Энергетические потери в голографических моделях с пространственной анизотропией и внешним магнитным полем

Павел Слепов

На основе работ с И.Я. Арефьевой, К. А. Ранну, А. Хаджилоу
JHEP 07, 161 (2021); ТМФ, 206 3 (2021), 400–409 [arXiv:2012.05758] и Eur. Phys. J. C 83, 12, 1143 (2023)

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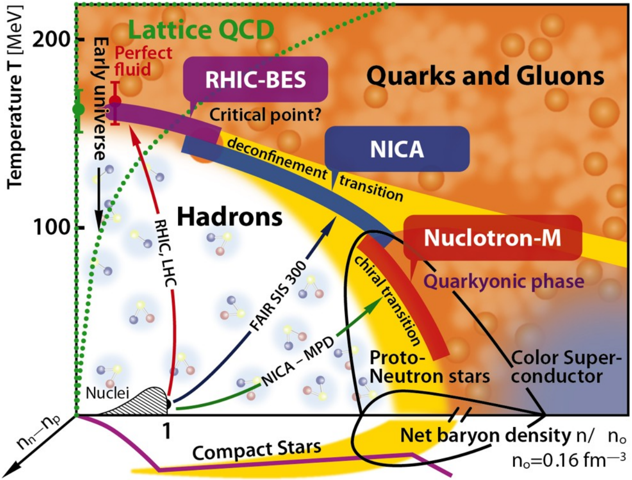
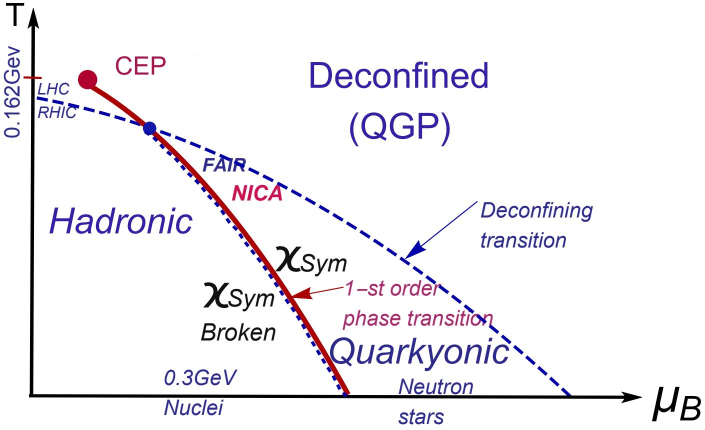
19.02.2025

Studies of QCD Phase Diagram is the main goal of new facilities



From: https://nica.jinr.ru/physics.php

Holographic QCD phase diagram for light quarks



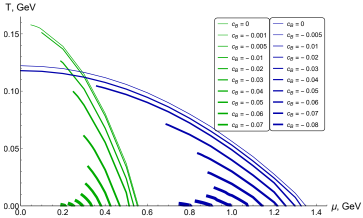
The main question to discuss today is: what directly measurable quantities indicate the presence of 1-st order phase transitions?

- Jet Quenching – I. Ya. Aref'eva's talk
- Direct photons – Ref.: I. Ya. Aref'eva, A. Ermakov and P. S., "Direct photons emission rate ... with first-order phase transition," EPJC **82** (2022) 85
- Energy loss – this talk
- Cross-sections – M.Usova's and A.Nikolaev's talks

- Details of the CEP locations – K.Rannu's talk

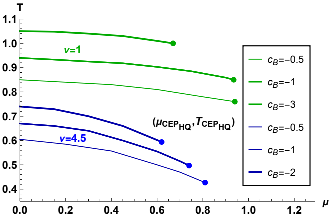
1-st order phase transition for “light” and “heavy” quarks in holography

Light quarks



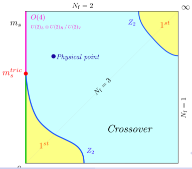
Aref’eva, Ermakov, Rannu, P.S., EPJC’23

Heavy quarks



Aref’eva, Hajilou, Rannu, P.S.,EPJC’23

- QCD Phase Diagram from Lattice
  Columbia plot
  Brown et al.’90 Philipsen, Pinke’16
- **Main problem on Lattice:** $\mu \neq 0$



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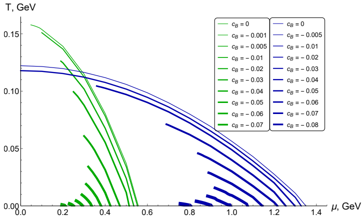

Aref’eva, Ermakov, Rannu, P.S., EPJC’23

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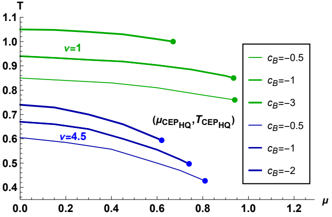

Aref’eva, Hajilou, Rannu, P.S.,EPJC’23

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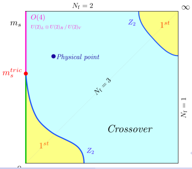


Holographic model of an anisotropic plasma in a magnetic field at a nonzero chemical potential

$$\mathcal{L} = R - \frac{f_0(\phi)}{4} F_0^2 - \frac{f_1(\phi)}{4} F_1^2 - \frac{f_3(\phi)}{4} F_3^2 - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi)$$

$A_\mu^0 = A_t(z)\delta_\mu^0$ $\qquad F_1 = q_1 \; dx^2 \wedge dx^3$ $\qquad F_3 = q_3 \; dx^1 \wedge dx^2$

$A_t(0) = \mu$ $\qquad g(0) = 1$ $\qquad$ *Dudal et al. (2019)*

$A_t(z_h) = 0$ $\qquad g(z_h) = 0$ $\qquad \phi(z_0) = 0 \;\rightarrow\; \sigma_{\text{string}}$

$$ds^2 = \frac{L^2}{z^2} b(z) \left[- g(z) \; dt^2 + dx_1^2 + \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dx_2^2 + e^{c_B z^2} \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dx_3^2 + \frac{dz^2}{g(z)} \right]$$

Aref'eva, Golubtsova (2014), Giataganas (2013) $\qquad$ *Gürsoy, Järvinen et al. (2019)*

$b(z) = e^{2\mathcal{A}(z)} \;\rightarrow\;$ quarks mass $\qquad$ "Bottom-up approach"

Heavy quarks (b, t):

$\mathcal{A}(z) = - \; cz^2/4$ $\qquad$ *Andreev, Zakharov'06*

$\mathcal{A}(z) = - \; cz^2/4 - (p - c_B q_3) z^4$ $\qquad$ *Aref'eva, Hajilou, Rannu, P.S.'23*

Light quarks (d, u)

$\mathcal{A}(z) = - \, a \ln(bz^2 + 1)$ $\qquad$ *Li, Yang, Yuan'17*

$\mathcal{A}(z) = - \, a \ln((bz^2 + 1)(dz^4 + 1))$ $\qquad$ *Zhu, Chen, Zhou, Zhang, Huang'25*

Modified warp-factor and twice anistropic holographic model for heavy quarks

NEW Warp-factor:

$\mathcal{A}(z) = -\,cz^2/4 - (p - c_B\, q_3)z^4$

Aref'eva, Hajilou, Rannu and P. S., Eur. Phys. J. C 83, 12, 1143 (2023)

$$g(z) = e^{c_B z^2}\left[1 - \frac{\tilde{I}_1(z)}{\tilde{I}_1(z_h)} + \frac{\mu^2\left(2R_{gg} + c_B(q_3 - 1)\right)\tilde{I}_2(z)}{L^2\left(1 - e^{(2R_{gg} + c_B(q_3 - 1))\frac{z_h^2}{2}}\right)^2}\left(1 - \frac{\tilde{I}_1(z)}{\tilde{I}_1(z_h)}\,\frac{\tilde{I}_2(z_h)}{\tilde{I}_2(z)}\right)\right],$$

$$\tilde{I}_1(z) = \int_0^z e^{(2R_{gg} - 3c_B)\frac{\xi^2}{2} + 3(p - c_B\, q_3)\xi^4}\,\xi^{1+\frac{2}{\nu}}\, d\xi,$$

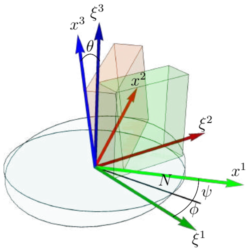
$$\tilde{I}_2(z) = \int_0^z e^{\left(2R_{gg} + c_B\left(\frac{q_3}{2} - 2\right)\right)\xi^2 + 3(p - c_B\, q_3)\xi^4}\,\xi^{1+\frac{2}{\nu}}\, d\xi.$$

Spatial Wilson loops. Parametrization*

To describe the nesting of the 2-dimensional world sheet in 5-dimensional spacetime we use

$$
\begin{aligned}
X^0(\xi) &= const, \\
X^i(\xi) &= \sum_{\alpha=1,2} a_{i\alpha}(\phi,\theta,\psi)\,\xi^\alpha, \qquad i=1,2,3, \\
X^4(\xi) &= z(\xi^1),
\end{aligned}
$$

x^i are spatial coordinates and $a_{ij}(\phi,\theta,\psi)$ are entries of the rotation matrix. Here ϕ is the angle between ζ^1-axis and the node line (N), θ is the angle between ζ^3 and x^3-axes, ψ is the angle between the node line N and x^1-axis.



$$
\begin{aligned}
a_{11}(\phi,\theta,\psi) &= \cos\phi\cos\psi - \cos\theta\sin\phi\sin\psi, \\
a_{12}(\phi,\theta,\psi) &= -\cos\psi\sin\phi - \cos\phi\cos\theta\sin\psi, \\
a_{13}(\phi,\theta,\psi) &= \sin\theta\sin\psi, \\
a_{21}(\phi,\theta,\psi) &= \cos\theta\cos\psi\sin\phi + \cos\phi\sin\psi, \\
a_{22}(\phi,\theta,\psi) &= \cos\phi\cos\theta\cos\psi - \sin\phi\sin\psi, \\
a_{23}(\phi,\theta,\psi) &= -\cos\psi\sin\theta, \\
a_{31}(\phi,\theta,\psi) &= \sin\phi\sin\theta, \\
a_{32}(\phi,\theta,\psi) &= \cos\phi\sin\theta. \\
a_{33}(\phi,\theta,\psi) &= \cos\theta.
\end{aligned}
$$

Nambu-Goto action for Spatial Wilson Loop

Spatial Wilson loop (SWL):

$$\mathcal{S}_{SWL} = \int_{\mathcal{W}} \left(\frac{L^2 b_s}{z^2}\right) \sqrt{\left(\mathfrak{g}_1\mathfrak{g}_2 a_{33}^2 + \mathfrak{g}_1\mathfrak{g}_3 a_{23}^2 + \mathfrak{g}_2\mathfrak{g}_3 a_{13}^2 + \frac{z'^2}{g}\,\bar{\mathfrak{g}}_{22}\right)}\, d\xi^1 d\xi^2$$

$$\mathcal{V}_{SWL}(z) = \left(\frac{L^2 b_s}{z^2}\right) \sqrt{\mathfrak{g}_1\mathfrak{g}_2 a_{33}^2 + \mathfrak{g}_1\mathfrak{g}_3 a_{23}^2 + \mathfrak{g}_2\mathfrak{g}_3 a_{13}^2}$$

Holographic entanglement entropy (HEE):

$$\mathcal{S}_{HEE} = \int_{\mathcal{P}} \left(\frac{L^2 b_s}{z^2}\right)^{3/2} \sqrt{\left(\mathfrak{g}_1\mathfrak{g}_2\mathfrak{g}_3 + \frac{z'^2}{g}\left(\bar{\mathfrak{g}}_{22}\bar{\mathfrak{g}}_{33} - \bar{\mathfrak{g}}_{23}^2\right)\right)}\, d\xi^1 d\xi^2 d\xi^3,$$

$$\mathcal{V}_{HEE}(z) = \left(\frac{L^2 b_s}{z^2}\right)^{3/2} \sqrt{\mathfrak{g}_1\mathfrak{g}_2\mathfrak{g}_3},$$

g, $\mathfrak{g}_1$, $\mathfrak{g}_2$, $\mathfrak{g}_3$ are functions of z and $\bar{\mathfrak{g}}_{22}$, $\bar{\mathfrak{g}}_{33}$, $\bar{\mathfrak{g}}_{23}$ are functions of z and the Euler angles:

$$\begin{aligned}
\bar{\mathfrak{g}}_{22}(z,\phi,\theta,\psi) &= \mathfrak{g}_1 a_{12}^2 + \mathfrak{g}_2 a_{22}^2 + \mathfrak{g}_3 a_{32}^2,\\
\bar{\mathfrak{g}}_{33}(z,\phi,\theta,\psi) &= \mathfrak{g}_1 a_{13}^2 + \mathfrak{g}_2 a_{23}^2 + \mathfrak{g}_3 a_{33}^2,\\
\bar{\mathfrak{g}}_{23}(z,\phi,\theta,\psi) &= \mathfrak{g}_1 a_{12} a_{13} + \mathfrak{g}_2 a_{22} a_{23} + \mathfrak{g}_3 a_{32} a_{33}
\end{aligned}$$

I. Y. Aref'eva, A. Patrushev, P.S. JHEP **07**, 043 (2020)

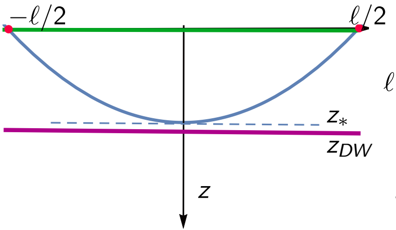
Born-Infeld type action (1-dim dynamic model):

$$\mathcal{S} = \int_{-\ell/2}^{\ell/2} M(z(\xi))\sqrt{\mathcal{F}(z(\xi)) + (z'(\xi))^2}\, d\xi, \quad V(z(\xi)) = M(z(\xi))\sqrt{\mathcal{F}(z(\xi))}$$

We have two options to have $\ell \to \infty$ O. Andreev, V. I. Zakharov, Phys. Lett. B **645**, 437 (2007)

Born-Infeld type action. First option.

1) The existence of a stationary point of $\mathcal{V}(z)$ for $0 < z < z_h$: $\mathcal{V}'\Big|_{z_{DW}} = 0$.



$$\ell \underset{z_* \to z_{DW}}{\sim} \frac{1}{\sqrt{F(z_{DW})}} \sqrt{\frac{\mathcal{V}(z_{DW})}{\mathcal{V}''(z_{DW})}} \log(z_{DW} - z_*),$$

$$\mathcal{S} \underset{z_* \to z_{DW}}{\sim} M(z_{DW}) \sqrt{\frac{\mathcal{V}(z_{DW})}{\mathcal{V}''(z_{DW})}} \log(z_{DW} - z_*).$$

$$\mathcal{S} \sim \sigma_{DW} \cdot \ell,$$

$$\sigma_{DW} = M(z_{DW}) \sqrt{F(z_{DW})}.$$

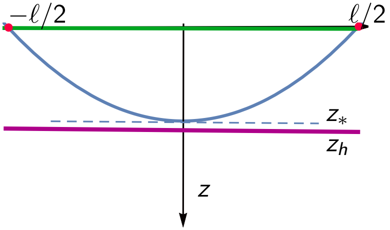
Born-Infeld type action. Second option.

2) There is no stationary point of $\mathcal{V}(z)$ in the region $0 < z < z_h$ and we suppose it to be near horizon

$$F(z) = \mathfrak{F}(z_h)(z_h - z) + \mathcal{O}((z_h - z)^2),$$

if $M(z) \underset{z \to z_h}{\to} \infty$ as

$$M(z) \underset{z \sim z_h}{\sim} \frac{\mathcal{M}(z_h)}{\sqrt{z_h - z}},$$



$$\ell \underset{z_* \to z_h}{\sim} \frac{1}{\sqrt{\mathfrak{F}(z_h)}} \frac{1}{\sqrt{-\frac{2\mathcal{V}'(z_h)}{\mathcal{V}(z_h)}}} \log(z_h - z_*),$$

$$\mathcal{S} \underset{z_* \to z_h}{\sim} \mathcal{M}(z_h) \frac{1}{\sqrt{-\frac{2\mathcal{V}'(z_h)}{\mathcal{V}(z_h)}}} \log(z_h - z_*).$$

$$\sigma_h = \mathcal{M}(z_h)\sqrt{\mathfrak{F}(z_h)} = M(z_h)\sqrt{F(z_h)}.$$

DW equations for SWLs

The equations for the DW for SWL in particular cases for different orientations:

$$xY_1 \text{ and } Xy_1: \quad \left.\frac{2b_s'(z)}{b_s(z)} + \frac{\mathfrak{g}_1'(z)}{\mathfrak{g}_1(z)} + \frac{\mathfrak{g}_2'(z)}{\mathfrak{g}_2(z)} - \frac{4}{z}\right|_{z=z_{DW}} = 0,$$

$$xY_2: \quad \left.\frac{2b_s'(z)}{b_s(z)} + \frac{\mathfrak{g}_1'(z)}{\mathfrak{g}_1(z)} + \frac{\mathfrak{g}_3'(z)}{\mathfrak{g}_3(z)} - \frac{4}{z}\right|_{z=z_{DW}} = 0,$$

$$y_1Y_2: \quad \left.\frac{2b_s'(z)}{b_s(z)} + \frac{\mathfrak{g}_2'(z)}{\mathfrak{g}_2(z)} + \frac{\mathfrak{g}_3'(z)}{\mathfrak{g}_3(z)} - \frac{4}{z}\right|_{z=z_{DW}} = 0.$$

String tension for SWLs vs drag forces

For solution $\mathfrak{g}_1 = 1$, $\mathfrak{g}_2 = (z/L)^{2-2/\nu}$, $\mathfrak{g}_3 = (z/L)^{2-2/\nu} e^{c_B z^2}$:

$$1) \quad \sigma_{xY_1} = \sigma_{Xy_1} = \left(\frac{L^2 b_s(z)}{z^2}\right) \sqrt{\mathfrak{g}_1\mathfrak{g}_2} = \left(\frac{L^{1+1/\nu} b_s(z)}{z^{1+1/\nu}}\right),$$

$$2) \quad \sigma_{xY_2} = \left(\frac{L^2 b_s(z)}{z^2}\right) \sqrt{\mathfrak{g}_1\mathfrak{g}_3} = \left(\frac{L^{1+1/\nu} b_s(z)}{z^{1+1/\nu}}\right) e^{c_B z^2/2},$$

$$3) \quad \sigma_{y_1 Y_2} = \left(\frac{L^2 b_s(z)}{z^2}\right) \sqrt{\mathfrak{g}_2\mathfrak{g}_3} = \left(\frac{L^{2/\nu} b_s(z)}{z^{2/\nu}}\right) e^{c_B z^2/2},$$

where $z = z_h$ or $z = z_{DW}$ (if the DW exists). The answers can be compared with drag forces

S. J. Sin and I. Zahed, Phys.Lett. B **648**, 318 (2007),
O. Andreev, Mod. Phys. Lett. A **33**, 06 (2018),
I. Aref'eva, Phys.Part.Nucl. **51** 4, 489-496 (2020).

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The drag forces for metric with $\mathfrak{g}_1 = 1$:

$$p_x = v_x \frac{b_s(z)}{z^2} \qquad p_{y_1} = v_{y_1} \frac{b_s(z)}{z^2} \mathfrak{g}_2(z) \qquad p_{y_2} = v_{y_2} \frac{b_s(z)}{z^2} \mathfrak{g}_3(z),$$

$v_x = v\sqrt{\mathfrak{g}_2}$, $\quad v_{y_1} = v\frac{\sqrt{\mathfrak{g}_3}}{\mathfrak{g}_2}$, $\quad v_{y_2} = v\frac{\sqrt{\mathfrak{g}_2}}{\sqrt{\mathfrak{g}_3}}$ with some constant v

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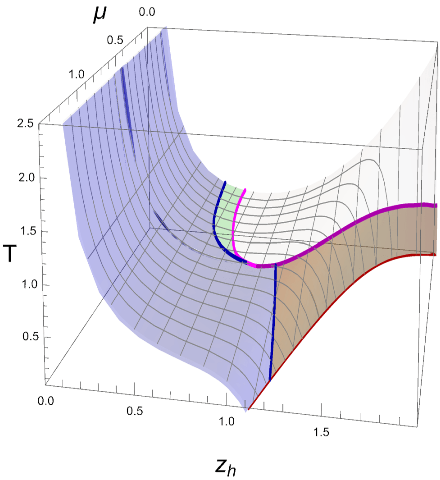
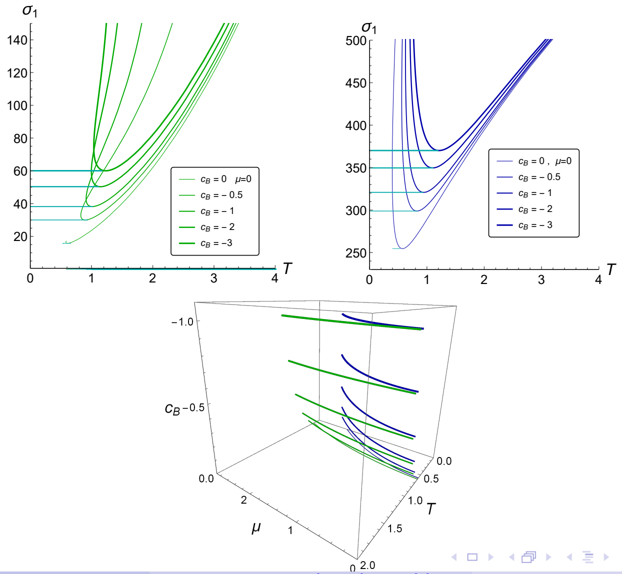
Temperature for heavy quarks model

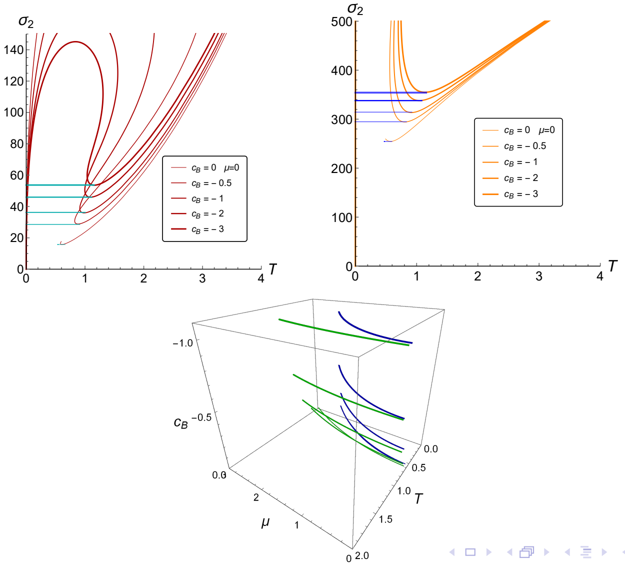


Aref'eva, Hajilou, P.S., Usova, PRD'24

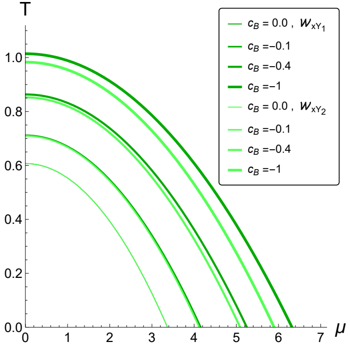
Phase transitions of $\mathcal{V}_1$ for $\nu = 1,\ 4.5$ in magnetic catalysis model with modified warp-factor



Phase transitions of $\mathcal{V}_2$ for $\nu = 1,\ 4.5$ in magnetic catalysis model with modified warp-factor



Phase transitions comparison in particular orientations



Conclusion

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- Under variation of thermodynamic parameters – temperature T, chemical potential μ and magnetic field – the string tensions undergo the phase transition.

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What's next?
Fully anisotropic hybrid model

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Thank you for your attention!

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