z}{\partial \Omega^2}\bigg|_{\Omega=0} = T\big(\langle\!\langle S_1^2 \rangle\!\rangle_T + \langle\!\langle S_2 \rangle\!\rangle_T \big)\,,$$

where $\langle\!\langle \mathcal{O} \rangle\!\rangle_T = \langle \mathcal{O} \rangle_T - \langle \mathcal{O} \rangle_{T=0}$.
Using the exact forms of S_1, S_2, we get

$$I = I_{\text{mech}} + I_{\text{magn}}$$

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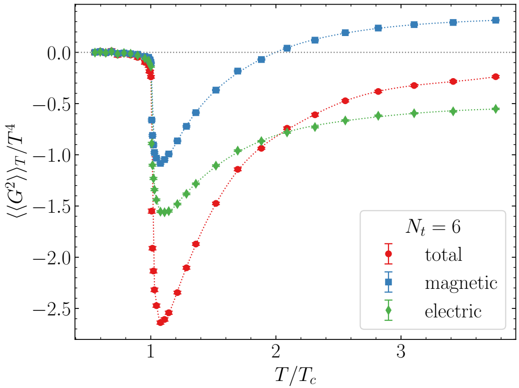

[V. V. Braguta et al., Phys. Rev. D **110**, 014511 (2024), arXiv:2310.16036 [hep-ph]]

- $\langle\!\langle \mathcal{B}^2 \rangle\!\rangle$ reverse its sign at $\sim 2T_c$.

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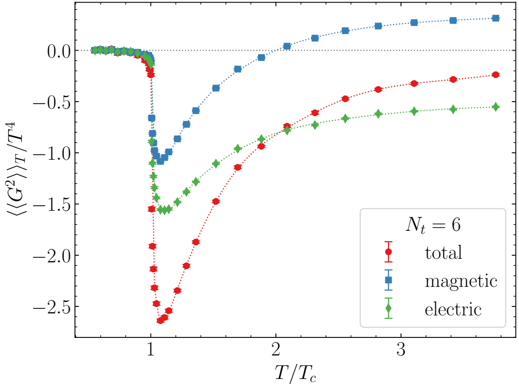


[V. V. Braguta et al., Phys. Rev. D **110**, 014511 (2024), arXiv:2310.16036 [hep-ph]]

- $\langle\!\langle \mathcal{B}^2 \rangle\!\rangle$ reverse its sign at $\sim 2T_c$.
- In QCD fermionis (J_ψ) contribute only to $I_{\rm mech}$.

Total angular momentum $\boldsymbol{J} = I\boldsymbol{\Omega}$ is a sum of the orbital and spin parts:

$$\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}\,, \tag{20}$$

and $I < 0$. The possible physical picture: instability, or *negative* Barnett effect for gluon.

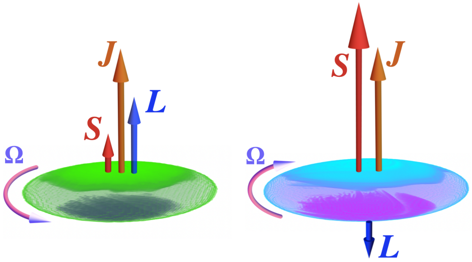
In the temperature range $T_c \lesssim T < T_s \simeq 1.5T_c$:

(i) a sizable fraction of the total angular momentum $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$ is accumulated in the spin of gluons $\boldsymbol{S}$;

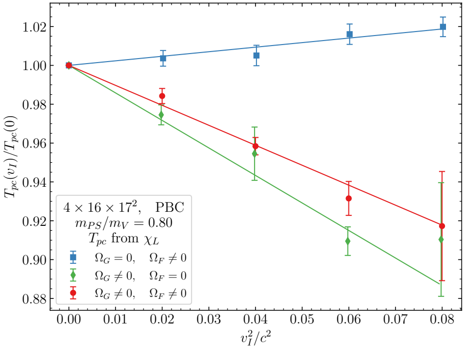
(ii) therefore, $\boldsymbol{S} \uparrow\uparrow \boldsymbol{J}$ and $\boldsymbol{S} \uparrow\downarrow \boldsymbol{L}$.

Let's introduce $\boldsymbol{L} = I_L\boldsymbol{\Omega}$, $\boldsymbol{S} = I_S\boldsymbol{\Omega}$, therefore

$$I_L > 0\,, \quad I_S < 0\,, \quad I = I_L + I_S < 0\,.$$



(left) usual Barnett effect
(right) negative Barnett effect
[V. V. Braguta et al., Phys. Rev. D **110**, 014511 (2024), arXiv:2310.16036 [hep-ph]]



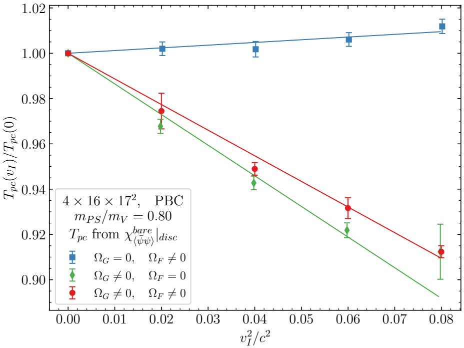


Figure: The (bulk-averaged) pseudo-critical temperature as a function of imaginary linear velocity on the boundary for various rotation regimes (full, only gluons, only fermions). [V. Braguta, A. Kotov, A. Roenko, and D. Sychev, PoS **LATTICE2022**, 190 (2023), arXiv:2212.03224 [hep-lat]]

QCD action: $S = S_G(\Omega_G) + S_F(\Omega_F)$

Rotation in fermionic and gluonic sectors have different influence on (bulk-averaged) T_{pc}. Gluons dominate.

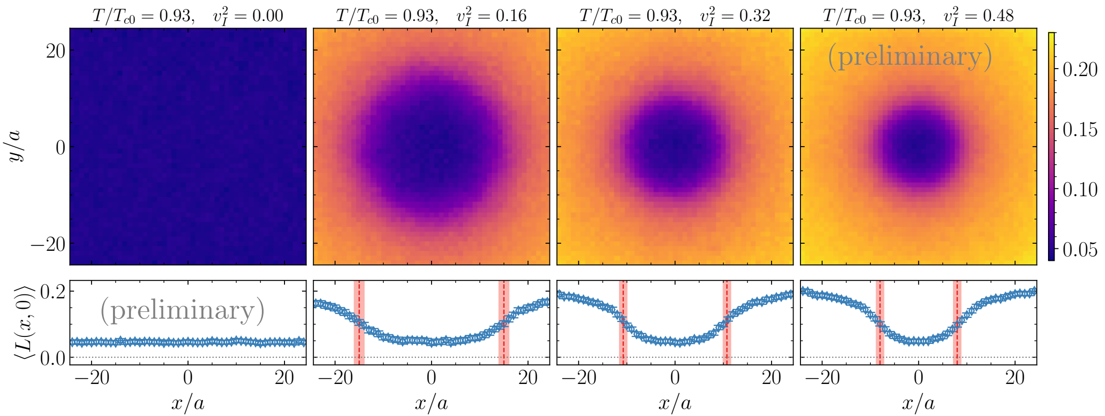


Figure: The distribution of the local Polyakov loop in x, y-plane for the lattice of size $4 \times 20 \times 49^2$ at the fixed temperature $T = 0.93\,T_{c0}$ and different v_I; QCD with Wilson fermions (Iwasaki action), $m_\pi/m_\rho = 0.80$.

- Mixed inhomogeneous phase takes place also in QCD! (work in progress . . .)

Conclusions

- Using lattice simulation with *imaginary* angular velocity, we found the mixed phase in rotating gluodynamics at thermal equilibrium. For *imaginary* rotation, it takes place for $T < T_{c0}$ with deconfinement (confinement) phase at the periphery (center).
- For *real* rotation, the inhomogeneous phase may arise for $T > T_{c0}$ with confinement at the periphery and deconfinement in the center.
- We demonstrate the validity of analytic continuation using Im2/Re2-regimes.
- The local critical temperature in rotating gluodynamics depends on the local velocity $u = \Omega r$:

$$\frac{T_c(r,\Omega)}{T_{c0}} = 1 + \kappa_2 (\Omega r)^2 \qquad \text{[bulk of full rotating system]}, \qquad (21)$$

$$\frac{T_c(u)}{T_{c0}} = 1 + k_2 u^2 + k_4 u^4, \quad \text{or} \quad \frac{T_c(u)}{T_{c0}} = \frac{1 + c_2 u^2}{1 - b_2 u^4}, \qquad \text{[local action]}, \qquad (22)$$

  The approximation of local thermalization gives consistent results. Note that $T_c(0) \approx T_{c0}$.
- The magnetovortical coupling generates asymmetry in the action for chromomagnetic fields. Linear coupling play subleading role near T_c. This mechanism can not be accounted for by TE.
- Gluon plasma has $I < 0$ below the supervortical temperature $T_s = 1.50(10) T_c$ (and $I > 0$ for $T > T_s$). Possible physical explanation: NBE. Results for a.c. from Ω_I and $\partial_\Omega|_{\Omega=0}$ are in agreement.
- We expect similar picture for QCD (work in progress).

Thank you for your attention!

Ehrenfest-Tolman effect: In gravitational field the temperature isn't a constant in space at thermal equilibrium, $T(r)\sqrt{g_{00}} = T_0 = \text{const}$. In the co-rotating reference frame:

$$T(r) = \frac{T_0}{\sqrt{1-\Omega^2 r^2}} = \frac{T_0}{\sqrt{1+\Omega_I^2 r^2}}\,. \tag{23}$$

TE law suggests that the rotation effectively heats the periphery. Let's derive $T_c^{TE}(u)$ from an assumption $T(r) = T_{c0}$, then the local critical temperature decreases:

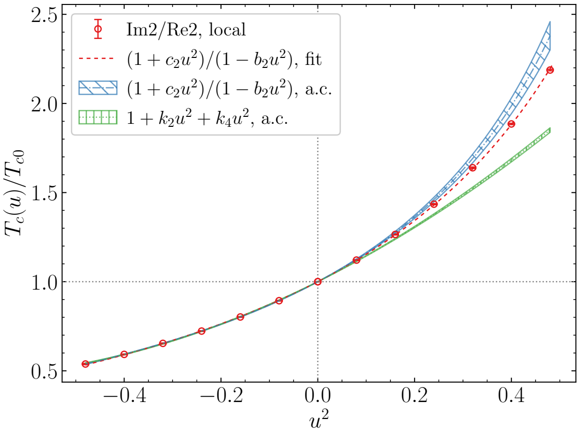
$$\frac{T_c^{TE}(u)}{T_{c0}} = \sqrt{1-u^2} \approx 1 - 0.5u^2 + \dots\,, \tag{24}$$

In the result, TE predicts confinement in the center and deconfinement at the periphery (for *real* rotation):

- 2+1 cQED: M. N. Chernodub, Phys. Rev. D **103**, 054027 (2021), arXiv:2012.04924 [hep-ph]
- Holography: N. R. F. Braga and O. C. Junqueira, Phys. Lett. B **848**, 138330 (2024), arXiv:2306.08653 [hep-th]

Lattice simulation gives opposite arrangement of the phases. Qualitatively consistent results:

- S. Chen, K. Fukushima, and Y. Shimada, Phys. Lett. B **859**, 139107 (2024), arXiv:2404.00965 [hep-ph]
- Y. Jiang, Phys. Rev. D **110**, 054047 (2024), arXiv:2406.03311 [nucl-th]



- The results for *local* action and for full system are in a good agreement with each other in all regimes.
- The data are well described by the polynomial:

$$\frac{T_c(u_I)}{T_{c0}} = 1 + k_2 u^2 + k_4 u^4\,, \qquad (25)$$

- Or, by the rational function:

$$\frac{T_c(u)}{T_{c0}} = \frac{1 + c_2 u^2}{1 - b_2 u^2}\,. \qquad (26)$$

- The function (26) better describe all data from regimes Im2/Re2.

Imaginary vs real rotation for different regimes

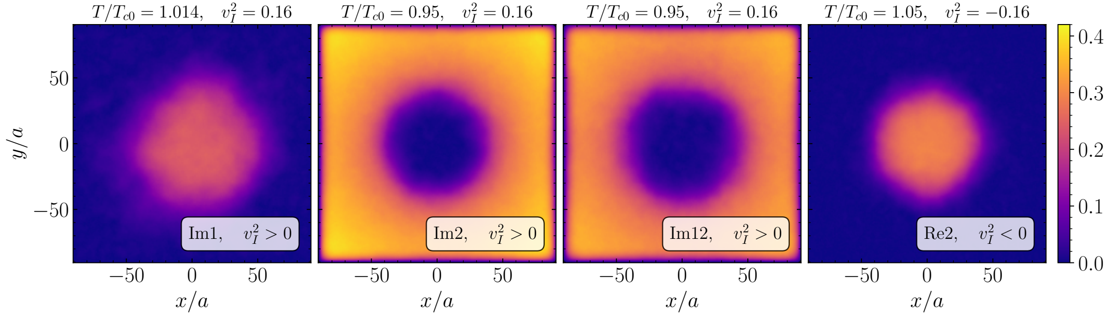
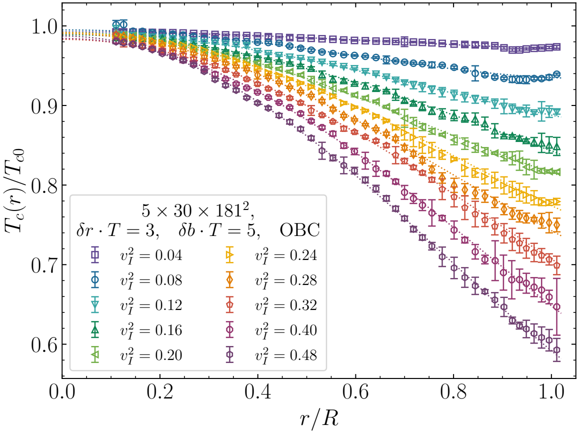


Figure: The distribution of the local Polyakov loop in x, y-plane for lattice size $5 \times 30 \times 181^2$, open boundary conditions (OBC) at fixed velocity $|v_I^2| = 0.16$ and different regimes. Temperature was chosen to see mixed phase.

- In the regimes Im1 and Re2, the rotation produces confinement phase in the outer region at $T > T_{c0}$. Regime Re2 realizes real rotation for S_2 system.
- Phase arrangement is the same in Im2- and Im12-regimes. The radius of the inner region in regime Im2 is slightly smaller, than in regime Im12.

Local critical temperature

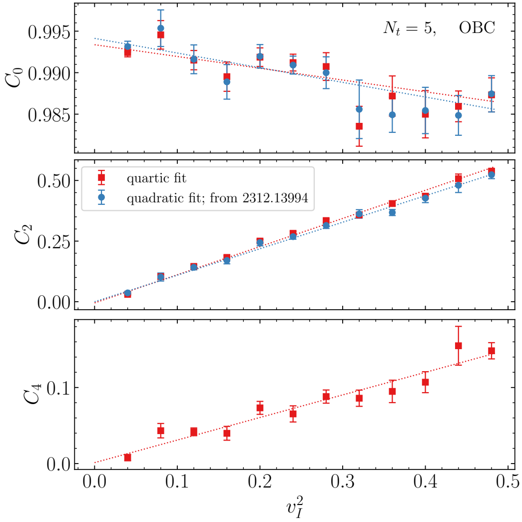


- The results in the whole region are well described by the quartic formula

$$\frac{T_c(r)}{T_{c0}} = C_0 - C_2\left(\frac{r}{R}\right)^2 + C_4\left(\frac{r}{R}\right)^4 . \quad (27)$$

- In the bulk, $r/R \lesssim 0.5$, quadratic fit is sufficient $(C_4 = 0)$.

Local critical temperature



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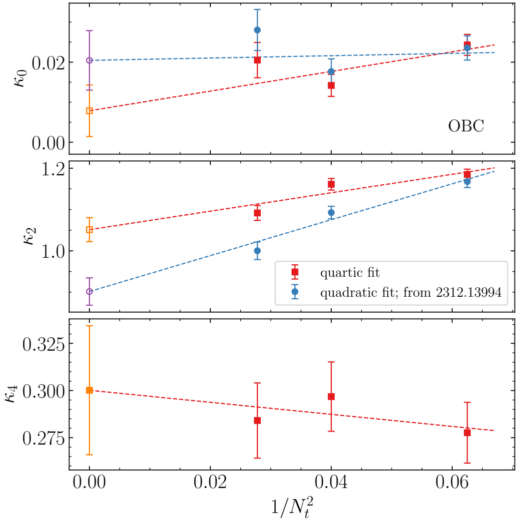
- We found numerically that

$$C_i(v_I^2) = a_i + \kappa_i v_I^2 . \quad (28)$$

- $T_c(0) \approx T_{c0}$ with few percent accuracy:
  - Effects of finite radius R.
  - Effects of averaging in layers of width δr.



- Results: The local critical temperature decreases with imaginary angular velocity.

$$\frac{T_c(r,\Omega_I)}{T_{c0}} = 1 - (\Omega_I r)^2 \left(\kappa_2 - \kappa_4 \left(\frac{r}{R}\right)^2\right) . \quad (29)$$

- The vortical curvature in continuum limit from quadratic fit ($r/R \lesssim 0.5$) is universal

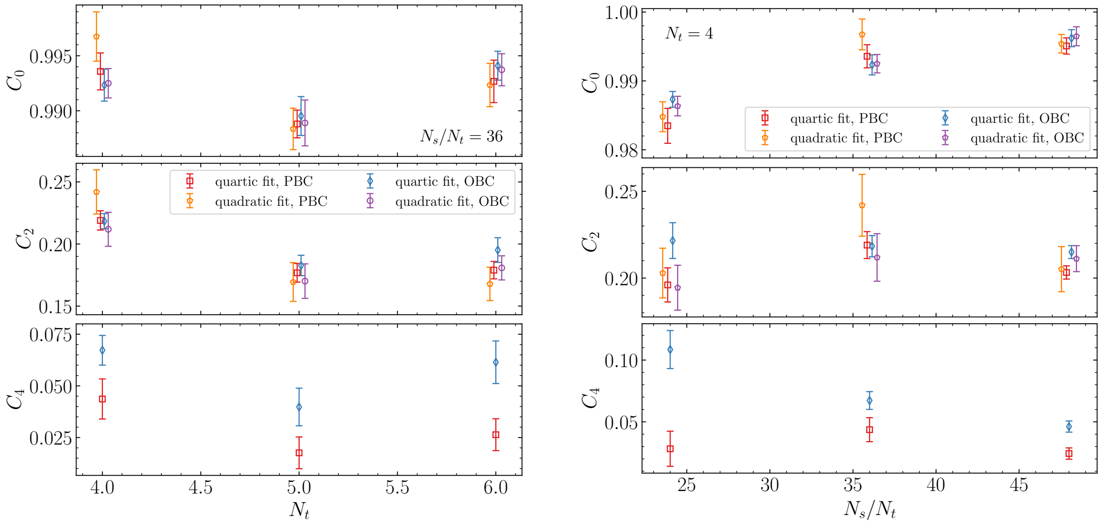
$$\kappa_2 = 0.902(33) , \quad (30)$$

- And from quartic fit (for OBC) there is

$$\kappa_2 = 1.051(29) , \qquad \kappa_4 = 0.300(34) , \quad (31)$$

where κ_4 term is a finite volume correction;

- We can not distinguish $\sim \Omega^4$ term.



C_0
0.995
0.990
$N_s/N_t = 36$
C_2
0.25
0.20
0.15
quartic fit, PBC
quadratic fit, PBC
quartic fit, OBC
quadratic fit, OBC
C_4
0.075
0.050
0.025
4.0
4.5
5.0
5.5
6.0
N_t
1.00
$N_t = 4$
C_0
0.99
0.98
quartic fit, PBC
quadratic fit, PBC
quartic fit, OBC
quadratic fit, OBC
C_2
0.25
0.20
C_4
0.10
0.05
25
30
35
40
45
N_s/N_t

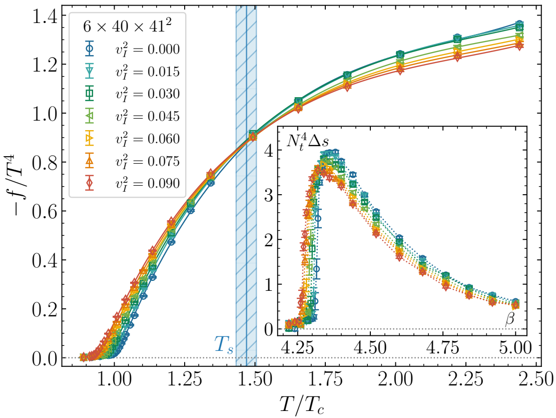
Symanzik gauge action; we calculate $f = F/V$ using standard relations

$$\frac{f(T)}{T^4} = -N_t^4 \int_{\beta_0}^{\beta} d\beta' \Delta s(\beta'),$$

where $\Delta s(\beta) = \langle s(\beta)\rangle_{T=0} - \langle s(\beta)\rangle_T \equiv -\langle\langle s\rangle\rangle$.

- $N_t \times 40 \times 41^2$ lattices with $N_t = 5, 6, 7, 8$;
- $N_t^{(T=0)} = 40$ for $T = 0$ subtraction;
- $v_I^2 \ll 1$, where $v_I = \Omega_I R$, $R = a(N_s - 1)/2$.
- $v_I = \text{const} \quad \Leftrightarrow \quad \Omega_I/T = v_I/RT = \text{const}$.
- $T_c \searrow$ with the imaginary angular velocity.
- Fit by the quadratic function ($f_0 = -p < 0$):

$$f(T, v_I) = f_0(T)\Big(1 - \frac{1}{2}K_2(T)v_I^2\Big).$$



[V. V. Braguta, M. N. Chernodub, A. A. Roenko, and D. A. Sychev, Phys. Lett. B **852**, 138604 (2024), arXiv:2303.03147 [hep-lat]]

- The moment of inertia of gluon plasma

$$I(T)|_{\Omega=0} = -K_2 F_0 R^2 \,,$$

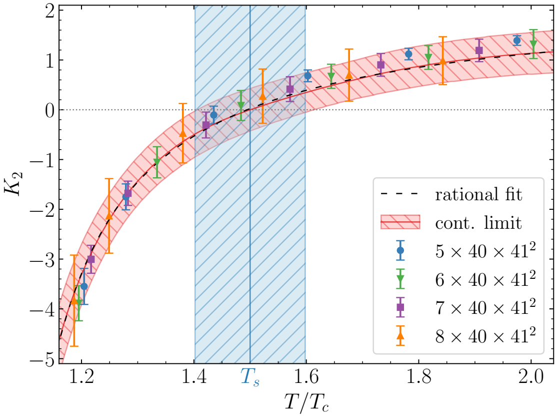
becomes zero at "supervortical" temperature

$$T_s = 1.50(10) T_c.$$

and it is negative for $T < T_s$.

- The result for the system with OBC is

$$T_s = 1.53(15) T_c$$



[V. V. Braguta, M. N. Chernodub, A. A. Roenko, and D. A. Sychev, Phys. Lett. B **852**, 138604 (2024), arXiv:2303.03147 [hep-lat]]

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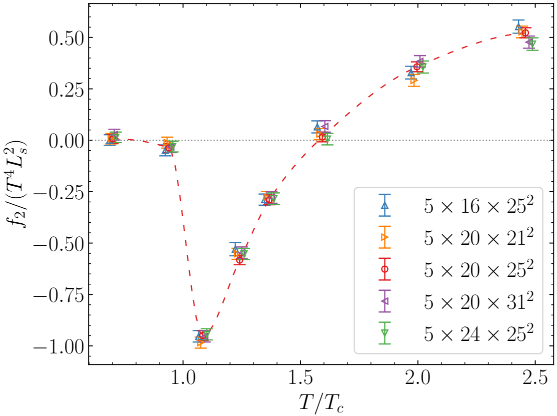
where $\langle\langle \mathcal{O} \rangle\rangle_T = \langle \mathcal{O} \rangle_T - \langle \mathcal{O} \rangle_{T=0}$ corresponds to the thermal contribution to $\langle \mathcal{O} \rangle$.

$$f_2/(T^4 L_s^2) \equiv \mathrm{i}_2/T^4 ,$$



[V. V. Braguta et al., JETP Lett. **117**, 639–644 (2023)]

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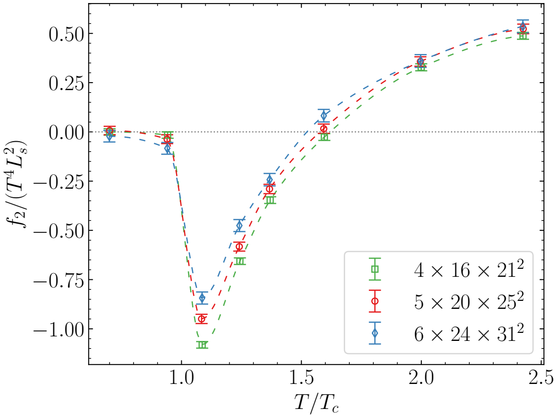


[V. V. Braguta et al., JETP Lett. **117**, 639–644 (2023)]

Results of lattice simulations with zero angular velocity

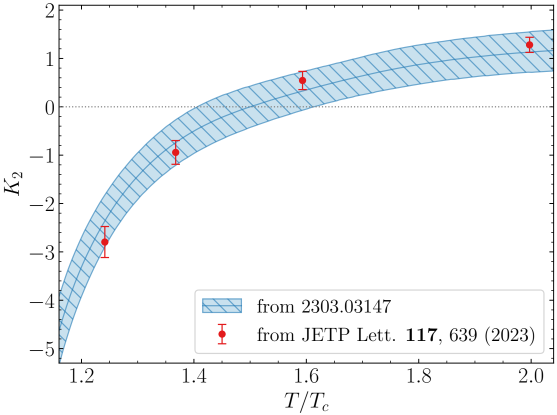
Taking the derivative at $\Omega = 0$, we obtain:

$$I = F_2 = T \frac{\partial^2 \log Z}{\partial \Omega^2}\bigg|_{\Omega=0} = T\big(\langle\langle S_1^2 \rangle\rangle_T + \langle\langle S_2 \rangle\rangle_T\big)\,,$$

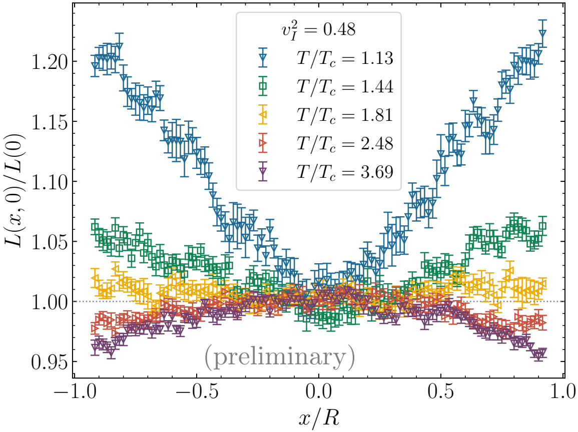
where $\langle\langle \mathcal{O} \rangle\rangle_T = \langle \mathcal{O} \rangle_T - \langle \mathcal{O} \rangle_{T=0}$ corresponds to the thermal contribution to $\langle \mathcal{O} \rangle$.

$$f_2/(T^4 L_s^2) \equiv \mathsf{i}_2/T^4\,, \qquad K_2 = \mathsf{i}_2/(-f_0)$$

Results of two methods (a.c. from Ω_I and $\partial_\Omega|_{\Omega=0}$) are in agreement.



[V. V. Braguta et al., PoS **LATTICE2023**, 181 (2024), arXiv:2311.03947 [hep-lat]]



- $T > T_s \simeq 1.5 T_{c0}$: $I > 0$
- $T \gtrsim 2T_{c0}$: $\langle\langle \mathcal{B}^2 \rangle\rangle > 0$
- Local Polyakov loop decreases with r at high temperatures $T \gtrsim 2T_{c0}$

  (local temperature from TE decreases with r for imaginary Ω_I)