On the semileptonic decays of the $B\bar{B}$ pairs and the branching ratio of the $\Upsilon(5S) \to B_s^{(*)}\bar{B}_s^{(*)}$ decay

A. Bondar, E. Karkaryan, A. Simovonian, M. Vysotsky;
arXiv:2412.06535

Сессия ОЯФ РАН, посвящённая 70-летию В.А.Рубакова
February 17-21 2025

$B_s \to \mu^+\mu^-$

This is a rare decay - so good place to look for New Physics.

Theory: $\mathcal{B}(B_s \to \mu^+\mu^-) = (3.63^{+0.15}_{-0.10}) \times 10^{-9}$.

This branching ratio was measured at LHC:
$\mathcal{B}(B_s \to \mu^+\mu^-) = (3.34 \pm 0.27) \times 10^{-9}$

Uncertainty in the number of B_s-mesons produced at LHC do not allow to measure $\mathcal{B}(B_s \to \mu^+\mu^-)$ with the desired small uncertainty.

$B_s \to D_s^- \pi^+$ as a normalization channel

We can find the value of $\mathcal{B}(B_s \to D_s^- \pi^+)$ at $\Upsilon(5S)$ (at Belle II) by the following relation:

$$\frac{N(B_s \to D_s^- \pi^+)}{N(B^0 \to D^- \pi^+)} \frac{N(B^0)}{N(B_s)} \mathcal{B}(B^0 \to D^- \pi^+) = \mathcal{B}(B_s \to D_s^- \pi^+).$$

Consideration of the time dependence of semileptonic decays of both B-mesons produced at $\Upsilon(5S)$ resonance at Belle II allows to determine the fraction of the $\Upsilon(5S) \to B_s^{(*)} \bar{B}_s^{(*)}$ decays with the percentage accuracy.
Thus we suggest the way to determine the probability of the rare decay $B_s \to \mu^+ \mu^-$ with the percentage accuracy at the LHC using $B_s \to D_s^- \pi^+$ as normalisation channel.

$\Upsilon(4S)$ and $\Upsilon(5S)$ decays

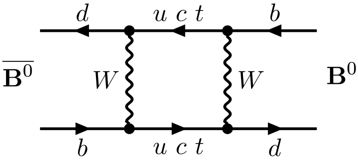
Pairs of B-mesons are produced in Υ decays; in asymmetric e^+e^- colliders time dependencies of semileptonic B decays are studied.

In $\Upsilon(4S)$ decays $B\bar{B}$ pairs are produced.
They are produced in C-odd state.

In $\Upsilon(5S)$ decays $B\bar{B}$ as well as $B_s\bar{B}_s$ pairs are produced.
They can be produced in C-even state as well:

$$\Upsilon(5S) \to B^*_{(s)}\bar{B}_{(s)}, \qquad B^*_{(s)} \to B_{(s)}\gamma.$$

$B^0 - \bar{B}^0$ mixing and oscillations



$$B_{\mathrm{L}} = pB^0 + q\overline{B}^0, B_{\mathrm{H}} = pB^0 - q\overline{B}^0,$$

$$|B^0(t)\rangle = e^{-iMt}e^{-(\Gamma/2)t}\left[\cos\left(\frac{\Delta m}{2}t\right)|B^0\rangle + i\frac{q}{p}\sin\left(\frac{\Delta m}{2}t\right)|\overline{B}^0\rangle\right],$$
$$|\overline{B}^0(t)\rangle = e^{-iMt}e^{-(\Gamma/2)t}\left[\cos\left(\frac{\Delta m}{2}t\right)|\overline{B}^0\rangle + i\frac{p}{q}\sin\left(\frac{\Delta m}{2}t\right)|B^0\rangle\right],$$

where $M = (m_{\mathrm{L}} + m_{\mathrm{H}})/2$ and $\Delta m = m_{\mathrm{H}} - m_{\mathrm{L}}$. Since the modulus of the ratio $|p/q| = 1 + \mathcal{O}(10^{-3})$, then in what follows we will assume $|p/q| = 1$.

$B\bar{B}$ from $\Upsilon(4S)$ decays

They are produced in C-odd state, so at the moment of semileptonic decay of the first one we know flavor of the second one:

$B \to l^+ \nu X$ - the second one is $\bar{B}$ (EPR paradox).

So the one-particle wave function is all what we need:

$$N_{++} = N_{--} = \frac{1}{2\Gamma} \int\limits_0^{+\infty} d(\Delta t) e^{-\Gamma \Delta t} \sin^2 \frac{\Delta m \Delta t}{2} = \frac{1}{4\Gamma^2} \frac{x^2}{1+x^2},$$

$$N_{+-} = N_{-+} = \frac{1}{2\Gamma} \int\limits_0^{+\infty} d(\Delta t) e^{-\Gamma \Delta t} \cos^2 \frac{\Delta m \Delta t}{2} = \frac{1}{4\Gamma^2} \frac{2+x^2}{1+x^2}.$$

At $\Delta t = 0$ $N_{++} = N_{--} = 0$.
The ratio of the number of decays into leptons of the same signs to the number of decays into leptons of the opposite signs in the case of a C-odd wave function is equal to

$$R_{\rm odd} = \frac{N_{++} + N_{--}}{N_{+-} + N_{-+}} = \frac{x^2}{2+x^2},$$

where the dimensionless parameter $x = \Delta m/\Gamma$ was introduced. In case $x = 0$ (no oscillatios) $N_{++} = N_{--} = 0$; in case $x >> 1$ (fast oscillations) $N_{++} = N+-$.

two-paricle wave functions

For semileptonic decays of C-even states this trick does not work and we need two-paricle wave function.

C-odd wave function:

$$|\psi(t_1,t_2)\rangle_{\rm odd} = |B^0(t_1)\rangle \otimes |\overline{B}^0(t_2)\rangle - |\overline{B}^0(t_1)\rangle \otimes |B^0(t_2)\rangle.$$

C-even wave function:

$$|\psi(t_1,t_2)\rangle_{\rm even} = |B^0(t_1)\rangle \otimes |\overline{B}^0(t_2)\rangle + |\overline{B}^0(t_1)\rangle \otimes |B^0(t_2)\rangle.$$

$B\bar{B}$ even state

$$N_{++} = N_{--} = \frac{1}{4\Gamma(1+x^2)} \int\limits_0^{+\infty} d(\Delta t) e^{-\Gamma \Delta t} \Big(1 + x^2 - \cos \Delta m \Delta t + $$

$$+ x \sin \Delta m \Delta t \Big) = \frac{1}{4\Gamma^2} \frac{3x^2 + x^4}{(1+x^2)^2},$$

$$N_{+-} = N_{-+} = \frac{1}{4\Gamma(1+x^2)} \int\limits_0^{+\infty} d(\Delta t) e^{-\Gamma \Delta t} \Big(1 + x^2 + \cos \Delta m \Delta t - $$

$$- x \sin \Delta m \Delta t \Big) = \frac{1}{4\Gamma^2} \frac{2 + x^2 + x^4}{(1+x^2)^2},$$

$$R_{\text{even}} = \frac{N_{++} + N_{--}}{N_{+-} + N_{-+}} = \frac{3x^2 + x^4}{2 + x^2 + x^4}.$$

Branchings of the $\Upsilon(5S) \to B_s^{(*)} \bar{B}_s^{(*)}$ decays

C-parity of $B\bar{B}$ pair	Decay modes				Branching notation
	$B^0\bar{B}^0$ in the final state				
C-odd state	$B^0\bar{B}^0$ $B^{0*}\bar{B}^{0*}\pi^0$	$B^{0*}\bar{B}^{0*}$	$B^0\bar{B}^0\pi^0$		$(\epsilon_{00})^{\mathrm{odd}}$
C-even state	$B^0\bar{B}^{0*}$ $B^0\bar{B}^{0*}\pi^0$	$B^{0*}\bar{B}^0$	$B^{0*}\bar{B}^0\pi^0$		$(\epsilon_{00})^{\mathrm{even}}$
	B^+B^- in the final state				
C-odd and C-even states	B^+B^- $B^{+*}B^{-*}\pi^0$	$B^{+*}B^{-*}$	$B^+B^-\pi^0$		ϵ_{+-}
	B^+B^{-*} $B^+B^{-*}\pi^0$	$B^{+*}B^-$	$B^{+*}B^-\pi^0$		
	$B_S\bar{B}_S$ in the final state				
C-odd state	$B_S\bar{B}_S$	$B_S^*\bar{B}_S^*$			$(\epsilon_{SS})^{\mathrm{odd}}$
C-even state	$B_S^*\bar{B}_S$	$B_S\bar{B}_S^*$			$(\epsilon_{SS})^{\mathrm{even}}$
	$B^{\pm}B^0$ in the final state				
No definite C-parity	$B^+\bar{B}^0\pi^-$	$B^{+*}\bar{B}^0\pi^-$	$B^+\bar{B}^{0*}\pi^-$	$B^{+*}\bar{B}^{0*}\pi^-$	ϵ_+
	$B^-B^0\pi^+$	$B^{-*}B^0\pi^+$	$B^-B^{0*}\pi^+$	$B^{-*}B^{0*}\pi^+$	ϵ_-

time dependence of the same sign dileptons production

$$\frac{dN_{++}}{d\Delta t} = L\sigma_{e^+e^-\to\Upsilon(5S)}\mathcal{B}^2(B^0 \to \ell^+\nu_\ell X)\frac{\Gamma}{2}e^{-\Gamma\Delta t}\Bigg[(\epsilon_{00})^{\rm odd}\sin^2\frac{\Delta m\Delta t}{2}+ \tag{1}$$

$$+\frac{(\epsilon_{00})^{\rm even}}{2(1+x^2)}\left(1+x^2-\cos\Delta m\Delta t + x\sin\Delta m\Delta t\right)+\frac{(\epsilon_{SS})^{\rm odd}+(\epsilon_{SS})^{\rm even}}{2}+$$

$$+\frac{\epsilon_+}{4+x^2}\left(2+x^2-2\cos\Delta m\Delta t + x\sin\Delta m\Delta t\right)\Bigg]$$

where L is a luminosity of the e^+e^- collider, $\sigma_{e^+e^-\to\Upsilon(5S)}$ is a cross section of the $\Upsilon(5S)$ production, $\mathcal{B}(B^0 \to \ell^+\nu_\ell X)$ is the branching ratio of the semileptonic B^0-meson decay and Γ is B^0 width.

Analogous expression holds for $dN_{--}/d\Delta t$.

time dependence of the opposite sign dileptons production

Analogous formulas are obtained for the case of dileptons of the opposite signs:

$$\frac{dN_{+-}}{d\Delta t} = L\sigma_{e^+e^-\to\Upsilon(5S)}\mathcal{B}^2(B^0\to\ell^+\nu_\ell X)\frac{\Gamma}{2}e^{-\Gamma\Delta t}\left[(\epsilon_{00})^{\mathrm{odd}}\cos^2\frac{\Delta m\Delta t}{2}+\right. \quad (2)$$

$$+\frac{(\epsilon_{00})^{\mathrm{even}}}{2(1+x^2)}\left(1+x^2+\cos\Delta m\Delta t-x\sin\Delta m\Delta t\right)+\epsilon_{+-}+\frac{(\epsilon_{SS})^{\mathrm{odd}}+(\epsilon_{SS})^{\mathrm{even}}}{2}+$$

$$\left.+\frac{\epsilon_++\epsilon_-}{2(4+x^2)}\left(6+x^2+2\cos\Delta m\Delta t-x\sin\Delta m\Delta t\right)\right].$$

It is convenient to form the following ratios from (1) and (2):

$$\frac{d(N_{+-}+N_{-+}-N_{--}-N_{++})/d\Delta t}{d(N_{+-}+N_{-+}+N_{--}+N_{++})/d\Delta t} = C + A\sin(\Delta m\Delta t+\varphi), \quad (3)$$

$$\frac{d(N_{++}+N_{--})/d\Delta t}{d(N_{+-}+N_{-+}+N_{--}+N_{++})/d\Delta t} = C' - \frac{A}{2}\sin(\Delta m\Delta t+\varphi), \quad (4)$$

where the parameters C, C', A and ϕ are equal to

$$C = \epsilon_{+-} + \left(\epsilon_{+} + \epsilon_{-}\right)\frac{2}{4+x^2},$$

$$C' = \frac{(\epsilon_{00})^{\rm odd} + (\epsilon_{00})^{\rm even} + (\epsilon_{SS})^{\rm odd} + (\epsilon_{SS})^{\rm even}}{2} + \left(\epsilon_{+} + \epsilon_{-}\right)\frac{1+x^2/2}{4+x^2},$$

$$A = -\left[\left((\epsilon_{00})^{\rm odd} + \frac{(\epsilon_{00})^{\rm even}}{1+x^2} + \frac{2}{4+x^2}(\epsilon_+ + \epsilon_-)\right)^2 + \right.$$
$$\left. + \left(\frac{x}{1+x^2}(\epsilon_{00})^{\rm even} + \frac{x}{4+x^2}(\epsilon_+ + \epsilon_-)\right)^2\right]^{1/2},$$

$$\phi = -\arcsin\left[\frac{(\epsilon_{00})^{\rm odd} + \frac{(\epsilon_{00})^{\rm even}}{1+x^2} + \frac{2}{4+x^2}(\epsilon_+ + \epsilon_-)}{-A}\right].$$

Experimental measurement of the time dependences of the left sides of (3) and (4) allows to obtain the branching ratio $\epsilon_{SS} = (\epsilon_{SS})^{\rm odd} + (\epsilon_{SS})^{\rm even}$:

$$2C' - C = \epsilon_{SS} + \Big(\epsilon_+ + \epsilon_-\Big)\frac{x^2}{4+x^2},$$

where we used the isotopic invariance: $\epsilon_{+-} = (\epsilon_{00})^{\mathrm{odd}} + (\epsilon_{00})^{\mathrm{even}}$.
Let us estimate the uncertainty in ϵ_{SS} determination which comes from the last term. The value of this term is

$$\frac{2}{3} \times (7 \pm 1)\% \times \frac{(0.77)^2}{4 + (0.77)^2} = (0.62 \pm 0.09)\%,$$

where factor 2/3 accounts for the decays with charged π mesons and x=0.77 is substituted. Comparing it with the approximate ϵ_{SS} value $(22 \pm 2)\%$ we see that the relative uncertainty due to this correction is less than 1%.

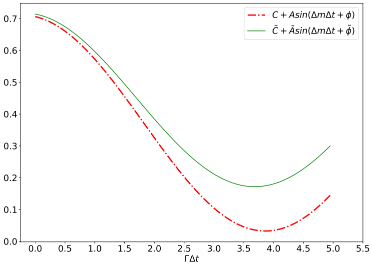
Conclusion

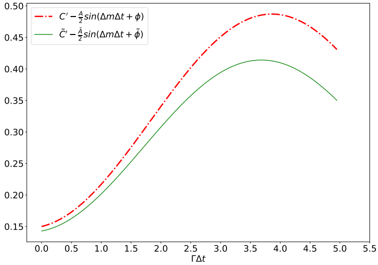
The fraction of B_s-mesons produced in $\Upsilon(5S)$ decays can be determined from the time dependences of semileptonic decays at Belle II.

It can be used to find the branching ratio of normalization channel $B_s \to D_s^- \pi^+$ at Belle II with the percentage accuracy openning the way to measure $\mathcal{B}(B_s \to \mu^+ \mu^-)$ at LHC with the same accuracy.

Corrections due to the longer lifetime of B^+ meson

While lifetimes of B^0 and B_s mesons coincide at a percent accuracy the lifetime of B^+ meson is considerably higher: $\tau_{B^+}/\tau_{B^0} = 1.076 \pm 0.004$.





$\Upsilon(4S) \to B\bar{B} \to J/\psi K_S \;\; J/\psi K_S$

Two-particle wave function is needed:

$$\langle J/\psi K_S, J/\psi K_S|\psi(t_1,t_2)\rangle_{\rm odd} = e^{-2iMt-\Gamma t}\left[A\cos\left(\frac{\Delta m}{2}t_1\right) + i\frac{q}{p}\overline{A}\sin\left(\frac{\Delta m}{2}t_1\right)\right] \times$$

$$\times\left[\overline{A}\cos\left(\frac{\Delta m}{2}t_2\right) + i\frac{p}{q}A\sin\left(\frac{\Delta m}{2}t_2\right)\right] - (t_1 \longleftrightarrow t_2) = e^{-2iMt-\Gamma t}\left[\left(i\frac{p}{q}A^2 -\right.\right.$$

$$\left.-i\frac{q}{p}\overline{A}^2\right)\cos\left(\frac{\Delta m}{2}t_1\right)\sin\left(\frac{\Delta m}{2}t_2\right) + \left(i\frac{q}{p}\overline{A}^2 - i\frac{p}{q}A^2\right)\sin\left(\frac{\Delta m}{2}t_1\right)\cos\left(\frac{\Delta m}{2}t_2\right)\Big]$$

$$= -e^{-2iMt-\Gamma t}\left(i\frac{p}{q}A^2\right)\left[1-\lambda^2\right]\sin\left(\frac{\Delta m\Delta t}{2}\right)$$

where $A(\bar{A})$ is an amplitude of $B^0(\bar{B}^0) \to J/\psi K_S$ decay and $\lambda = (q/p)(\overline{A}/A)$ was introduced. In the last line the fact that $A = \bar{A}$ was used. The ratio q/p can be expressed via the angle β of the unitarity triangle: $q/p = e^{-2i\beta}$. Then the probability of the decay equals

$$\mathbb{P}(J/\psi K_S, J/\psi K_S) = e^{-2\Gamma t} A^4 [1 - e^{4i\beta}][1 - e^{-4i\beta}] \sin^2\left(\frac{\Delta m \Delta t}{2}\right) =$$
$$= e^{-2\Gamma t} A^4 \cdot 4 \sin^2(2\beta) \sin^2\left(\frac{\Delta m \Delta t}{2}\right).$$

It equals zero for $\Delta t = 0$ due to Bose statistics, for $\Delta m = 0$ because of absence of oscillations and for $\beta = 0$ due to CP conservation. The point is that $\Upsilon(4S)$ is CP even while final state is CP odd.
Let us estimate the amount of $\Upsilon(4S) \to J/\psi K_S J/\psi K_S$ decays which could be detected at Belle II. The total number of $\Upsilon(4S)$ which should be produced at SuperKEKB with integrated luminosity $L = 50$ ab^{-1} equals $\approx 10^{11}$. For the number of decays $\Upsilon(4S) \to J/\psi K_S J/\psi K_S$ we get:

$$N(\Upsilon(4S) \to J/\psi K_S J/\psi K_S) = \epsilon \cdot N(\Upsilon(4S)) \cdot \mathcal{B}(\Upsilon(4S) \to J/\psi K_S J/\psi K_S) \approx 150,$$

where $\epsilon = 0.04$ is an efficiency of Belle II detector in the $\Upsilon(4S) \to J/\psi K_S J/\psi K_S$ channel. Taking into account excited charmonium states this number can be doubled.