Аксиальная аномалия, запутанность и поляризация в соударениях тяжелых ионов
Axial anomaly, entanglement and polarization in Heavy-Ion Collisions

17 Февраля 2025 г
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ОИЯИ

Main Topics

- Axial anomaly in HIC: anomalous transport and polarization
- Gravity in HIC: Highest vorticity and acceleration and Gravitational anomaly

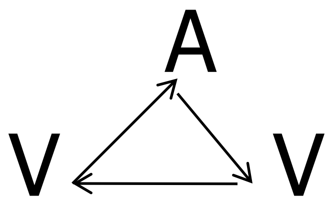
- Axial anomaly and EPR(B) effect: connecting the two discoveries by J.S. Bell
- Anomaly and entanglement in tensor polfrization in HIC

- Conclusions and Outlook

Anomaly in Heavy-Ion Collisions

Delicate QFT phenomenon

Triangle VVA diagram



Free of perturbative (Adler-Bardeen theorem) and non-perturbative (t'Hooft consistency principle) corrections

HIC: various vertices

Vertices in HIC

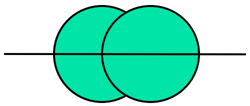
V- external real or virtual photon, current, external potential and magnetic field, medium velocity and vorticity (in dense medium with finite chemical potential) μrot v

A- topological field, axial chemical potential, polarization (helicity, chirality)

Chiral Magnetic (Vortical, Separation) Effects

Global polarization

- Global polarization of hyperons (self – analyzing!) normal to REACTION plane



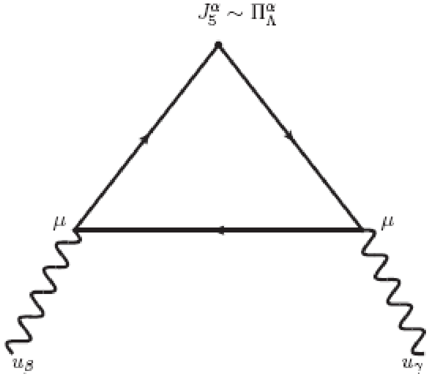
- Predictions (Z.-T.Liang et al.): large orbital angular momentum -> large polarization
- Search by STAR (Selyuzhenkov et al.'07) : polarization NOT found at % level!
- Maybe due to locality of LS coupling while large orbital angular momentum is distributed
- How to transform rotation to spin?

Anomalous mechanism – polarization similar to CM(V)E

- 4-Velocity is also a GAUGE FIELD (V.I. Zakharov et al): $\mu\,\rho = \mu\,J_0\,V^0 \;\text{->}\; \mu\,J_\gamma\,V^\gamma$

$$e_j A_\alpha J^\alpha \Rightarrow \mu_j V_\alpha J^\alpha$$

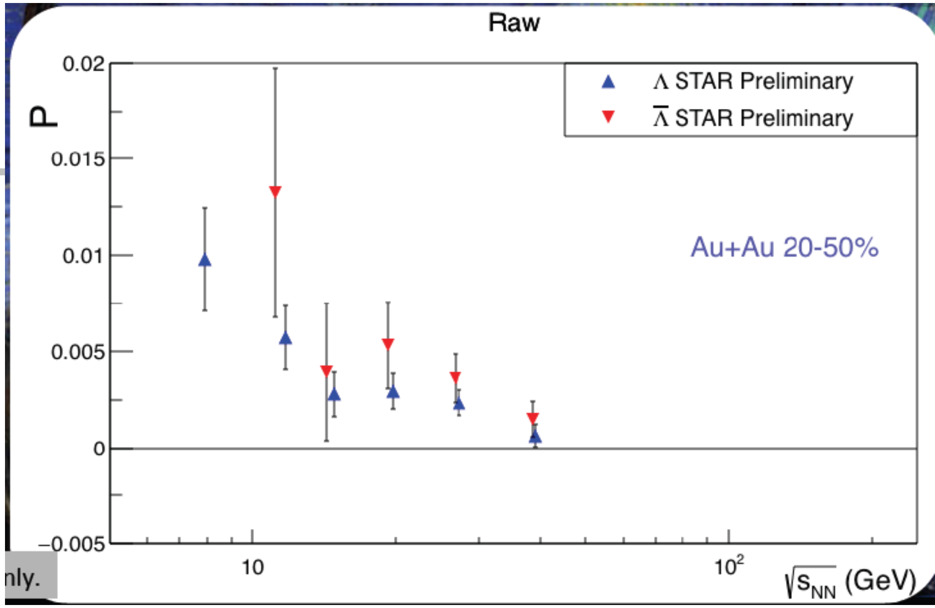
- Triangle anomaly leads to polarization of quarks and hyperons (Rogachevsky, Sorin, OT ’10)
- Analogous to anomalous gluon contribution to nucleon spin (Efremov,OT’88)
- 4-velocity instead of gluon field!

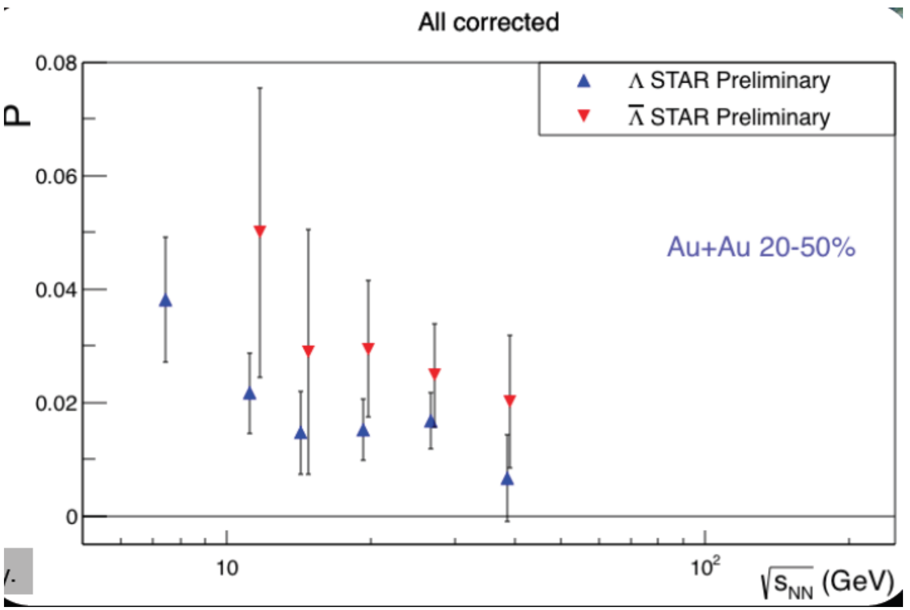


STAR, **Nature 548 (2017) 62-65**

One might compare the prediction below with the right panel figures

O. Rogachevsky, A. Sorin, O. Teryaev
Chiral vortaic effect and neutron asymmetries in heavy-ion collisions
PHYSICAL REVIEW C 82, 054910 (2010)





One would expect that polarization is proportional to the anomalously induced axial current [7]

$$j_A^\mu \sim \mu^2 \left(1 - \frac{2\mu n}{3(\epsilon + P)}\right) \epsilon^{\mu\nu\lambda\rho} V_\nu \partial_\lambda V_\rho, \qquad (6)$$

where n and ϵ are the corresponding charge and energy densities and P is the pressure. Therefore, the μ dependence of polarization must be stronger than that of the CVE, leading to the effect's increasing rapidly with decreasing energy.

This option may be explored in the framework of the program of polarization studies at the NICA [17] performed at collision points as well as within the low-energy scan program at the RHIC.

“Anomalous” mechanism

O. Rogachevsky, A. Sorin, O. Teryaev
Chiral vortaic effect and neutron asymmetries in heavy-ion collisions
PHYSICAL REVIEW C 82, 054910 (2010)

- Prediction of decrease with energy (due to chemical potential)

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- Prediction of P~1%

BAZNAT, GUDIMA, SORIN, AND TERYAEV

PHYSICAL REVIEW C **88**, 061901(R) (2013)

$$\langle P_\Lambda \rangle \sim \frac{\langle \mu^2 \rangle N_c H}{2\pi^2 \langle N_\Lambda \rangle}.$$

For numerical estimate at NICA energies, we take (see Fig. 3) $H = 30\ \mathrm{fm}^2 (c = 1)$ and, as typical values, $\langle \mu^2 \rangle = 900\ \mathrm{MeV}^2$, $\langle N_\Lambda \rangle = 15$ to get $\langle P_\Lambda \rangle \sim 0.8\%$. This value is

- Postdiction of larger polarization of antilambdas

ALEXANDER SORIN AND OLEG TERYAEV

PHYSICAL REVIEW C **95**, 011902(R) (2017)

The proportionality of the polarization to the square of the chemical potential related to C-even parity of axial current leads to the same sign of polarization of Λ and $\bar{\Lambda}$ hyperons. The smaller number of the latter should result in a larger fraction of the axial charge, corresponding to each antihyperon and to a larger absolute value of polarization. Detailed numerical sim-

Effective Gravity in HIC

Anomalous transport: emergent gravity in heavy-ion collisions

[R. Khakimov, G. Prokhorov, O. Teryaev and V. Zakharov, Phys. Rev. D 109, no.10, 105001 (2024), arXiv:2401.09247]

- The relationship between phenomena in a **relativistic fluid** and in **curved space** is shown.

Relativistic fluid (*flat space*)

Examples: matter in collisions of heavy ions (QGP, hadron plasma)

Features: has vorticity ω_μ and acceleration a_μ, but space is flat

Considered phenomena:

- Novel nondissipative transport effect in the axial current

$$j^A_\mu = \lambda_1 \omega^2 \omega_\mu + \lambda_2 a^2 \omega_\mu$$

«Kinematical Vortical Effect» (KVE)

Curved space

Examples: early Universe, black holes

Features: finite curvature

Considered phenomena:

- Famous gravitational chiral anomaly

$$\nabla j_A = \mathcal{N} \epsilon^{\mu\nu\alpha\beta} R_{\mu\nu\rho\sigma} R_{\alpha\beta}{}^{\rho\sigma}$$

- New effect of **cosmological constant** Λ

$$j^A_\mu = \lambda_\Lambda \Lambda \omega_\mu$$

- The **gravitational chiral anomaly** controls the effect in the **vortical accelerated fluid**:

  (generalization of ***[Prokhorov, Teryaev, Zakharov, PRL, 2022]*** *to the case with* ***nonzero Ricci tensor****)*

- The relationship between the effects of **acceleration** and **cosmological constant**:

Dualities for conductivities λ:

$$\lambda_1 - \lambda_2 = 32\mathcal{N}$$

$$\lambda_2 = -3\lambda_\Lambda$$

"Эквивалентная" гравитация
HIC: Strong interactions and superstrong "EP-gravity"

- $E_{EM}/E_G \sim e^2/(m/M_{Pl})^2$ $\qquad M_{Pl} \sim 10^{18}$ GeV
- For 2 particles with M_{Pl} mass at Compton wavelength distance $(1/M_{Pl})$: $E_G \sim (G = 1/M_{Pl}^2)\, M_{Pl}^2 / (1/M_{Pl}) = M_{Pl}$
  $g \sim (G = 1/M_{Pl}^2)\, M_{Pl} / (1/M_{Pl})^2 = M_{Pl}$
- Gravitational interaction is strongly suppressed wrt strong one $\sim (\Lambda/M_{Pl})^2$
- Equivalence Principle
- I: Acceleration <-> Gravity
- HIC: $a \sim \Lambda$, $a/g \sim \frac{c^2}{v_\oplus^2} \cdot \frac{R_\oplus}{R_A} \sim 10^{30}$

$$\eta_{\rm rot} = \frac{\Omega}{\omega_\oplus} = \frac{c}{R_A} \cdot \frac{T_\oplus}{2\pi} = \frac{1}{2\pi} \cdot \frac{cT_\oplus}{R_A} \approx 10^{27}$$

- M_{Pl} -> Λ ("GeV Gravity" to be compared with 'TeV gravity" from extra dimensions)

Axial anomaly and entanglement in HIC (OT, in preparation)

Two famous works by J.S. Bell

EPR(BA) – spin correlations of photons in pion decay

$$M^{P\to\gamma\gamma} = C\varepsilon^{e_1^* e_2^* kp} = Cq^2\varepsilon^{e_1^* e_2^*}$$

1 ⟵——— P ———⟶ 2 $|M^{P\to\gamma\gamma}|^2 = N(1-|(e_1^* e_2)|^2) = N(1 - Tr(\rho_1\rho_2))$

Final and detected density matrices $dw \sim Tr(\rho_a \rho_a^f)$

$$\rho_1^f = I - \rho_2, \ \ \rho_2^f = I - \rho_1 \qquad \rho_a = \frac{1}{2}(I + \sigma_i \xi_{i,a}) \qquad \xi_{i,1}^f = -\xi_{i,2}, \ \ \xi_{i,2}^f = -\xi_{i,1}$$

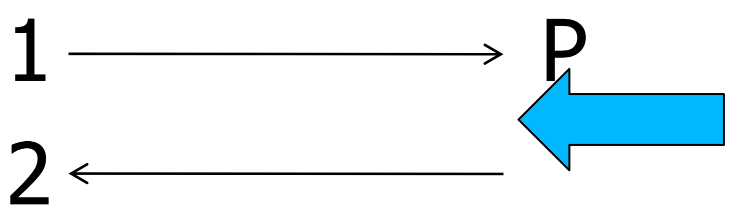
Measurement defines state of other photon before its measurement

Acausality of EPR

Lorentz invariance – no causal connection

Circular polarization – (nonlocal!) angular momentum conservation

Covariant description of EPR – "time-like" entanglement: virtual pion in Breit frame (say, in Sullivan process)



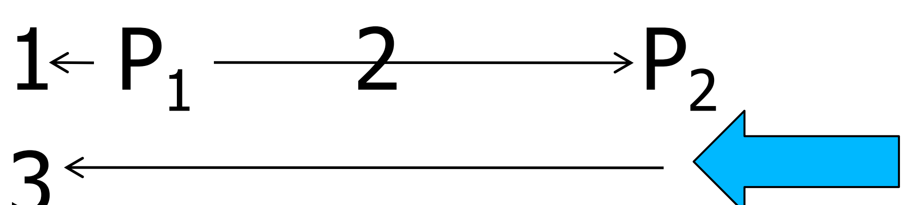
Time-like EPR: backward measurement

Density matrices $M^{\gamma P \to \gamma} = C \varepsilon^{e_1 e_2^* k p} = C q^2 \varepsilon^{e_1 e_2^*}$

$$|M^{\gamma P \to \gamma}|^2 = N(1 - |(e_1 e_2)|^2) = N(1 - Tr(\rho_1^T \rho_2)) \qquad \xi^f_{i,2} = -\xi_{i,2}(i = 1, 3), \;\; \xi^f_{2,2} = \xi_{2,1}$$

Polarization plane rotation and angular momentum conservation

Measurement of final (3)circular polarization measures also the initial (2) one besides "usual" EPR(BA) 1 !



Entanglement in HIC

- $V(H/\mu \text{rot}\, v)V(\gamma^* \to l^+ l^-)A(P)$
- Pseudoscalar P decay to lepton pair in magnetized (or vortical) medium)
- EPR: longitudinal polarization wrt to normal to reaction plane (some indications in STAR/ALICE data)
- Measurement of lepton helicity: backward meaurement of γ^* helicity
- Smallness in single event: suppressed as λ/L : ratio of de Broglie length to the typycal scale of magnetic field variation in detector (can smallness of L in HIC be used?!)

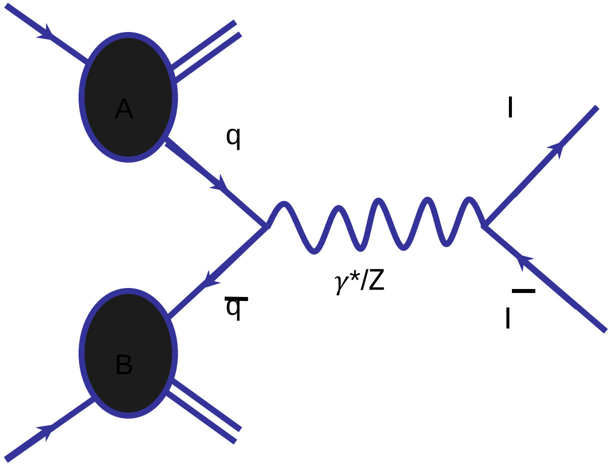
Conclusions/Outlook

- Anomaly in HIC is sensitive to effective and "Equivalent" gravity: HIC experiments (in particular at NICA) are also this gravity labs
- Anomalous pseudoscalar decay combines two famous works of J.S. Bell
- EPR(BA) effect is acausal which is especially clear in covariant description: measurement of the past!
- Entanglement in HIC: decay P-> l^+l^-
- Longitudinal polarization and principle possibility of backward measurement

BACKUP

Angular distribution

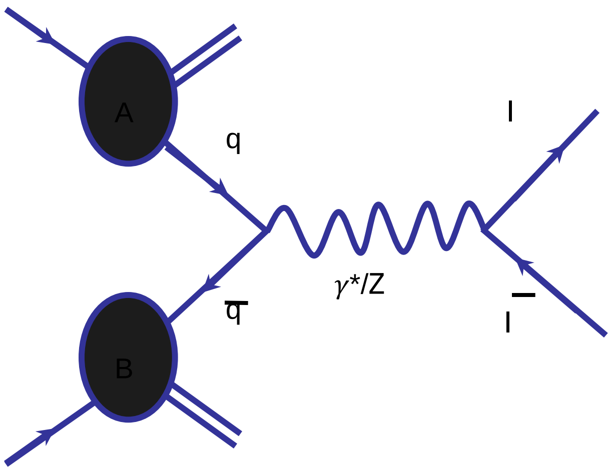
general form of angular distribution (wrt particular frame)



$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi}\frac{1}{3+\lambda_\theta}\left(1+\lambda_\theta\cos^2\theta+\lambda_{\theta\phi}\sin 2\theta\cos\phi\right.$$
$$+\ \lambda_\phi\sin^2\theta\cos 2\phi+\lambda_{\perp\phi}\sin^2\theta\sin 2\phi$$
$$+\ \lambda_{\perp\theta\phi}\sin 2\theta\sin\phi+2A_\theta\cos\theta$$
$$\left.+\ 2A_\phi\sin\theta\cos\phi+2A_{\perp\phi}\sin\theta\sin\phi\right)$$

Angular distribution

general form of angular distribution :



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$$+ \lambda_{\perp\theta\phi}\sin 2\theta\sin\phi+2A_\theta\cos\theta$$
$$\left.+ 2A_\phi\sin\theta\cos\phi+2A_{\perp\phi}\sin\theta\sin\phi\right)$$

Invariants

➔ Facilitate comparison b/w experiments, theory and experiment

➔ Reveal systematic biases

Positivity for dilepton angular distribution

- Angular distribution

$$d\sigma \propto 1 + \lambda \cos^2\theta + \mu \sin 2\theta \cos\phi + \frac{\nu}{2}\sin^2\theta \cos 2\phi + \rho \sin 2\theta \sin\phi + \sigma \sin^2\theta \sin 2\phi$$

- Positivity of the matrix (= hadronic tensor in dilepton rest frame)

$$M_0 = \begin{pmatrix} \frac{1-\lambda}{2} & \mu & \rho \\ \mu & \frac{1+\lambda-\nu}{2} & \sigma \\ \rho & \sigma & \frac{1+\lambda+\nu}{2} \end{pmatrix}$$

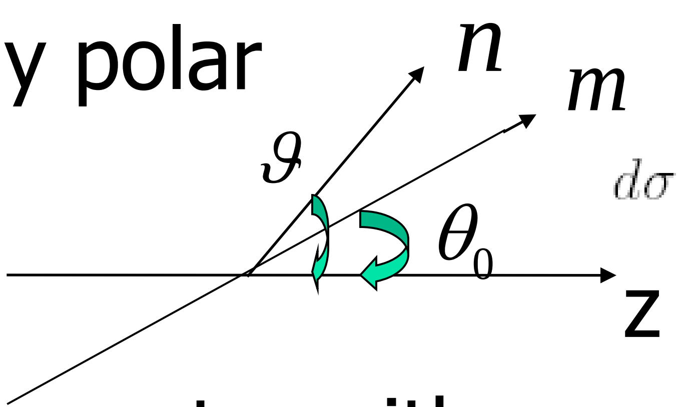
$$|\lambda| \le 1,\ |\nu| \le 1+\lambda,\ \mu^2 \le \frac{(1-\lambda)(1+\lambda-\nu)}{4}$$

$$\rho^2 \le \frac{(1-\lambda)(1+\lambda+\nu)}{4},\ \sigma^2 \le \frac{(1+\lambda)^2-\nu^2}{4}$$

- + cubic – det $M_0 > 0$
- 1st line – Lam&Tung by SF method

Kinematic azimuthal asymmetry from polar one

Only polar



$$d\sigma \propto 1 + \lambda_0 (\vec{n}\vec{m})^2 = 1 + \lambda_0 \cos^2 \theta_{nm}^2$$

asymmetry with respect to m!

$$\cos\theta_{nm} = \cos\theta \cos\theta_0 + \sin\theta \sin\theta_0 \cos\phi$$

- azimuthal angle appears with new

$$\lambda = \lambda_0 \frac{2 - 3\sin^2\theta_0}{2 + \lambda_0 \sin^2\theta_0}$$

$$\nu = \lambda_0 \frac{2 sin^2 \theta_0}{2 + \lambda_0 \sin^2\theta_0}$$

Matching with pQCD results (J. Collins, PRL 42,291,1979)

- Direct comparison: $\tan^2 \theta_0 = (k_T/Q)^2$
- New ingredient – expression for μ
- Linear in k_T
- Saturates positivity constraint!
- Extra probe of transverse momentum

Generalized Lam-Tung relation (OT'05)

- Relation between coefficients (high school math sufficient!)

$$\lambda_0 = \frac{\lambda + \frac{3}{2}\nu}{1 - \frac{1}{2}\nu}$$

- Reduced to standard LT relation for transverse polarization (λ_0 =1)
- LT - contains two very different inputs: kinematical asymmetry+transverse polarization

Generalized Lam-Tung relation

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- LT - contains two very different inputs: kinematical asymmetry+transverse polarization
- Non-coplanarity – violation of LT (talk of W.-C. Chang)

General approach: the number of independent invariants

- → 8 parameters describe distribution
- → 3 Euler angles parameterize rotation → **8 - 3 = 5 independent invariants**

Handedness: directly observable P-odd momentum correlations

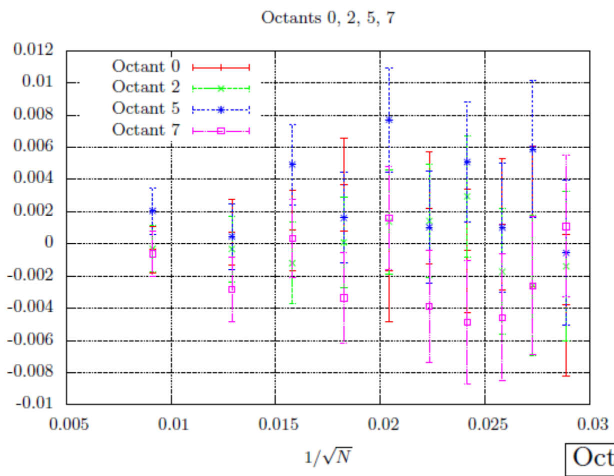
- Found in jets in e^+e^- annihilation (LEP, BELLE)
- First attempt in HIC: OT, Usubov, arXiv:1406.4451 (to appear in PRC, poster by Rahim Usubov)
- Average =0: Phase space – 8 octants

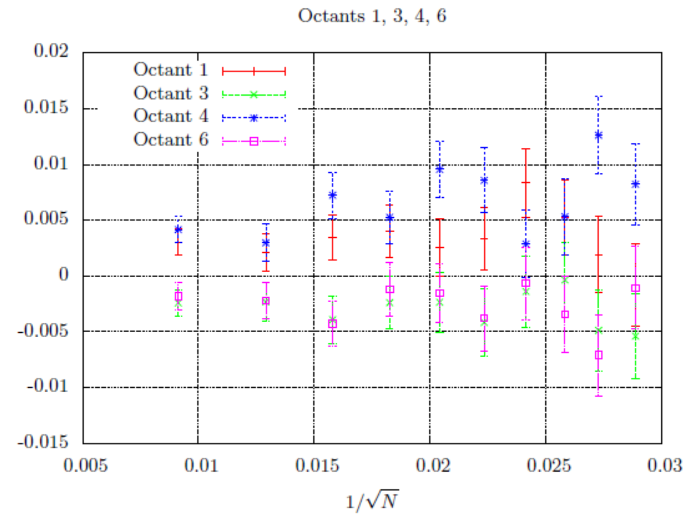
Octant	Momentum
0	$p_x > 0, p_y > 0, p_z > 0$
1	$p_x > 0, p_y > 0, p_z \leq 0$
2	$p_x > 0, p_y \leq 0, p_z > 0$
3	$p_x > 0, p_y \leq 0, p_z \leq 0$
4	$p_x \leq 0, p_y > 0, p_z > 0$
5	$p_x \leq 0, p_y > 0, p_z \leq 0$
6	$p_x \leq 0, p_y \leq 0, p_z > 0$
7	$p_x \leq 0, p_y \leq 0, p_z \leq 0$

$$\eta = \frac{\sum(\vec{p_3}, \vec{p_2}, \vec{p_1})}{\sum |(\vec{p_3}, \vec{p_2}, \vec{p_1})|}$$

Handedness separation

- Indication for small separation effect in some of the octants





Octant	Momentum
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3	$p_x > 0, p_y \leq 0, p_z \leq 0$
4	$p_x \leq 0, p_y > 0, p_z > 0$
5	$p_x \leq 0, p_y > 0, p_z \leq 0$
6	$p_x \leq 0, p_y \leq 0, p_z > 0$
7	$p_x \leq 0, p_y \leq 0, p_z \leq 0$

From (chiral) quarks to hadrons: quark-hadron duality via axial charge

- Induced axial charge

$$c_V = \frac{\mu_s^2 + \mu_A^2}{2\pi^2} + \frac{T^2}{6}, \quad Q_5^s = N_c \int d^3x\, c_V \gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$

- Neglect axial chemical potential
- $\mu_{s(q)}$ -> $\mu_B/3 - \mu_S$
- T-dependent term (Landsteiner's gravity anomaly); no π^2 in denominator : "hint" for role of Unruh effect ($T=a/2\pi$; poster #130 by G. Prokhorov)
- Lattice simulations: suppressed by order of magnitude due to collective effects – responsible for RHIC/LHC polarization?

Energy dependence

- Coupling -> chemical potential

$$Q_5^s = \frac{N_c}{2\pi^2}\int d^3x\, \mu_s^2(x)\gamma^2\epsilon^{ijk}v_i\partial_j v_k$$

- Field -> velocity; (Color) magnetic field strength -> vorticity;
- Topological current -> hydrodynamical helicity
- Large chemical potential: appropriate for NICA/FAIR energies

Microworld: where is the fastest possible rotation?

- Non-central heavy ion collisions (Angular velocity ~ c/Compton wavelength)
- ~25 orders of magnitude faster than Earth's rotation
- Differential rotation – vorticity
- P-odd :May lead to various P-odd effects
- Calculation in kinetic quark - gluon string model (DCM/QGSM) – Boltzmann type eqns + phenomenological string amplitudes): Baznat,Gudima,Sorin,OT, PRC'13,16

Rotation in HIC and related quantities

- Non-central collisions – orbital angular momentum
- $L = \Sigma\, r \times p$
- Differential pseudovector characteristics – vorticity
- ϖ = curl v
- Pseudoscalar – helicity
- H ~ <(v curl v)>
- Maximal helicity – Beltrami chaotic flows v || curl v

Simulation in QGSM (First calculation of vorticity in kinetic model; Baznat, Gudima, Sorin, OT, PRC'13)

$50 \times 50 \times 100$ cells $\quad dx = dy = 0.6$ fm, $dz = 0.6/\gamma$ fm

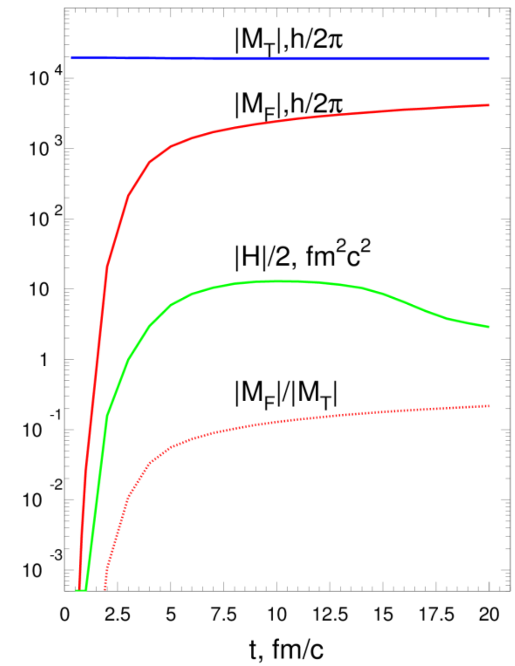
- Velocity

$$\vec{v}(x, y, z, t) = \frac{\sum_i \sum_j \vec{P}_{ij}}{\sum_i \sum_j E_{ij}}$$

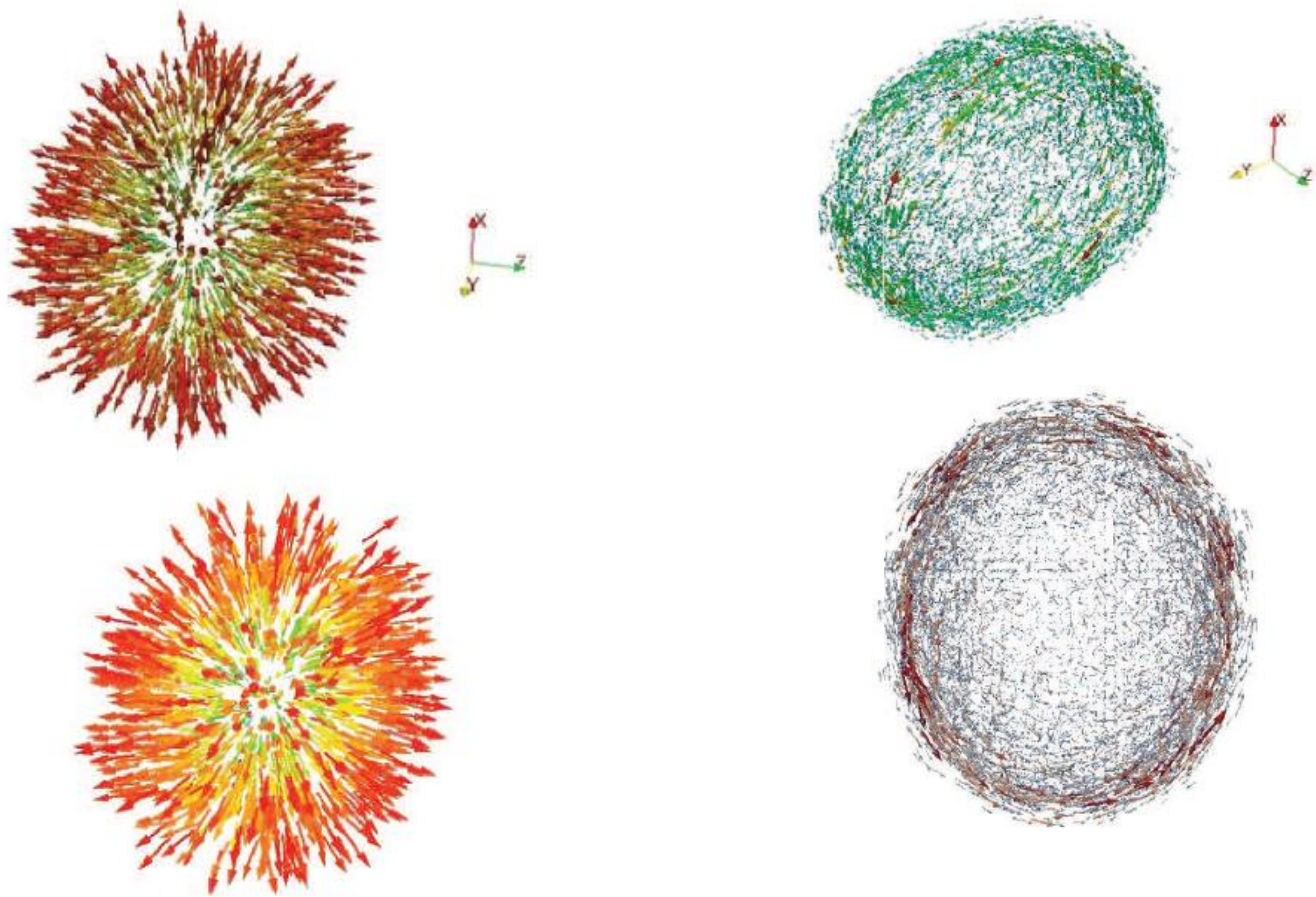
- Vorticity – from discrete partial derivatives

Angular momentum conservation and helicity

- Helicity vs orbital angular momentum (OAM) of fireball
- (~10% of total)
- Conservation of OAM with a good accuracy!

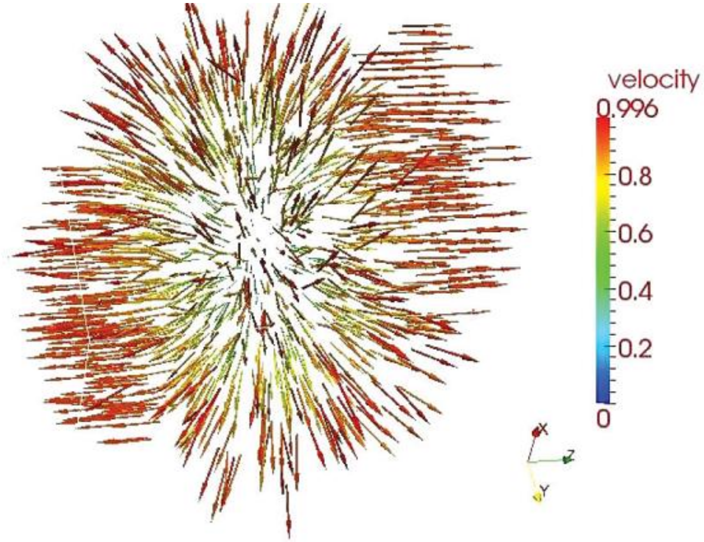


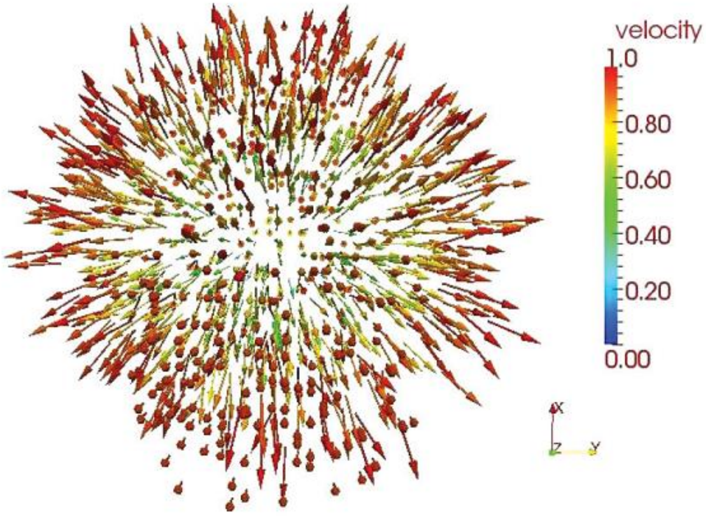
Structure of velocity and vorticity fields (NICA@JINR-5 GeV/c)



Distribution of velocity ("Little Bang")

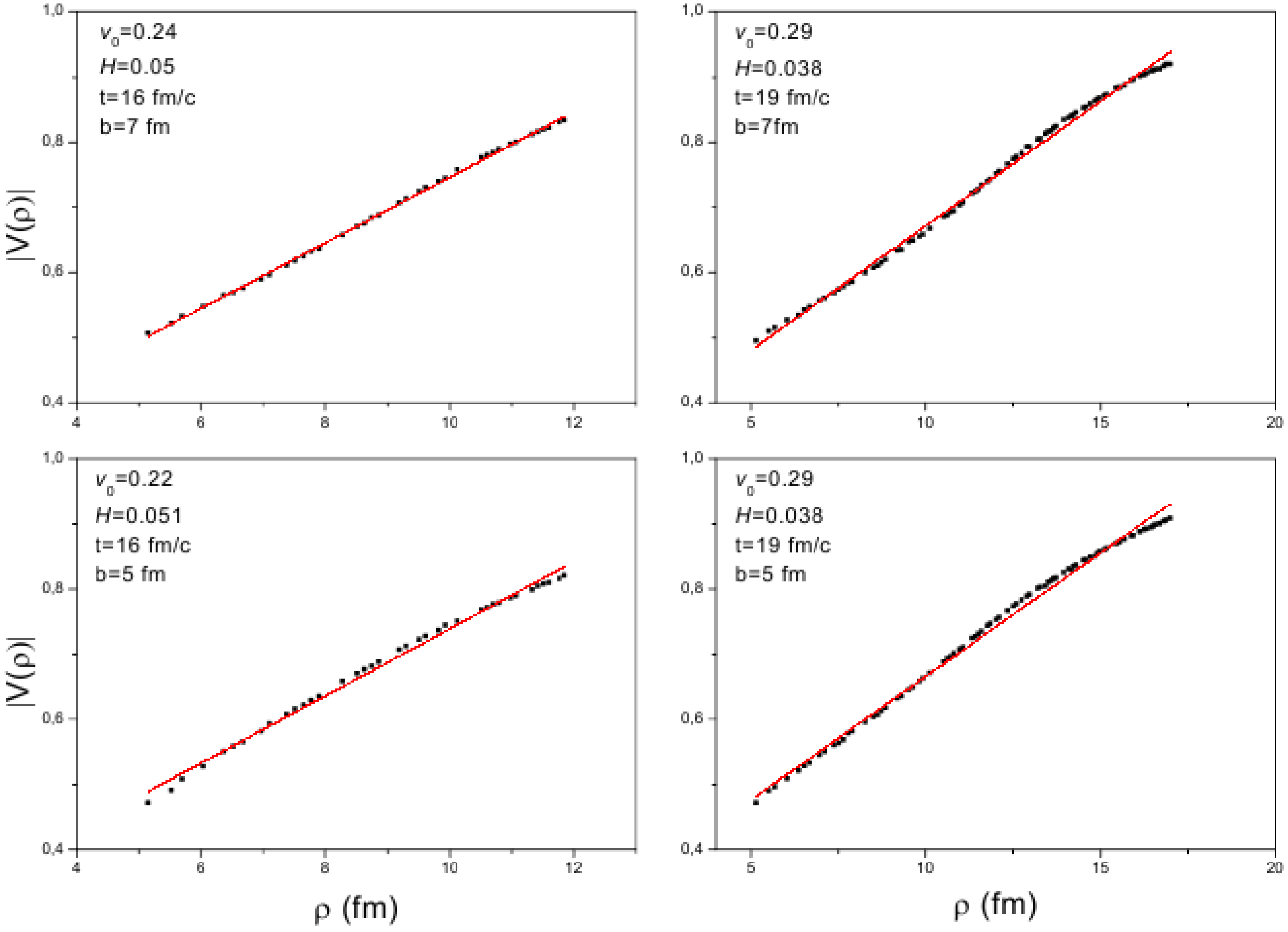
- 3D/2D projection
- z-beams direction
- x-impact paramater





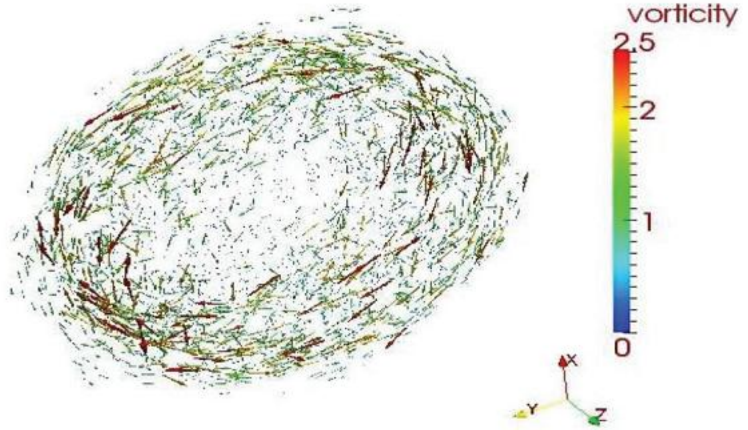
New: "Little Hubble" in PHSD (Baznat,Sorin,OT,Zinchenko, in progress)

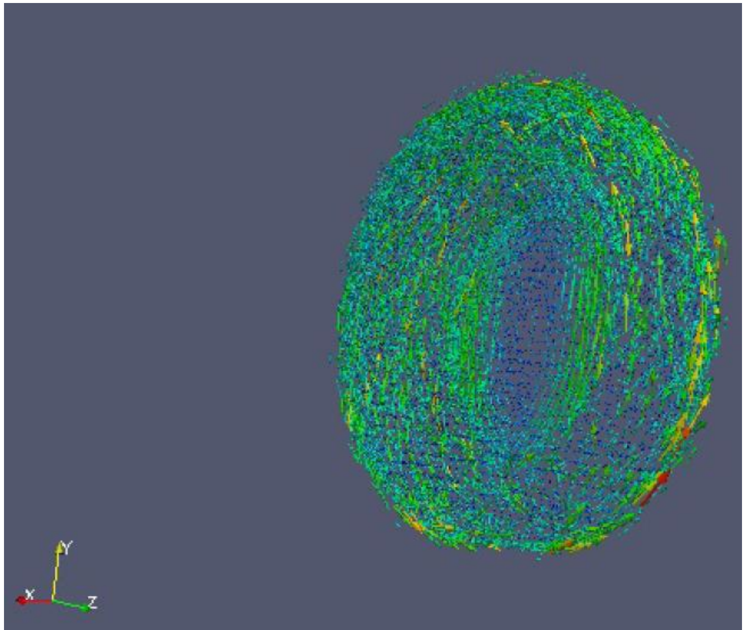
$$v = v_0 + H\rho$$

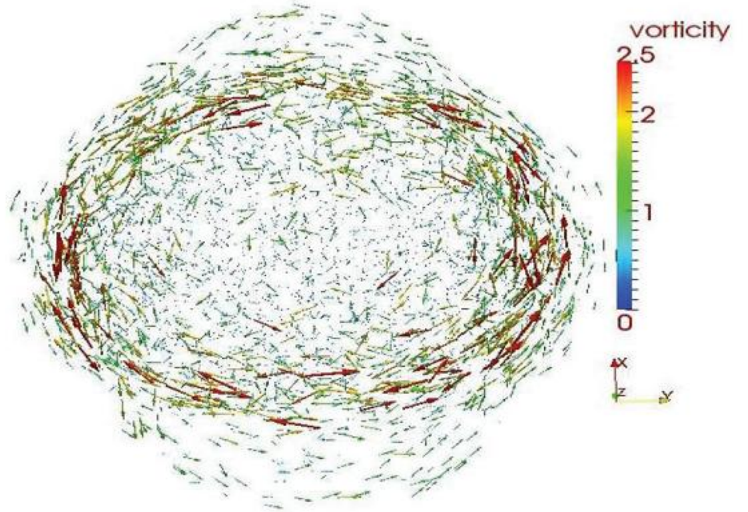


Distribution of vorticity (“Femto-cyclones”, “Little galaxies”)

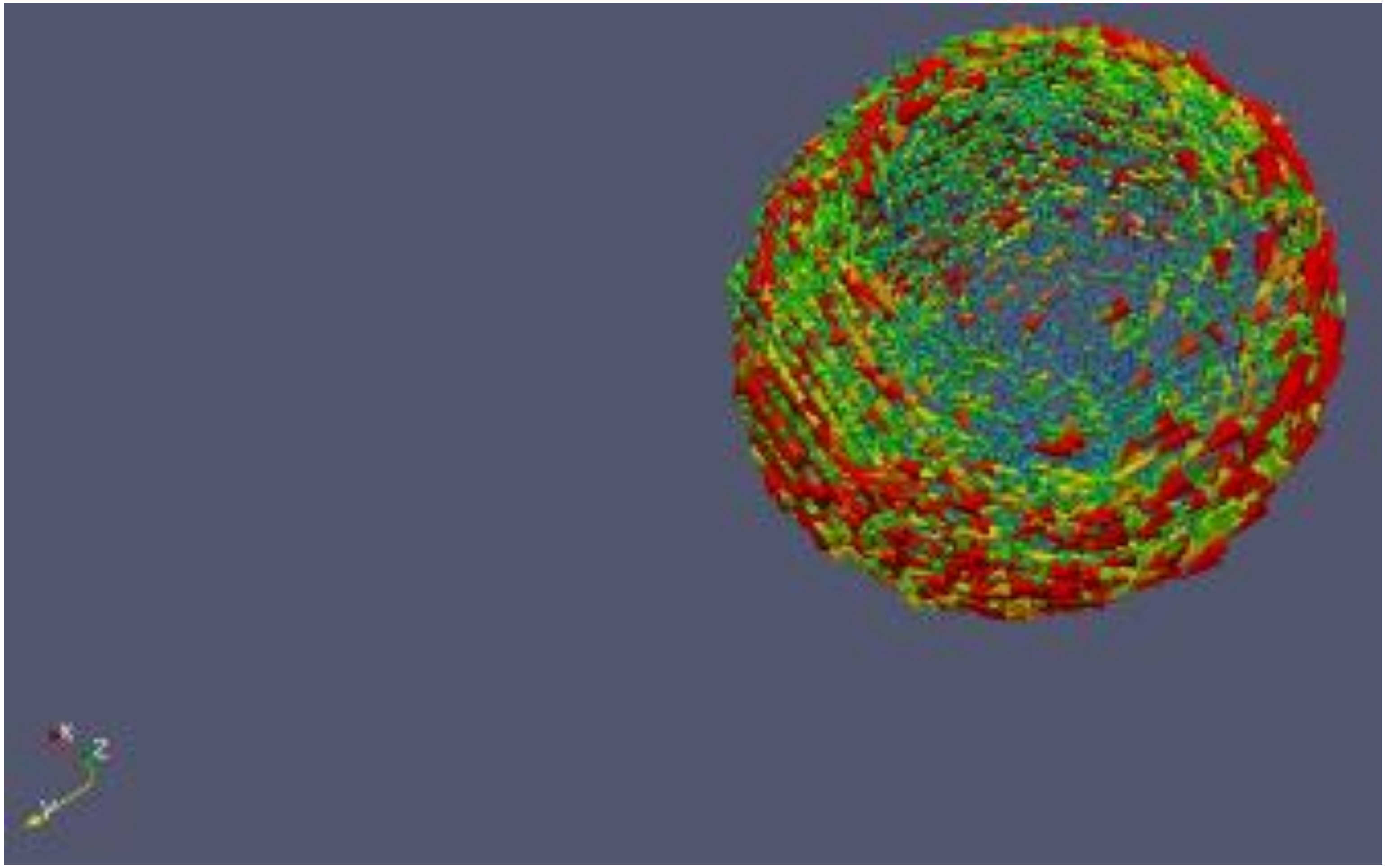
- Layer (on core - corona borderline) patterns



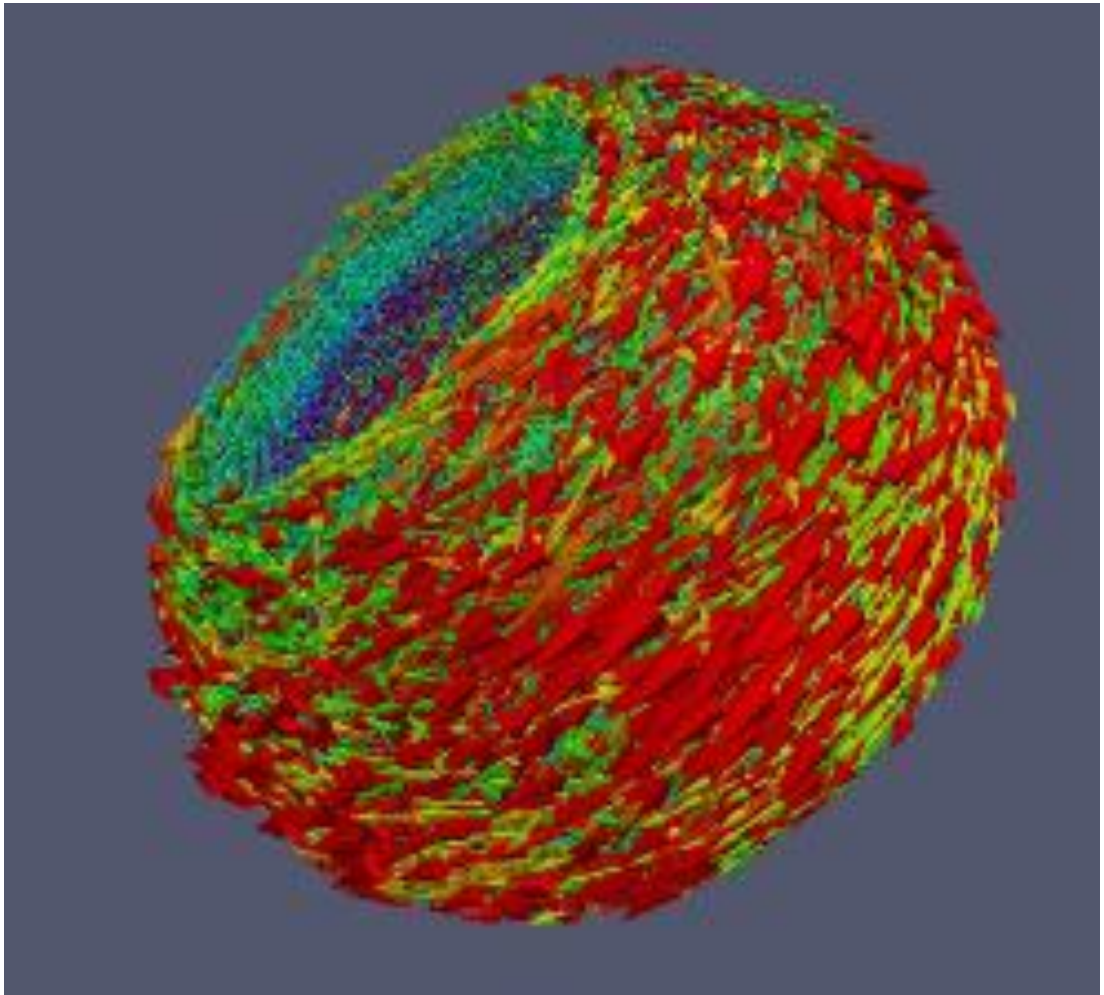




Vortex sheet (Femto-cyclone) with fixed direction of L

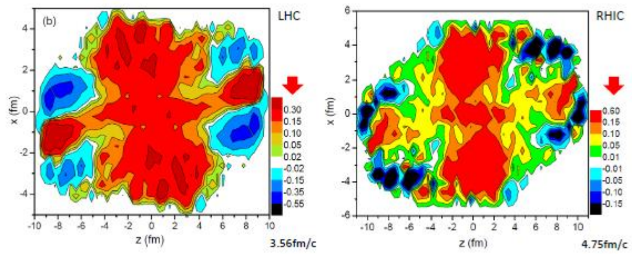


Vortex sheet （ Average over L directions ）

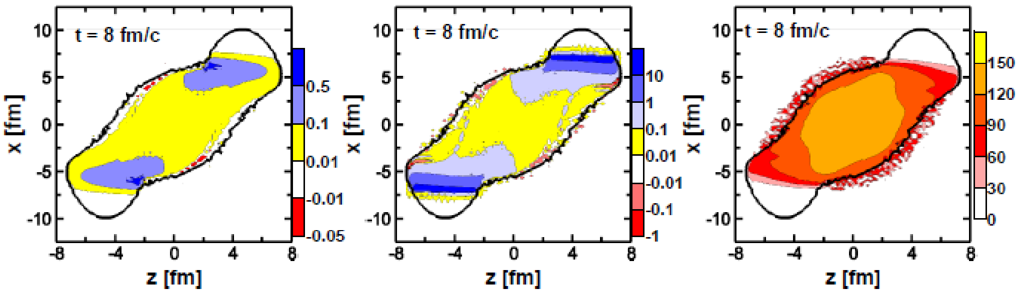


Vortex sheets

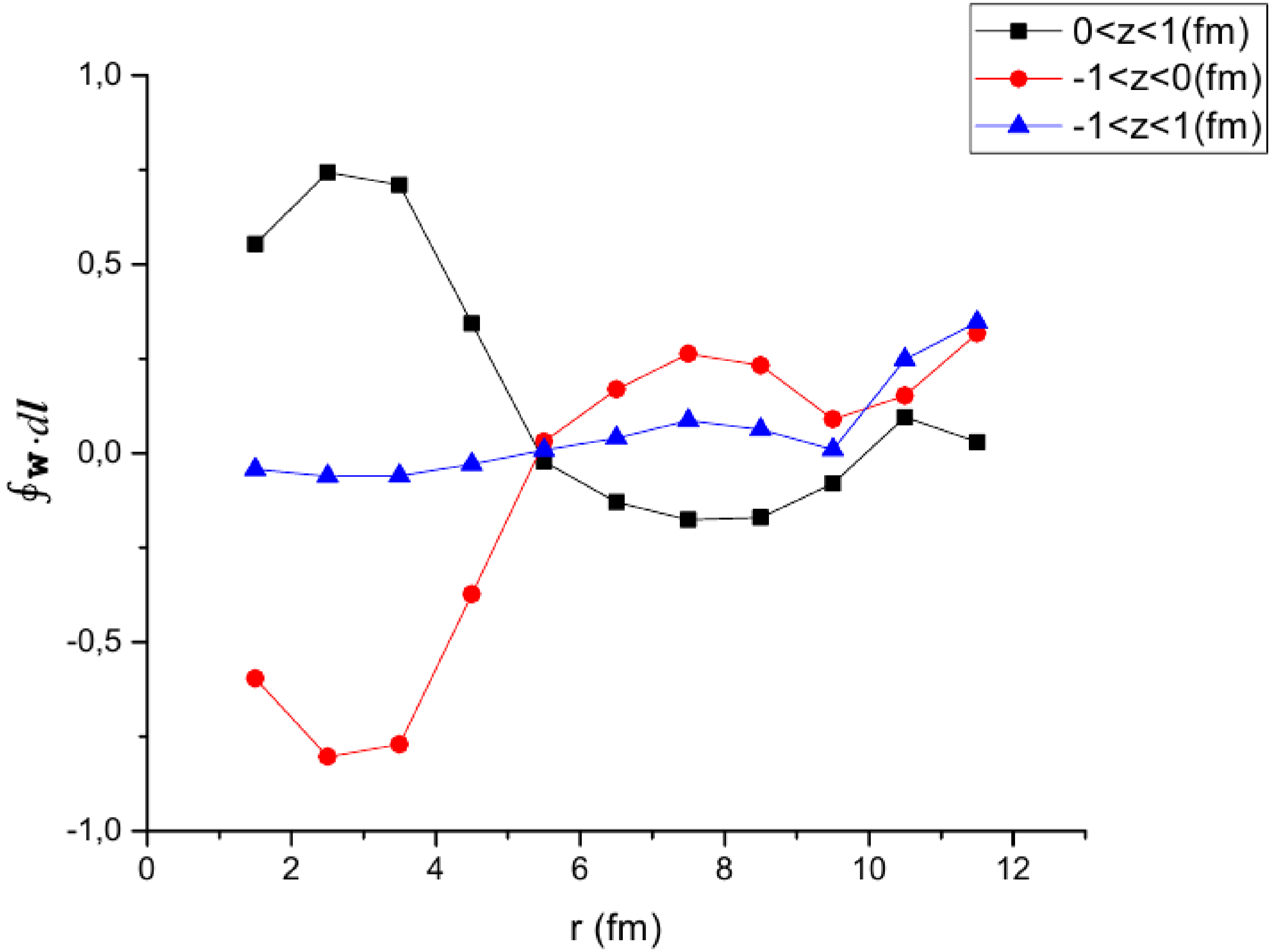
- Naturally appears in kinetic models
- Absent in viscous HD (L. Csernai et al)



- Appears in 3 fluid dynamics model (Yu. Ivanov, A. Soldatov,PRC'17)

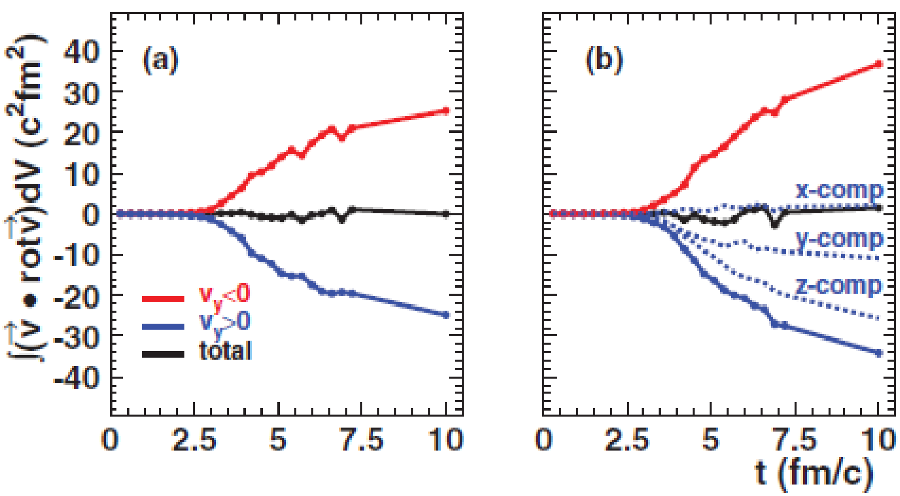


New: Mirror vortex rings in PHSD

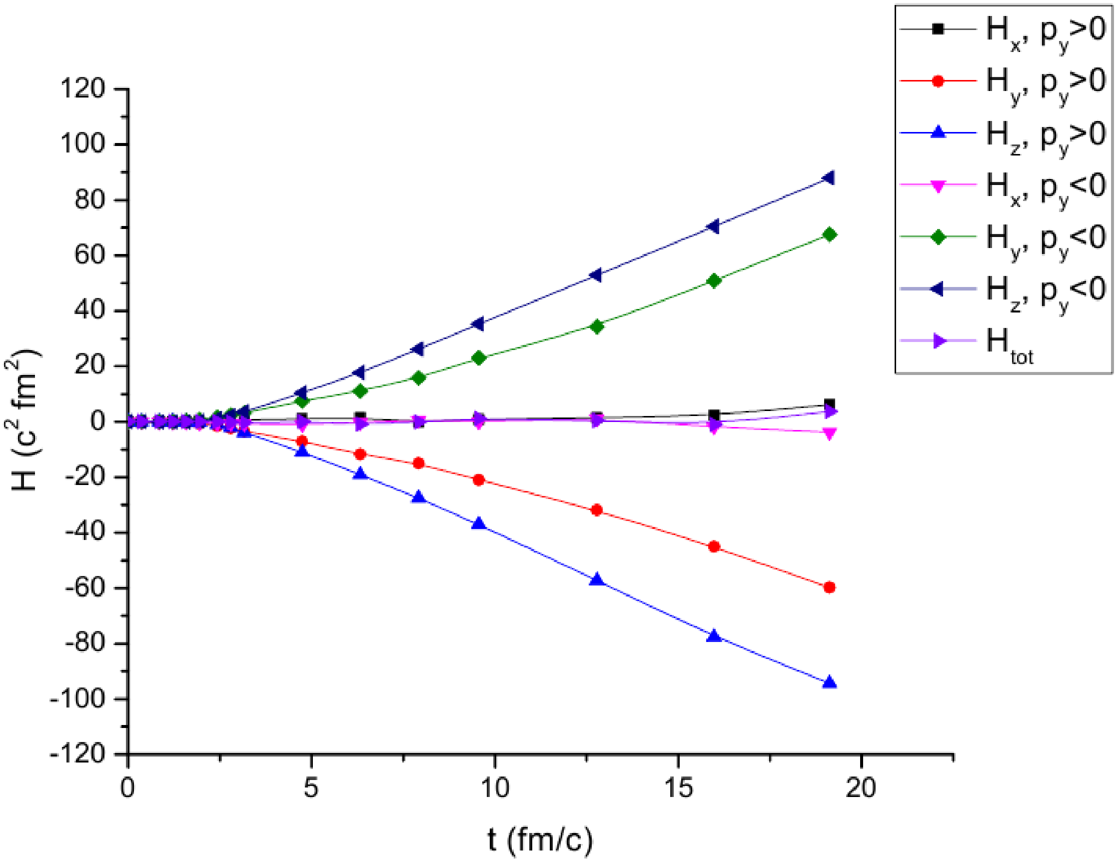


Helicity separation in QGSM **PRC88 (2013) 061901**

- Total helicity integrates to zero BUT
- Mirror helicities below and above the reaction plane
- Confirmed in HSD (OT, Usubov, PR**C92 (2015) 014906**

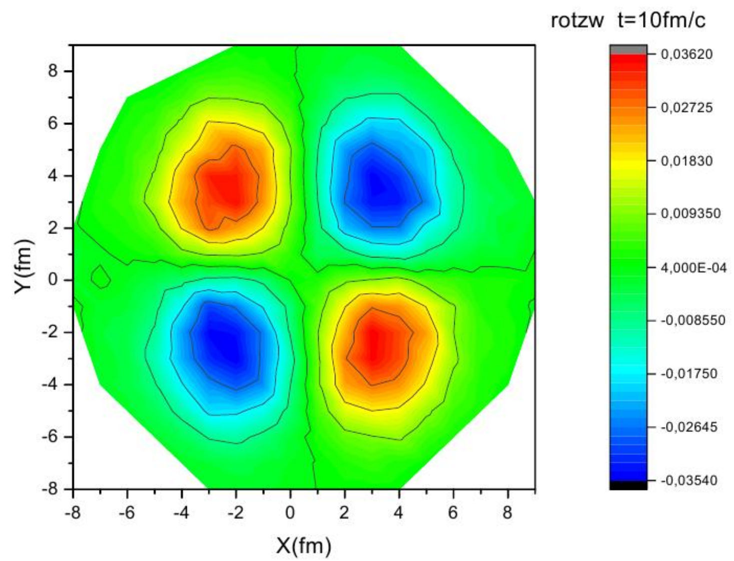


New: Helicity@PHSD

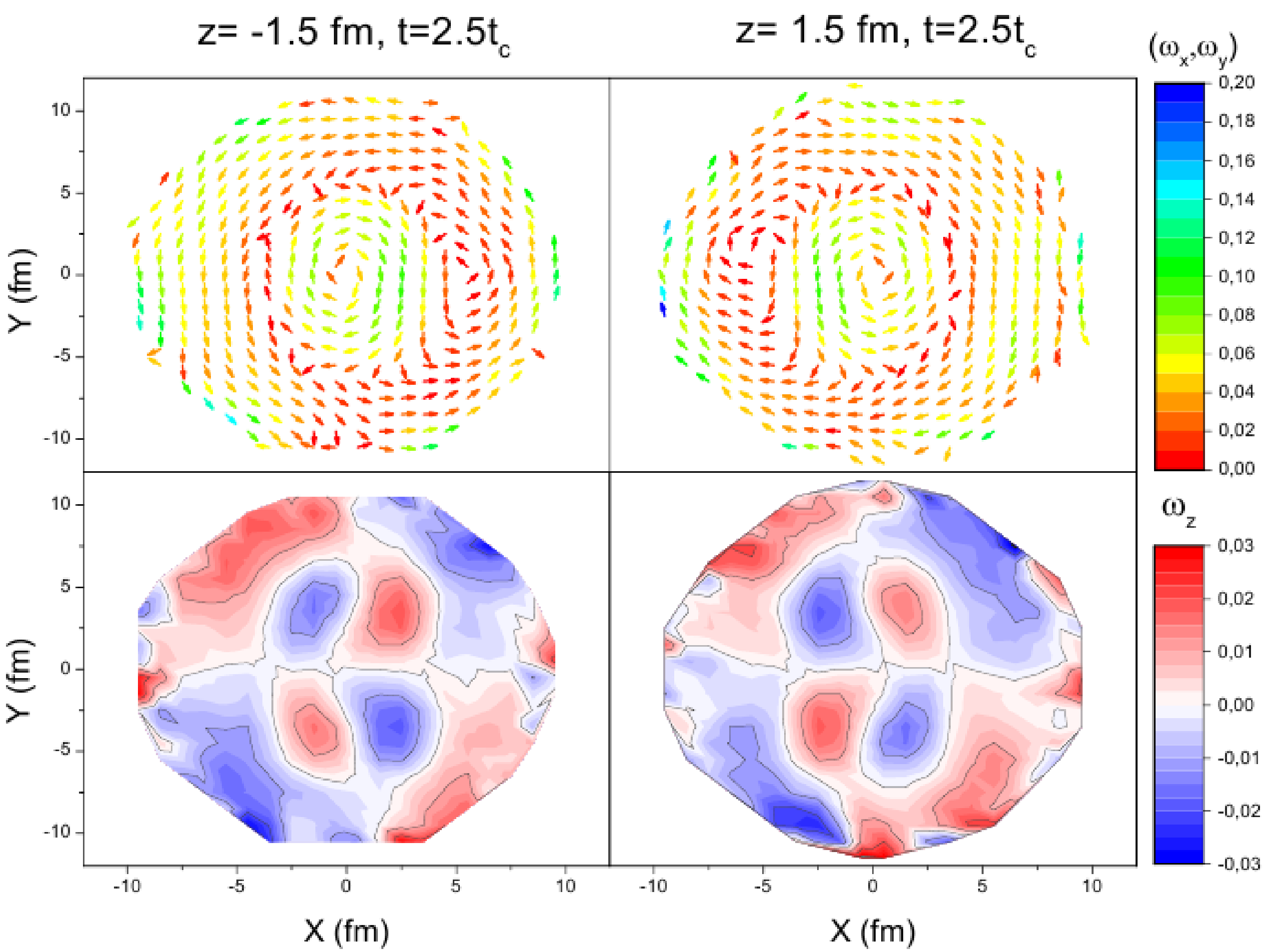


Structure of vorticity (Baznat, Gudima, Sorin, OT'17)

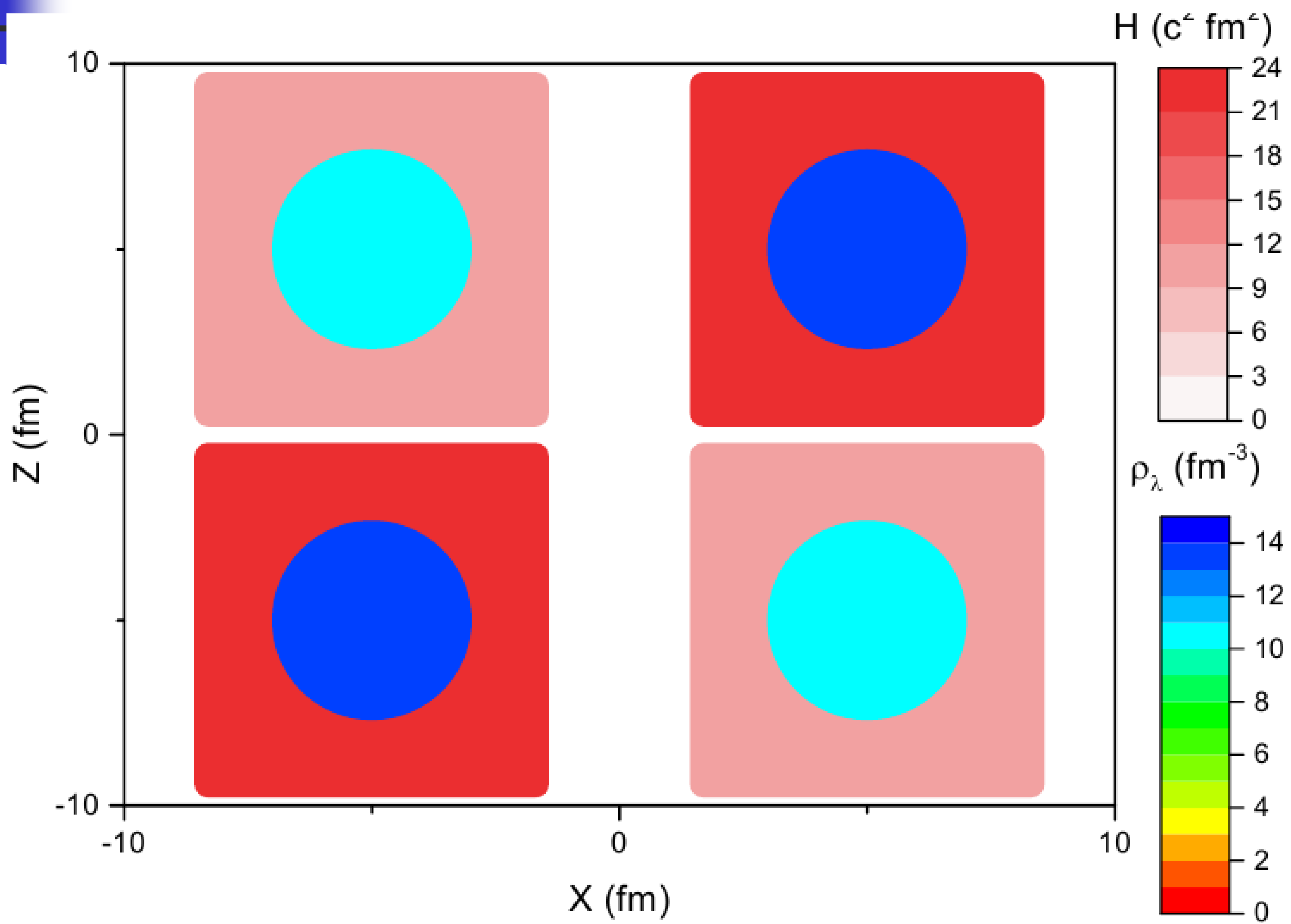
- y-component: constant vorticity, velocity changes sign
- **z-component: quadrupole structure of vorticity**



New: PHSD: 2nd Quadrupole Structure

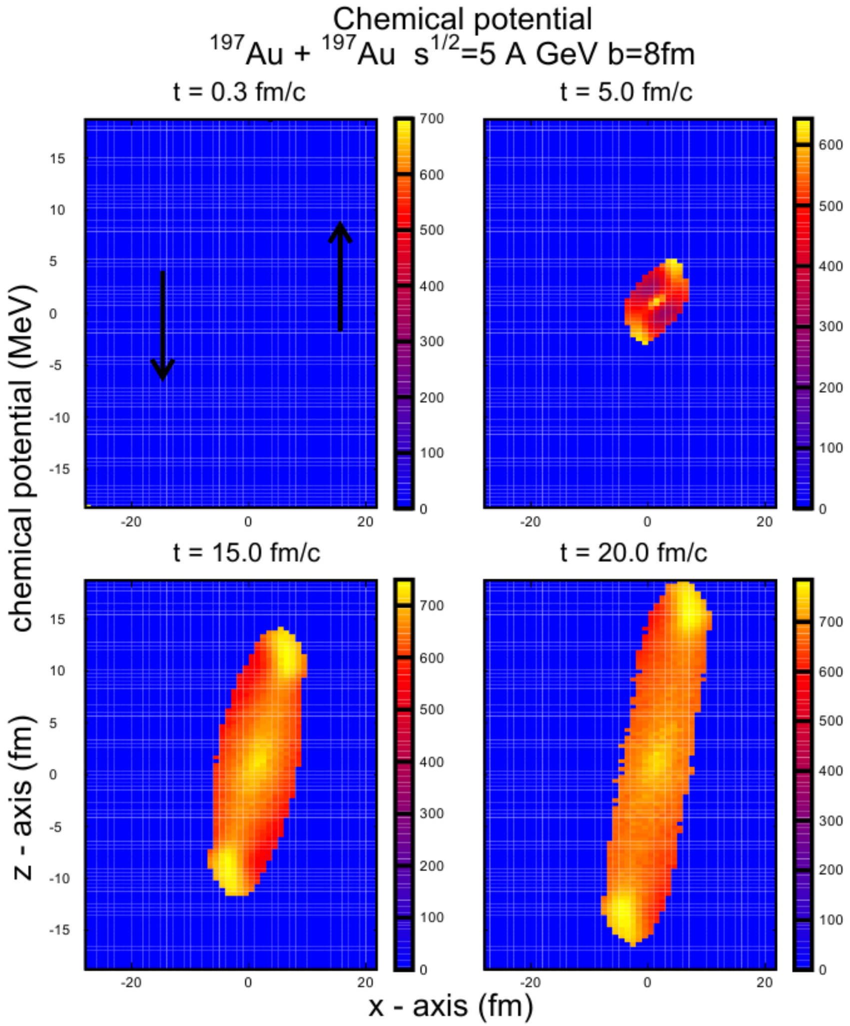


New: XZ- structure of helicity and polarization

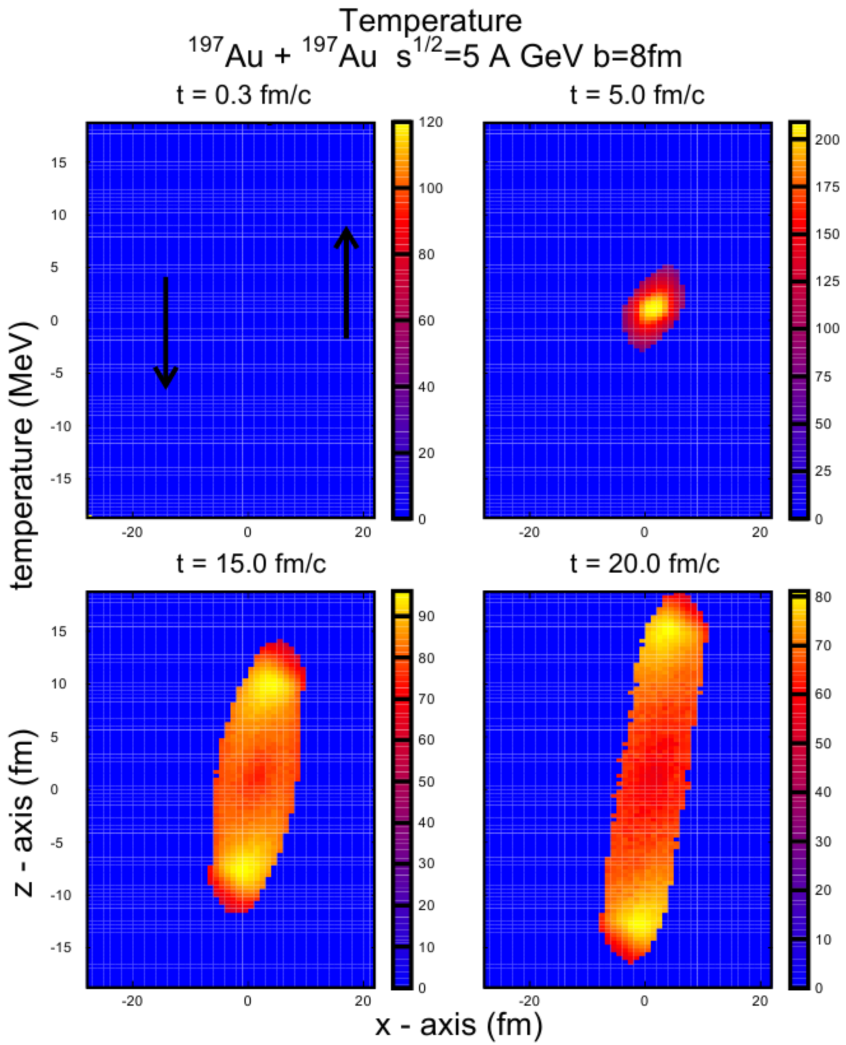


Chemical potential : Kinetics -> TD

- TD and chemical equilibrium
- Conservation laws
- Chemical potential from equilibrium distribution functions
- 2d section: y=0



Temperature



From axial charge (analog of Cooper-Frye) to polarization and from quarks to confined hadrons (Sorin,OT'17)

- Analogy of matrix elements and classical averages

$$< p_n|j^0(0)|p_n >= 2p_n^0 Q_n \qquad < Q >\equiv \frac{\sum_{n=1}^N Q_n}{N} = \frac{\int d^3x\, j^0_{class}(x)}{N}$$

- Axial current: charge -> polarization vector
- Lorentz boost: compensates the sign change of helicity "below" and "above" the RP

$$\Pi^{\Lambda,lab} = (\Pi_0^{\Lambda,lab}, \Pi_x^{\Lambda,lab}, \Pi_y^{\Lambda,lab}, \Pi_z^{\Lambda,lab}) = \frac{\Pi_0^\Lambda}{m_\Lambda}(p_y, 0, p_0, 0)$$

$$< \Pi_0^\Lambda >= \frac{m_\Lambda \Pi_0^{\Lambda,lab}}{p_y} =< \frac{m_\Lambda}{N_\Lambda p_y} > Q_5^s \equiv < \frac{m_\Lambda}{N_\Lambda p_y} > \frac{N_c}{2\pi^2}\int d^3x\, \mu_s^2(x)\gamma^2\epsilon^{ijk}v_i\partial_j v_k$$

Axial charge and properties of polarization

- Antihyperons : same sign (C-even axial charge) and larger value (smaller N)
- More pronounced at lower energy. Baryon/antibaryon splitting due to magnetic field – increase (?!) with energy. Non-linear effects in H may be essential, cf vector mesons on the lattice: Luschevskaya*, Solovjeva, OT: **JHEP 1709 (2017) 142**

***RFBR Megascience program**

Lambda vs Antilambda and role of vector mesons

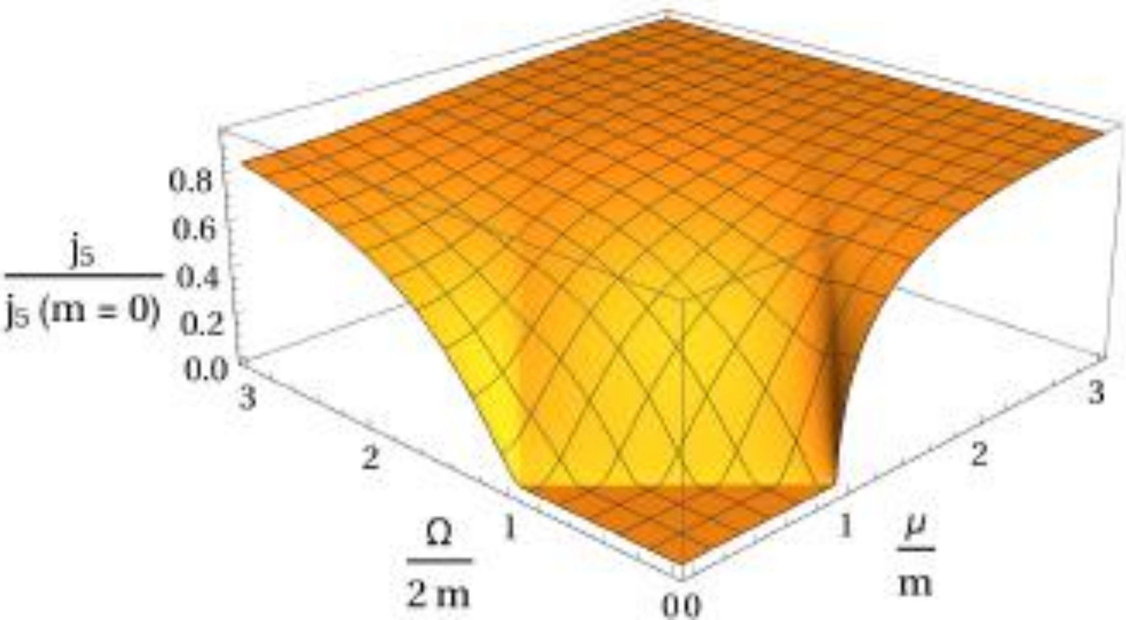
- Difference at low energies too large – same axial charge carried by much smaller number
- Strange axial charge may be also carried by K* mesons
- Λ - accompanied by (+,anti 0) K* mesons with two sea quarks – small corrections
- Anti Λ – more numerous (-,0) K* mesons with single (sea) strange antiquark
- Dominance of one component of spin results also in tensor polarization (P-even source like H^2: implied by positivityfor large polarization) –revealed in dilepton anisotropies (Bratkovskaya, Toneev,OT'95)

Chemical potential and flavour dependence

- Way via axial current/charge (TD: chemical potential) differs from "direct" TD (F. Becattini et al.: also for orbital/spin momentum; problems with symmetric EMT)
- TD-Universal, "flavor-blind" (only mass-dependent) polarization of universal sign
- Axial current: polarization depends on baryon structure
- Most pronounced at low energies
- Comparison of hyperons polarization (c.f. hadronic collisions)

Axial current in TD approach: Role of mass effects (Prokhorov, OT, Zakharov, PRD98 (2018), 071901)

- Threshold effects in chemical potential and angular velocity; 1906.03529: acceleration (important for longitudinal polarization)

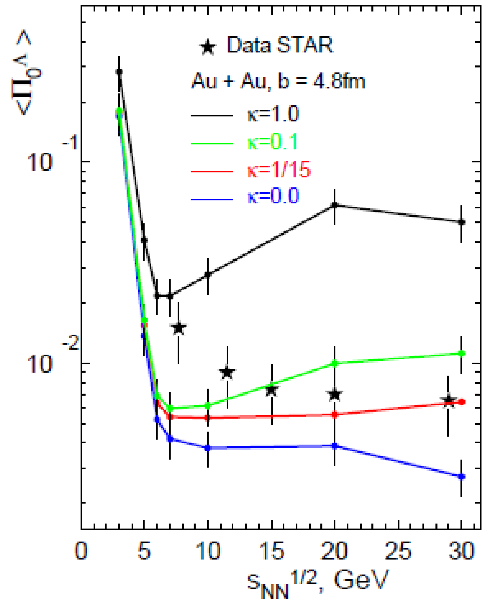


“Hidden anomaly”

- Anomalous current recovered in chiral limit and integration over all momenta
- W (x,p) → J(x)
- S(p) <S>

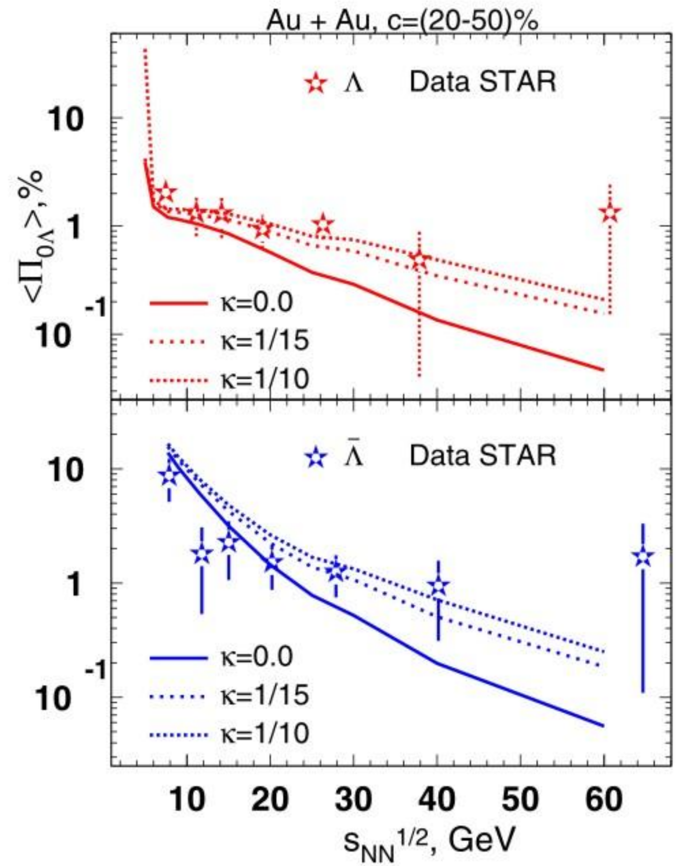
The role of (gravitational anomaly related) T^2 term

- Different values of coefficient probed



- LQCD suppression by collective effects supported

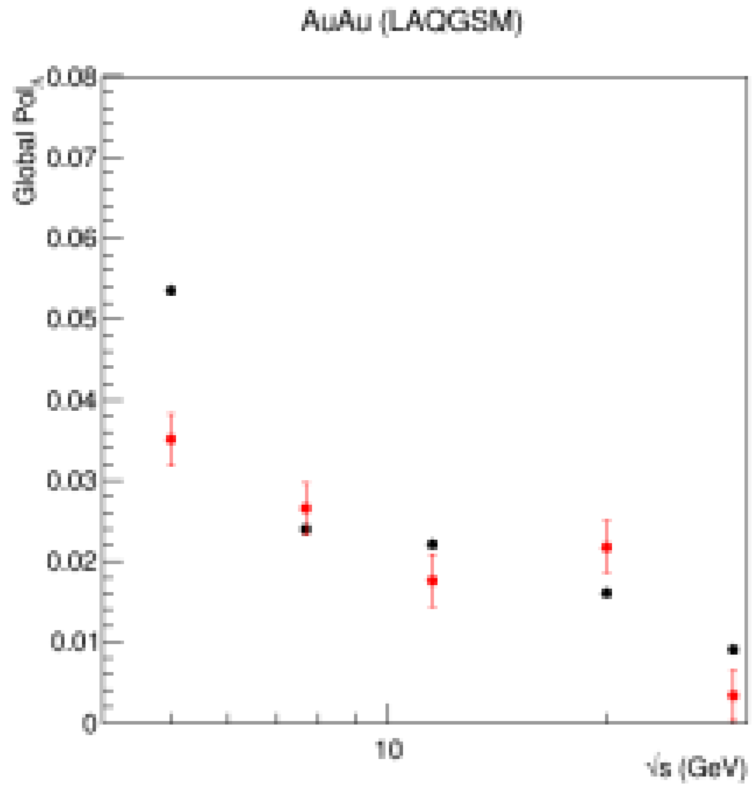
Λ vs Anti Λ



Au + Au, c=(20-50)%
Λ Data STAR
$\bar{\Lambda}$ Data STAR
κ=0.0
κ=1/15
κ=1/10
κ=0.0
κ=1/15
κ=1/10
<Π0Λ>,%
10
1
10-1
10
1
10-1
10
20
30
40
50
60
sNN1/2, GeV

Polarization at NICA/MPD (A. Kechechyan)

- QGSM Simulations and recovery accounting for MPD acceptance effects



Conclusions

- Anomaly induced polarization: energy dependence predicted and confirmed
- Gravity anomaly contributes in large energy limit
- Femtoscopic vortical structures emerge: conformity of Nature at bery large range of scales
- Interplay with TD mechanism: details to be studied in flavour dependence

Outlook

- Studies of vorticity in various models (Hydro, PHSD, UrQMD,…)-> MPD Root
- Comparison of TD and anomalous mechanisms
- Interplay with inertial effects (Unruh radiation etc.)
- Interplay of inverse vorticity/femtoscopy radii
- Polarization in pp,dp,pA,AA – small systems: cf Decrease of polarization with energy in Regge theory (cuts). Relation to SPD program
- Polarization at HADES -> Theory for lower energy and test@(BM@N)
- Start of activity in studies of vorticity and hyperon polarization in the framefwork of MPD PWG3

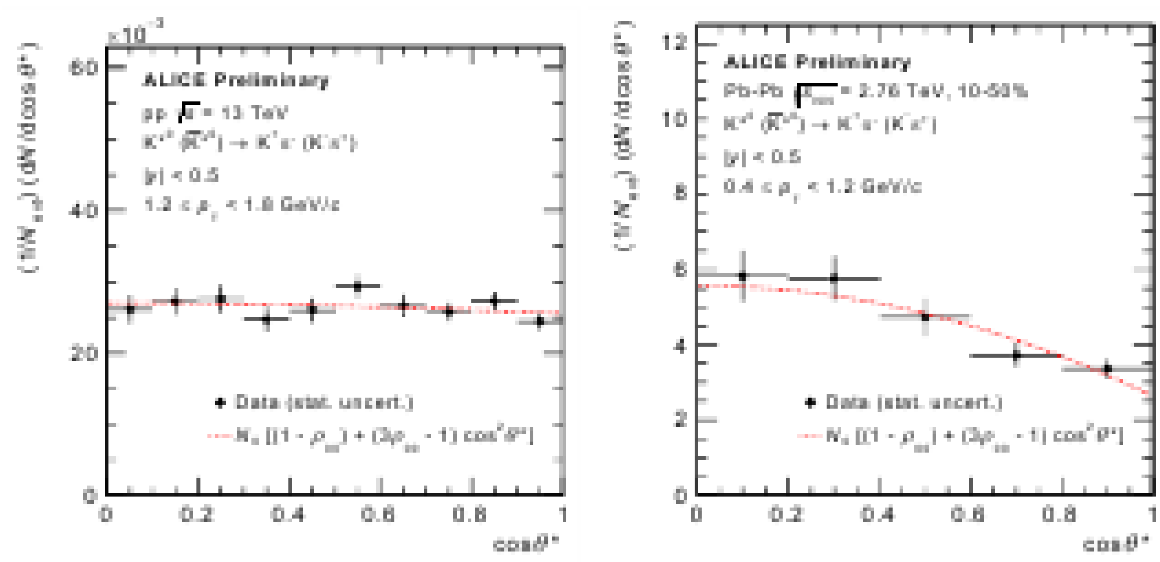
BACKUP

Tensor polarization

- P-even: quadratic effect of vorticity and/or magnetic field (tensor polarizability)
- Lattice: low invariant mass- longitudinal polarization (Buividovich, Polikarpov, OT'12)
- Lattice for vector mesons - non-trivial dependence on magnetic field (Lushevskaya, Slovjeva, OT'16)

Tensor polarization@ALICE

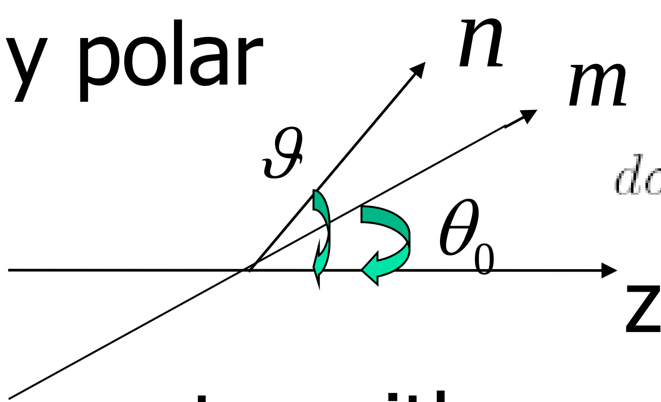
- Protons vs heavy ions



- Large energy - magnetic field effect?

Polar and azimuthal asymmetries: kinematically related

Only polar



$$d\sigma \propto 1 + \lambda_0 (\vec{n}\vec{m})^2 = 1 + \lambda_0 \cos^2 \theta_{nm}^2$$

asymmetry with respect to m!

$$\cos\theta_{nm} = \cos\theta\cos\theta_0 + \sin\theta\sin\theta_0\cos\phi$$

- azimuthal

angle dependence appears with new

$$\lambda = \lambda_0 \frac{2 - 3\sin^2\theta_0}{2 + \lambda_0 \sin^2\theta_0}$$

$$\nu = \lambda_0 \frac{2 sin^2 \theta_0}{2 + \lambda_0 \sin^2\theta_0}$$

Generalized Lam-Tung relation (OT'05)

- Relation between coefficients (high school math sufficient!)

$$\lambda_0 = \frac{\lambda + \frac{3}{2}\nu}{1 - \frac{1}{2}\nu}$$

- Reduced to standard LT relation for transverse polarization (λ_0 =1)
- LT - contains two very different inputs: kinematical asymmetry+transverse polarization

Properties of GLT

- Provides rotation-invariant observable (Faccioli,Lourenco,Seixas'10)
- Unknown collision axis – non-coplanarity (Peng, Chang, McClellan, OT'15, talk of W.-C. Chang) – violation of LT even for $\lambda_0=1$
- Quarkonia production, HIC… – different λ_0
- HIC – two natural axes: momentum direction in medium rest frame and normal to reaction plane – extra non-coplanarity appears if reaction plane is known only approximately

Conclusions/Outlook

- Polarization – new probe of anomaly (analogous to gluon polarization in nucleon) in quark-gluon matter: to be studied at NICA
- Generated by femto-vortex sheets
- Energy dependence predicted and confirmed
- Same sign and larger magnitude of antihyperon polarization
- Polarization - from core of vortices in pionic superfluid
- Dileptons – first results – to be compared with hadronic collisions

Are hadronic and nuclear polarizations so different?

- Decrease of polarization with energy – natural in Regge theory
- Vorticity in p(π $\bar{p}$)A collisions? Angular momentum is smaller, but vorticity is local?
- Gauge links with velocity fields?!
- ………………….?!

Properties of SSA

The same for the case of initial or final state polarization.

Various possibilities to measure the effects: change sign of $\vec{n}$ or $\vec{P}$: left-right or up-down asymmetry.

Qualitative features of the asymmetry

Transverse momentum required (to have $\vec{n}$)

Transverse polarization (to maximize $(\vec{P}\vec{n})$)

Interference of amplitudes

IMAGINARY phase between amplitudes - absent in Born approximation

Phases and T-oddness

Clearly seen in relativistic approach:

$\rho = \frac{1}{2}(\hat{p} + m)(1 + \hat{s}\gamma_5)$

Than: $d\sigma \sim Tr[\gamma_5 ....] \sim im\varepsilon_{sp_1p_2p_3}...$

Imaginary parts (loop amplitudes) are required to produce real observable.

$\varepsilon_{abcd} \equiv \varepsilon^{\alpha\beta\gamma\delta} a_\alpha b_\beta c_\gamma d_\delta$ each index appears once: $P-$ (compensate S) and $T-$ odd.

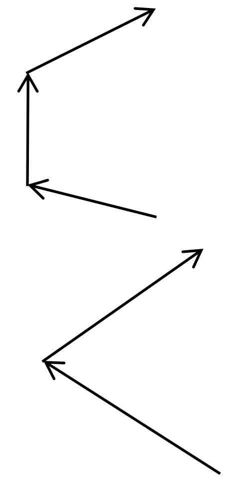
However: no real $T-$violation: interchange $|i> \leftrightarrow |f>$ is the nontrivial operation in the case of nonzero phases of $< f|S|i>^* = < i|S|f>$.

SSA - either T-violation or the phases.

DIS - no phases ($Q^2 < 0$)- real T-violation.

Correlations of jets handedness

- LEP – quarks are polarized due to weak interaction
- BUT – how to ditinguish quark/antiquark jets?
- 2 jets - correlation of helicities – correlation of handedness
- Hadronic collisions – for jets from the same quark-antiquark pair



Spin-gravity/rotation (~ 25 orders of magnitude slower!) interactions

- How to describe hadron spin/gravity(inertia) couplings?
- Matrix elements of Energy-Momentum Tensor
- May be studied in non-gravitational experiments/theory
- Simple interpretation in comparison to EM field case

Gravitational Formfactors

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle = \bar{u}(p')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}p^{\nu)} + B_{q,g}(\Delta^2)P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M]u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \qquad A_q(0) + A_g(0) = 1$$

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right] \qquad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe interaction with both classical and TeV gravity

Generalized Parton Diistributions (related to matrix elements of non local operators) – models for both EM and Gravitational Formfactors (Selyugin,OT '09)

- Smaller mass square radius (attraction vs repulsion!?)

$$\rho(b) = \sum_q e_q \int dx q(x,b) \qquad = \int d^2 q F_1(Q^2 = q^2) e^{i\vec{q}\vec{b}}$$

$$= \int_0^\infty \frac{qdq}{2\pi} J_0(qb) \frac{G_E(q^2) + \tau G_M(q^2)}{1+\tau}$$

$$\rho_0^{\mathrm{Gr}}(b) = \frac{1}{2\pi} \int_\infty^0 dq q J_0(qb) A(q^2)$$

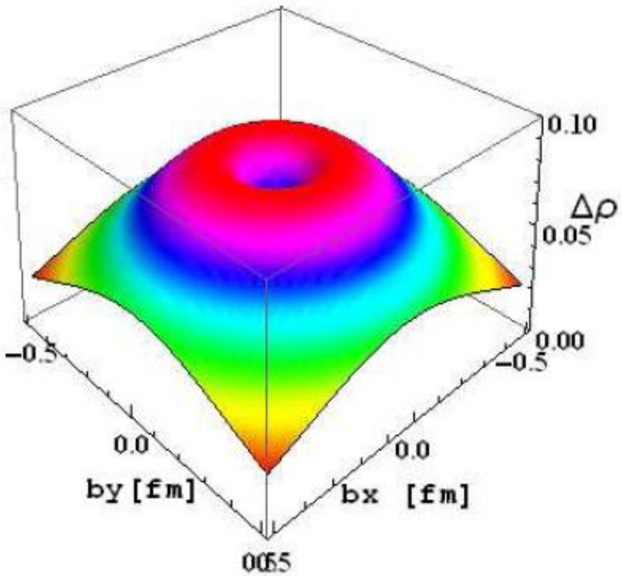


FIG. 17: Difference in the forms of charge density F_1^P and "matter" density (A)

Electromagnetism vs Gravity

- Interaction – field vs metric deviation

$$M = \langle P' | J_q^\mu | P \rangle A_\mu(q) \qquad M = \frac{1}{2} \sum_{q,G} \langle P' | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu}(q)$$

- Static limit

$$\sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle = 2 P^\mu P^\nu$$

$$\langle P | J_q^\mu | P \rangle = 2 e_q P^\mu$$

$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P | J_q^\mu | P \rangle A_\mu = 2 e_q M \phi(q) \qquad M_0 = \frac{1}{2} \sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle h_{\mu\nu} = 2M \cdot M \phi(q)$$

- Mass as charge – equivalence principle (Einstein '10-11, Praha)

Equivalence principle

- Newtonian – "Falling elevator" – well known and checked with high accuracy (also for elementary particles)
- Post-Newtonian – gravity action on SPIN – known since 1962 (Kobzarev and Okun' ZhETF paper contains acknowledgment to Landau: probably his last contribution to theoretical physics before car accident); rederived from conservarion laws - Kobzarev and Zakharov
- Anomalous gravitomagnetic (and electric-CP-odd) moment iz ZERO or
- Classical and QUANTUM rotators behave in the SAME way

- For GEDM –checked with sometimes controversial results
- For AGM not checked on purpose but in fact checked in the same atomic spins experiments at % level (Silenko,OT'07)

Gravitomagnetism

- Gravitomagnetic field (weak, except in gravity waves) – action on spin from $M = \frac{1}{2}\sum_{q,G}\langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$

  $\vec{H}_J = \frac{1}{2} rot\vec{g};\ \vec{g}_i \equiv g_{0i}$

  spin dragging twice smaller than EM
- Lorentz force – similar to EM case: factor ½ cancelled with 2 from $h_{00} = 2\phi(x)$ Larmor frequency same as EM

  $\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L\ \vec{H}_L = rot\vec{g}$
- Orbital and Spin momenta dragging – the same - Equivalence principle

Experimental test of PNEP

- Reinterpretation of the data on G(EDM) search

PHYSICAL REVIEW
LETTERS

VOLUME 68 | 13 JANUARY 1992 | NUMBER 2

Search for a Coupling of the Earth's Gravitational Field to Nuclear Spins in Atomic Mercury

B. J. Venema, P. K. Majumder, S. K. Lamoreaux, B. R. Heckel, and E. N. Fortson

Physics Department, FM-15, University of Washington, Seattle, Washington 98195

(Received 25 September 1991)

- If (CP-odd!) GEDM=0 -> constraint for AGM (Silenko, OT'07) from Earth rotation – was considered as obvious (but it is just EP!) background

$$\mathcal{H} = -g\mu_N \boldsymbol{B}\cdot\boldsymbol{S} - \zeta\hbar\boldsymbol{\omega}\cdot\boldsymbol{S}, \quad \zeta = 1+\chi$$

$$|\chi(^{201}\mathrm{Hg}) + 0.369\chi(^{199}\mathrm{Hg})| < 0.042 \quad (95\%\mathrm{C.L.})$$

Equivalence principle for moving particles

- Compare gravity and acceleration: gravity provides EXTRA space components of metrics $h_{zz} = h_{xx} = h_{yy} = h_{00}$
- Matrix elements DIFFER

$$\mathcal{M}_g = (\epsilon^2 + p^2) h_{00}(q), \qquad \mathcal{M}_a = \epsilon^2 h_{00}(q)$$

- Ratio of accelerations: $R = \frac{\epsilon^2 + p^2}{\epsilon^2}$ - confirmed by explicit solution of Dirac equation (Silenko, OT, '05)
- Arbitrary fields – Obukhov, Silenko, OT '09,'11,'13

Gravity vs accelerated frame for spin and helicity

- Spin precession – well known factor 3 (Probe B; spin at satellite – probe of PNEP!) – smallness of relativistic correction ($\sim \mathbf{P}^2$) is compensated by $1/\mathbf{P}^2$ in the momentum direction precession frequency
- Helicity flip – the same!
- No helicity flip in gravitomagnetic field – another formulation of PNEP (OT'99)

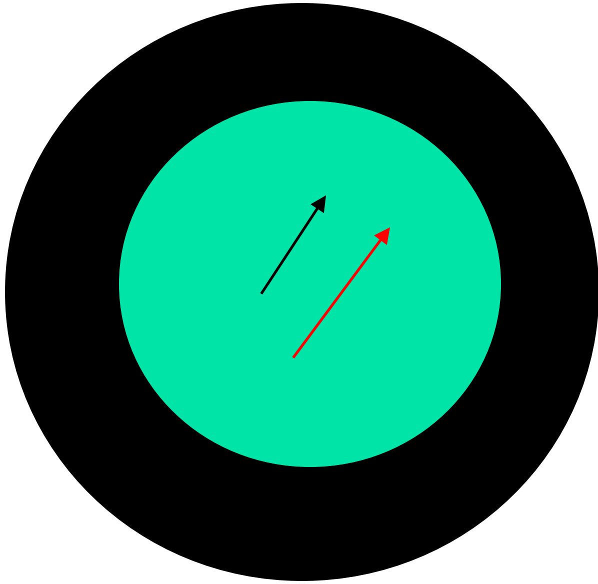
Gyromagnetic and Gravigyromagnetic ratios

- Free particles – coincide
- $\langle P+q|T^{mn}|P-q\rangle = P^{\{m}\langle P+q|J^{n\}}|P-q\rangle/e$ up to the terms linear in q
- Special role of g=2 for any spin (asymptotic freedom for vector bosons)

- Should Einstein know about PNEP, the outcome of his and de Haas experiment would not be so surprising
- Recall also g=2 for Black Holes. Indication of "quantum" nature?!

Cosmological implications of PNEP

- Necessary condition for Mach's Principle (in the spirit of Weinberg's textbook) -
- Lense-Thirring inside massive rotating empty shell (=model of Universe)
- For flat "Universe" - precession frequency equal to that of shell rotation
- Simple observation-Must be the same for classical and quantum rotators – PNEP!
- More elaborate models - Tests for cosmology ?!



Torsion – acts only on spin (violates EP)

Dirac eq+FW transformation-Obukhov,Silenko,OT, arXiv:1410.6197

- Hermitian Dirac Hamiltonian

$$e_i^{\widehat{0}} = V\,\delta_i^0, \qquad e_i^{\widehat{a}} = W^{\widehat{a}}{}_b\left(\delta_i^b - cK^b\,\delta_i^0\right)$$

$$ds^2 = V^2c^2dt^2 - \delta_{\widehat{a}\widehat{b}}W^{\widehat{a}}{}_cW^{\widehat{b}}{}_d\,(dx^c - K^c cdt)\,(dx^d - K^d cdt)$$

$$\mathcal{H} = \beta mc^2V + q\Phi + \frac{c}{2}\left(\pi_b\,\mathcal{F}^b{}_a\alpha^a + \alpha^a\mathcal{F}^b{}_a\pi_b\right) + \frac{c}{2}\left(\boldsymbol{K}\cdot\boldsymbol{\pi} + \boldsymbol{\pi}\cdot\boldsymbol{K}\right) + \frac{\hbar c}{4}\left(\Xi\cdot\Sigma - \Upsilon\gamma_5\right),$$

$$\mathcal{F}^b{}_a = VW^b{}_{\widehat{a}}, \qquad \Upsilon = V\epsilon^{\widehat{a}\widehat{b}\widehat{c}}\Gamma_{\widehat{a}\widehat{b}\widehat{c}}, \qquad \Xi^a = \frac{V}{c}\epsilon^{\widehat{a}\widehat{b}\widehat{c}}\left(\Gamma_{\widehat{0}\widehat{b}\widehat{c}} + \Gamma_{\widehat{b}\widehat{c}\widehat{0}} + \Gamma_{\widehat{c}\widehat{0}\widehat{b}}\right)$$

- Spin-torsion coupling

$$-\frac{\hbar cV}{4}\left(\Sigma\cdot\check{T} + c\gamma_5\check{T}^{\hat{0}}\right)$$

$$\check{T}^\alpha = -\frac{1}{2}\eta^{\alpha\mu\nu\lambda}T_{\mu\nu\lambda}$$

- FW – semiclassical limit - precession

$$\Omega^{(T)} = -\frac{c}{2}\check{T} + \beta\frac{c^3}{8}\left\{\frac{1}{\epsilon'},\left\{p,\check{T}^{\hat{0}}\right\}\right\} + \frac{c}{8}\left\{\frac{c^2}{\epsilon'(\epsilon'+mc^2)},\left(\{p^2,\check{T}\} - \{p,(p\cdot\check{T})\}\right)\right\}$$

Experimental bounds for torsion

- Magnetic field+rotation+torsion

$$H = -g_N \frac{\mu_N}{\hbar} \boldsymbol{B} \cdot \boldsymbol{s} - \boldsymbol{\omega} \cdot \boldsymbol{s} - \frac{c}{2} \check{\boldsymbol{T}} \cdot \boldsymbol{s}$$

- Same '92 EDM experiment

$$\frac{\hbar c}{4} |\check{T}| \cdot |\cos\Theta| < 2.2 \times 10^{-21}\,\mathrm{eV}, \qquad |\check{T}| \cdot |\cos\Theta| < 4.3 \times 10^{-14}\,\mathrm{m}^{-1}$$

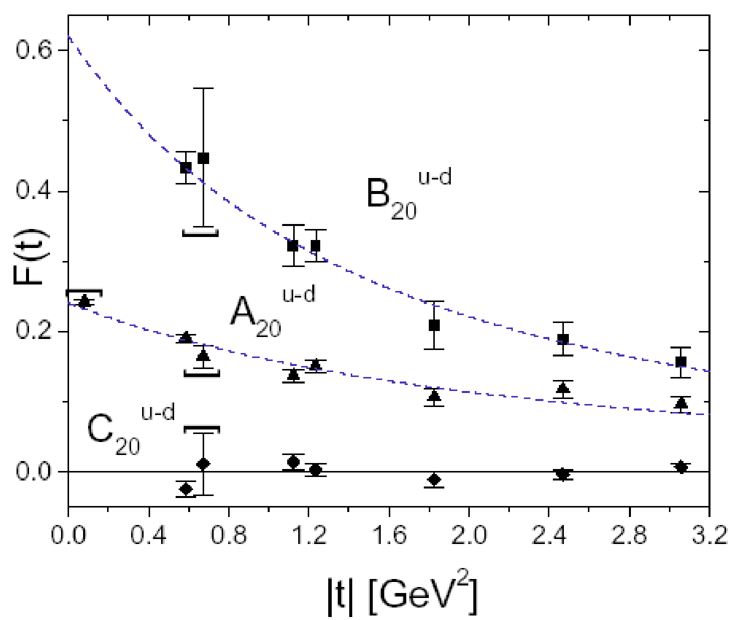
- New(based on Gemmel et al '10)

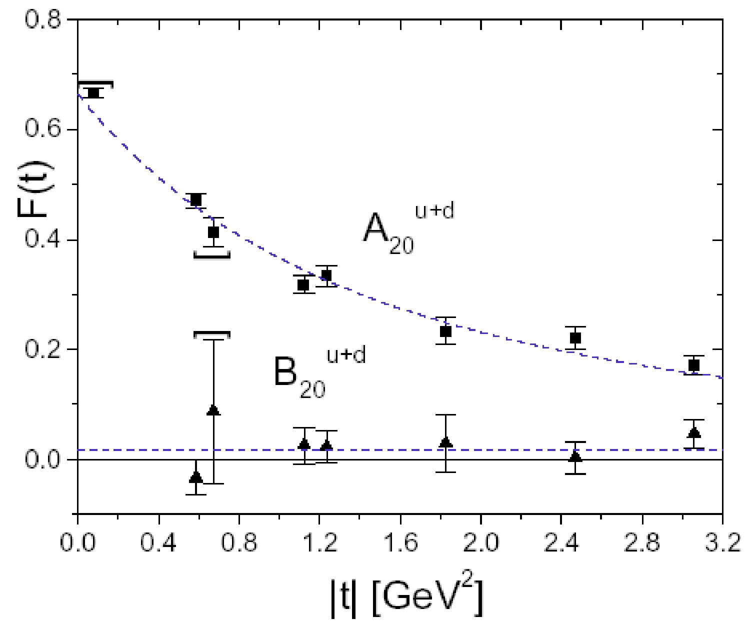
$$\frac{\hbar c}{2} |\check{T}| \cdot |(1-\mathcal{G})\cos\Theta| < 4.1 \times 10^{-22}\,\mathrm{eV}, \qquad |\check{T}| \cdot |\cos\Theta| < 2.4 \times 10^{-15}\,\mathrm{m}^{-1}$$

$$\mathcal{G} = g_{He}/g_{Xe}$$

Generalization of Equivalence principle

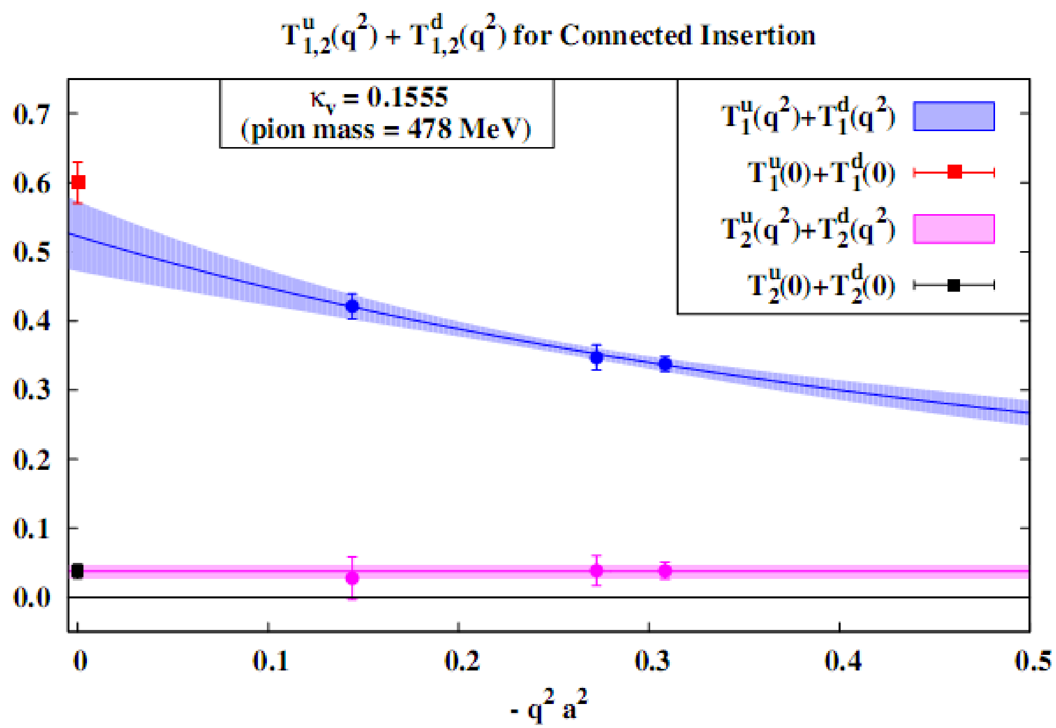
- Various arguments: AGM ≈0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)





Recent lattice study (M. Deka et al. arXiv:1312.4816)

- Sum of u and d for Dirac (T1) and Pauli (T2) FFs



Extended Equivalence Principle=Exact EquiPartition

- In pQCD – violated
- Reason – in the case of ExEP- no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Gravity-proof confinement (should the hadrons survive enetering Black Hole?)?!

Another manifestation of post-Newtonian (E)EP for spin 1 hadrons

- Tensor polarization - coupling of gravity to spin in forward matrix elements - inclusive processes
- Second moments of tensor distributions should sum to zero

$$\langle P,S|\bar{\psi}(0)\gamma^{\nu}D^{\nu_1}...D^{\nu_n}\psi(0)|P,S\rangle_{\mu^2} = i^{-n}M^2 S^{\nu\nu_1}P^{\nu_2}...P\nu_n \int_0^1 C_q^T(x)x^n dx$$

$$\sum_q \langle P,S|T_i^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^{\mu}P^{\nu}(1-\delta(\mu^2)) + 2M^2 S^{\mu\nu}\delta_1(\mu^2)$$

$$\langle P,S|T_g^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^{\mu}P^{\nu}\delta(\mu^2) - 2M^2 S^{\mu\nu}\delta_1(\mu^2)$$

$$\sum_q \int_0^1 C_i^T(x)x dx = \delta_1(\mu^2)$$ =0 for ExEP

HERMES – data on tensor spin structure function

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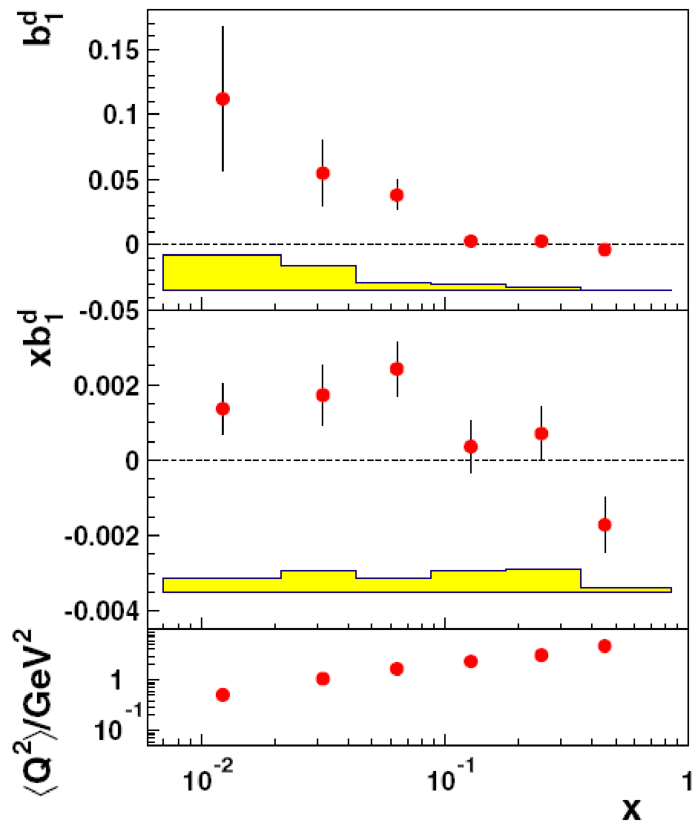
- Isoscalar target – proportional to the sum of u and d quarks – combination required by EEP
- Second moments – compatible to zero better than the first one (collective glue << sea) – for valence:



$$\int_0^1 C_i^T(x)dx = 0$$

Conclusions (slow rotation)

- Probe of equivalence principle for spin
- May be tested in EDM search experiments
- Extension of EP –validity separately for quarks and gluons

Sum rules for EMT (and OAM)

- First (seminal) example: X. Ji’s sum rule (’96). Gravity counterpart – OT’99
- Burkardt sum rule – looks similar: can it be derived from EMT?
- Yes, if provide correct prescription to gluonic pole (OT’14)

Pole prescription and Burkardt SR

- Pole prescription (dynamics!) provides ("T-odd") symmetric part!
  $$\sum \int \int dx_1 dx_2 \frac{T(x_1, x_2)}{x_1 - x_2 + i\varepsilon} = 0$$
- SR: $\sum \int dx T(x, x) = 0$ (but relation of gluon Sivers to twist 3 still not founs – prediction!)
- Can it be valid separately for each quark flavour: nodes (related to "sign problem")?
- Valid if structures forbidden for TOTAL EMT do not appear for each flavour
- Structure contains besides S gauge vector n: If GI separation of EMT – forbidden: SR valid separately!

Are more accurate data possible?

- HERMES – unlikely

- JLab may provide information about collective sea and glue in deuteron and indirect new test of Equivalence Principle

CONCLUSIONS

- Spin-gravity interactions may be probed directly in gravitational (inertial) experiments and indirectly – studing EMT matrix element
- Torsion and EP are tested in EDM experiments
- SR's for deuteron tensor polarization-indirectly probe EP and its extension separately for quarks and gluons

EEP and AdS/QCD

- Recent development – calculation of Rho formfactors in Holographic QCD (Grigoryan, Radyushkin)
- Provides g=2 identically!
- Experimental test at time –like region possible