Double Assisted Schwinger effect

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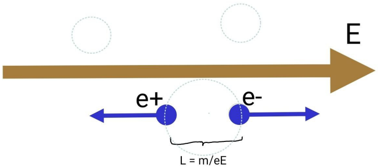
The Schwinger effect



Sauter 1931, Euler Heisenberg 1936, Schwinger 1951

$$\boxed{\Gamma = VT \cdot \frac{(eE)^2}{(2\pi)^3}\, e^{-\frac{\pi m^2}{eE}}}$$

Nonperturbative over small parameter e, $\quad E_S = m^2/e = 10^{18}\, V/m$

$$E_{LAB} \sim 10^{16}\ V/m, \qquad e^{-\pi E_S/E_{LAB}} \sim e^{-90}$$

No chance of detection? Need to enhance the process!

Sauter pulse $E = E_0/ch^2(\omega t)$

$$\Gamma \propto \exp\left(-\frac{2\pi m^2}{eE\left(\sqrt{1+\gamma^2}+1\right)}\right), \qquad \gamma = \frac{\omega m}{eE} \text{ -- Keldysh parameter}$$

$\gamma \gg 1 \ \rightarrow$ less suppressed

Nikishov; Nikishov, Ritus 1970

Dynamical assistance//Catalysis for the Schwinger effect

2 components of EM field $\to$ Schwinger p.p. less suppressed

Dynamically assisted Schwinger mechanism

Schutzhold Gies Dunne PRL 2008 arXiv:0807.0754

$$\vec{E}(t) = \frac{E_0}{cosh(\Omega t)}\vec{e}_z + \frac{\varepsilon}{cosh(\omega t)}\vec{e}_z$$

where $E_S \gg E_0 \gg \varepsilon > 0, \qquad m \gg \omega \gg \Omega > 0, \qquad \gamma = \frac{m\omega}{eE_0}$

$$\Gamma \propto \exp\left[-\frac{m^2}{eE_0}\left(2\arcsin\frac{\pi}{2\gamma} + \frac{\pi^2}{2\gamma^2}\sqrt{4\gamma^2-\pi^2}\right)\right], \qquad \Omega \to 0$$

Catalysis of Schwinger Vacuum Pair Production

= Photon decay — Dunne Gies Schutzhold PRD 2009 arXiv:0908.0948

Worldline Inst: — Monin, Voloshin PRD 2010 arXiv:1001.3354

$\Gamma \propto \exp\left(-\frac{m^2}{eE}\left[\left(2+\frac{\omega^2}{2m^2}\right)\arctan\left(\frac{2m}{\omega}\right) - \frac{\omega}{m}\right]\right) \to \exp\left(-\frac{8m^2}{3eE}\frac{m}{\omega}\right), \omega \gg 1$

Less suppressed at $\omega > m$

Assistance + Catalysis = Double Assistance
Time-dependent field vs plane wave

Torgrimsson Oertel Schützhold PRD 2016 arXiv:1607.02448
Torgrimsson PRD 2019 arXiv:1812.04607

Double assistance:
Const electric field E + photon $\Omega \sim m$ + intermediate field $\varepsilon E(t)$,
$E(t) = E/ch^2(\omega t)$ or $E(t) = E\sin(\omega t)$.
$\varepsilon \sim 10^{-3} - 10^{-2}$, $\omega \sim (10^{-3} - 10^{-2})m$ – freq. of X-ray laser

Torgrimsson Schneider Schützhold PRD 2018 arXiv:1712.08613
Dynamical assistance (not double!) with plane wave $\varepsilon E \cos\omega(t - x)$

Double assistance with photon and plane wave?

Worldline formalism

Scalar QED: $S_E = \int d^4x \left(-\frac{1}{4}F_{\mu\nu}^2 + |D_\mu\phi|^2 + m^2|\phi|^2\right).$

A_μ — classical EM field. $Z[A_\mu] = \int D\phi^* D\phi \, e^{-S_E[A_\mu,\phi^*,\phi]} = e^{-W[A_\mu]}.$

Effective action:

$$W[A_\mu] = \int d^4x \left(-\frac{1}{4}F_{\mu\nu}^2 - \log\det\left(-D_\mu^2 + m^2\right)\right).$$

Schwinger *proper time* representation: Schwinger, 1951

$$W[A_\mu] = \int d^4x \left(-\frac{1}{4}F_{\mu\nu}^2 + \int_0^\infty \frac{ds}{s} e^{-m^2 s} \mathrm{Tr}\left(e^{D_\mu^2 s}\right)\right).$$

Operator $\left(-D_\mu^2\right)$ can be interpreted as QM Hamiltonian.

$$\mathrm{Tr}\left(e^{sD_\mu^2}\right) = \int d^4x \langle x_\mu | e^{-s\left(-D_\mu^2\right)} | x_\mu \rangle = \int_{p.b.c.} Dx_\mu e^{-\int_0^s d\tau\left(\frac{\dot{x}_\mu^2}{4} + ieA_\mu \dot{x}_\mu\right)}.$$

The Schwinger effect with Worldline Instantons

Affleck, Alvarez, Manton, 1982

The Schwinger effect (vacuum pair production): $\Gamma = 2\,\mathrm{Im}\, W[A_\mu]$

$$\Gamma = 2\,\mathrm{Im} \int_0^\infty \frac{ds}{s} e^{-m^2 s} \int_{p.b.c.} Dx_\mu e^{-\int_0^1 d\tau \left(\frac{\dot{x}_\mu^2}{4s} + ieA_\mu \dot{x}_\mu\right)}.$$

Take integrals over x_μ and s in the saddle point approximation

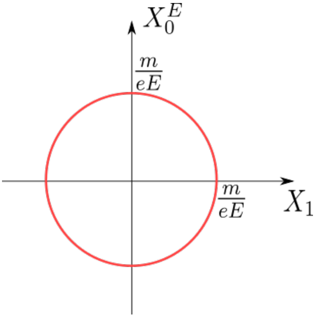
EOMs: $\frac{\ddot{x}_\mu}{2s} - eF_{\mu\nu}\dot{x}_\nu = 0$ and $-m^2 + \frac{\int_0^1 \dot{x}_\mu^2 d\tau}{4s^2} = 0.$

Uniform constant electric field E.

The leading order closed solution is a circle:

$$x_0^{cl} = \frac{m}{eE}\sin(2\pi\tau), \qquad x_1^{cl} = \frac{m}{eE}\cos(2\pi\tau),$$

$$x_2^{cl} = x_3^{cl} = 0, \qquad s = \frac{2\pi}{eE}.$$

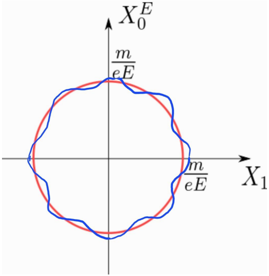


The action on the solution x_μ^{cl} is $S[x_\mu^{cl}] = \frac{\pi m^2}{eE}$.

$S[x_\mu^{cl}] \propto$ area

Condition: $S[x_\mu^{cl}] \gg 1, \qquad \Gamma \propto e^{-S[x_\mu^{cl}]}.$

- Pre-exponential factor: integrate over quadratic fluctuations δx_μ near classical solution
  $x_\mu = x_\mu^{cl} + \delta x_\mu$
- Zero mode: 4-volume VT
- Negative mode: $\delta x_\mu \propto x_\mu^{cl}$
  i in pre-exp factor $\to \Gamma = 2\,\mathrm{Im}\,W[A_\mu] \neq 0$.



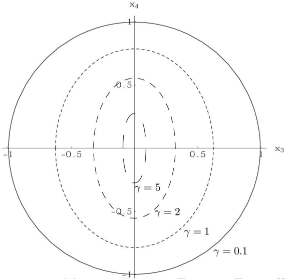
$$\Gamma = VT \cdot \frac{(eE)^2}{(2\pi)^3}\, e^{-\frac{\pi m^2}{eE}}$$

Sauter pulse $E = E_0/ch^2(\omega t)$

WInst - Dunne Schubert PRD 2005 hep-th/0507174

$$\Gamma \propto \exp\left(-\frac{2\pi m^2}{eE\left(\sqrt{1+\gamma^2}+1\right)}\right)$$

$$\gamma = \frac{\omega m}{eE}$$ — Keldysh parameter



How to deal with external photons in Worldline formalism

Each photon vertex — functional dervative over A_μ. In scalar QED

$$V[k_\mu, \varepsilon_\mu(k)] = \int_0^1 d\tau\, \varepsilon_\mu(k)\dot{x}_\mu(\tau)e^{ik^\mu x^\mu(\tau)}.$$

N-point photon amplitude   Schubert, 2001, hep-th/0101036

$$M[k_1, \varepsilon(k_1); \ldots ; k_N, \varepsilon(k_N)] = (-ie)^N \int_0^\infty \frac{ds}{s} e^{-m^2 s}(4\pi s)^{-2}\langle V[k_1, \varepsilon(k_1)] \ldots V[k_N, \varepsilon(k_N)]\rangle$$

Here $\langle V_1 \ldots V_N\rangle \equiv \int_{p.b.c} Dx_\mu \,[V_1 \ldots V_N] \exp\left(-\int_0^s d\tau \left(\frac{\dot{x}_\mu^2}{4} + ieA_\mu \dot{x}_\mu\right)\right).$

New extra terms $ik_j^\mu x^\mu(\tau_j)$ in the exponent from V_j, $j = 1..N$.

The Optical theorem

Cross-sections of pair production are proportional to the imaginary part of the corresponding amplitude (depending on the number of initial photons):

- The photon decay: $\Gamma_{\gamma\to e^+e^-} = \frac{1}{2\omega} \,\mathrm{Im}\, \mathcal{M}\,[k, \varepsilon(k); k, \varepsilon^*(k)]\,.$
- 2γ scattering: $\sigma_{\gamma\gamma\to e^+e^-} \propto \mathrm{Im}\, \mathcal{M}\,[k_1, \varepsilon(k_1); k_2, \varepsilon(k_2); k_1, \varepsilon^*(k_1); k_2, \varepsilon^*(k_2)]\,.$
- ...

Photon decay $\gamma \to e^+e^-$ in EM background

Decay width: $\Gamma_{\gamma\to e^+e^-} = \frac{1}{2\omega}\varepsilon_\mu(k)\varepsilon^*_\nu(k)\mathrm{Im}\,\Pi_{\mu\nu}(k).$

$$\mathrm{Im}\,\Pi_{\mu\nu}(k) = \mathrm{Im}\int_0^\infty \frac{ds}{s}\int_{p.b.c.} Dx_\mu \oint d\tau_1 \oint d\tau_2\, \dot{x}_\mu(\tau_1)\dot{x}_\nu(\tau_2)e^{-S_m[x_\mu;\tau_1,\tau_2]},$$

where

$$S_m[x_\mu;\tau_1,\tau_2] = m^2 s + \int_0^1 d\tau\left(\frac{\dot{x}_\mu^2}{4s} + ieA_\mu\dot{x}_\mu\right) - ik_\mu\left(x_\mu(\tau_1) - x_\mu(\tau_2)\right).$$

The saddle point over s: $m^2 - \frac{\int_0^1 d\tau \dot{x}_\mu^2}{4s^2} = 0.$

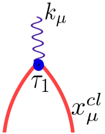
EOMs: $\frac{\ddot{x}_\mu}{2s} - ieF_{\mu\nu}\dot{x}_\nu = -ik_\mu\left(\delta(\tau-\tau_1) - \delta(\tau-\tau_2)\right).$

Break for the derivatives: $\frac{\dot{x}_\mu(\tau_1+0)-\dot{x}_\mu(\tau_1-0)}{2s} = -ik_\mu$

Saddle points $\tau_{1(2)}$ for the classical trajectory $x_\mu^{cl}(\tau)$

The strength of the break proportional to k_μ

Closed trajectories with saddle points both in E and H



Worldline instantons for the photon decay $\gamma \to e^+ e^-$ in electric and magnetic backgrounds

Ext. photon $k_\mu = (\omega, 0, \omega, 0)$

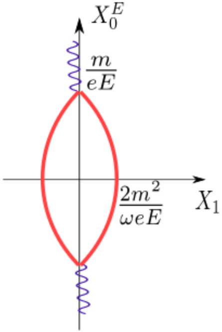
Electric field $(E, 0, 0)$ or
Magnetic field $(0, 0, H)$.

Classical solution —
two arcs of circle (electric)
two hyperbolas (magnetic)
connected at $\tau_1 = 0, \tau_2 = 0.5$

$$\Gamma \propto e^{-S}.$$

In the limit $\omega \gg 2m$

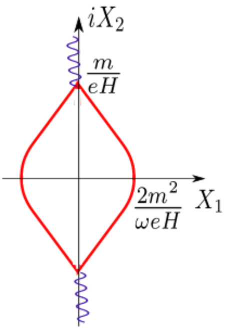
Electric field



$$S = \frac{8m^3}{3\omega eE}$$

Monin, Voloshin, 2010

Magnetic field



(projection on (ix_2, x_1))

$$S = \frac{8m^3}{3\omega eH}$$

PS 2013

Breit-Wheeler process $\gamma\gamma \to e^+e^-$ in external electric field

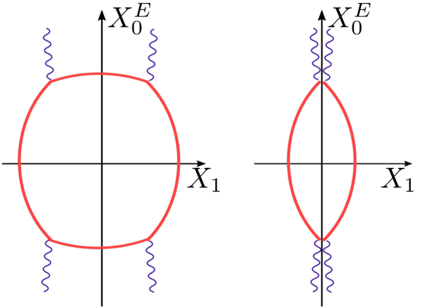
Breit-Wheeler cross-section is proportional to the imaginary part of 4-point photon amplitude:

$$\sigma_{\gamma\gamma\to e^+e^-}(k_1,k_2) \propto \mathrm{Im} \int_0^\infty \frac{ds}{s} e^{-m^2 s} \int_{p.b.c} Dx_\mu \prod_{j=1}^{4} \left(\oint d\tau_j \dot{x}_\mu \varepsilon_\mu(k_j) \right) \cdot$$

$$\cdot \exp\left(- \int_0^s d\tau \left(\frac{\dot{x}_\mu^2}{4} + ieA_\mu \dot{x}_\mu \right) - ik_1^\mu x_\mu(\tau_1) - ik_2^\mu x_\mu(\tau_2) + ik_1^\mu x_\mu(\tau_3) + ik_2^\mu x_\mu(\tau_4) \right).$$

General solution of E.O.M.s – 4 arcs of a circle, connected at 4 points $\tau_1 \ldots \tau_4$, see Fig, left panel.

On E.O.M.s up & down arcs shrink into points, see Fig, right panel.
(the same for N photons in the initial state)

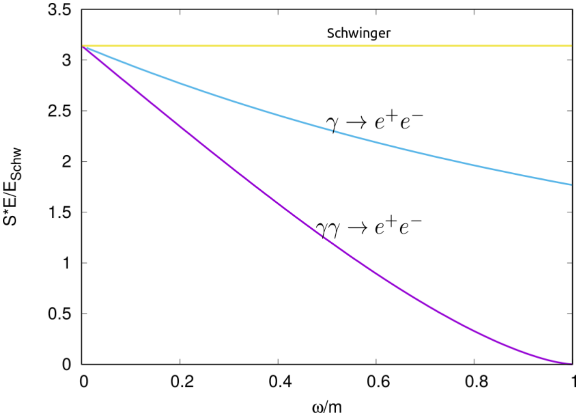


PS 2018 Quarks18 Proc

Breit-Wheeler pair production. Head-on collision.

Configuration: $k_1^\mu = (\omega, 0, \omega, 0)$, $k_2^\mu = (\omega, 0, -\omega, 0)$, $\vec{E} = (E, 0, 0)$.

Suppression exponent: $$S[x_\mu^{cl}] = \frac{m^2}{eE}\left[\pi - 2\arcsin\frac{\omega}{m} - 2\frac{\omega}{m}\sqrt{1-\left(\frac{\omega}{m}\right)^2}\right]$$



2γ – less suppressed than 1γ

$\omega/m = 1$ – threshold for Breit-Wheeler without external field

Pair production in N-photon interaction

Configuration: $K^\mu = (\omega_\Sigma = \sum_{i=1}^N \omega_i,\ \vec{k}_\Sigma = \sum_{i=1}^N \vec{k}_i)$, $\vec{E} = (E, 0, 0)$.

$$P^{non-pert}_{N\gamma\to e^+e^-} \propto \mathrm{Im} M_{2N} \sim e^{-S_N}$$

$$S_N = \frac{m^2}{eE}\left(\left(2 + \frac{k_\Sigma^2}{2m^2}\right)\arctan\frac{\sqrt{4m^2 - \omega_\Sigma^2 + k_\Sigma^2}}{\omega_\Sigma} - \frac{\omega\sqrt{4m^2 - \omega_\Sigma^2 + k_\Sigma^2}}{2m^2}\right)$$

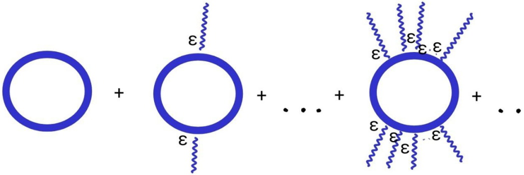
in agreement with Torgrimsson Schneider Oertel Schuetzhold JHEP 2017 (WKB method)

Dynamical assistance by a plane wave

Torgrimsson Schneider Schuetzhold PRD 2018

$$\vec{E} = E_0 \vec{e}_z + \varepsilon E_0 \cos \omega(t - x) \vec{e}_z$$

perturbation theory over ε



$$P = \sum_{N=1}^{\infty} \varepsilon^{2N} P_N, \qquad P_N : \omega_\Sigma = N\omega,\ k_\Sigma = N\Omega$$

$$\varepsilon^{2N} P_N = \exp\left(2N \log \varepsilon - \frac{m^2}{eE}\left[\left(2 + \frac{N^2\omega^2}{2m^2}\right) \arctan\left(\frac{2m}{N\omega}\right) - \frac{N\omega}{m}\right]\right)$$

Dynamical assistance by a plane wave

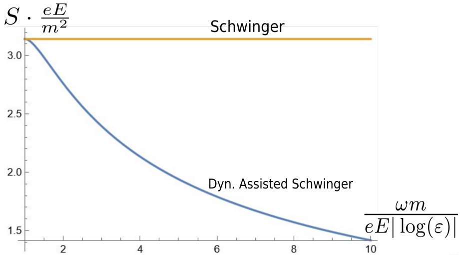
Which term gives the maximal contribution?

$$\text{saddle point:} \qquad \left.\frac{d(\varepsilon^{2N} P_N)}{dN}\right|_{N=N_*} = 0$$

$$1 - \frac{N_* \omega}{2m} \arctan \frac{2m}{N_* \omega} = \frac{|\log \varepsilon|}{m\omega/eE} \equiv \frac{\gamma_{crit}}{\gamma}$$

$$\gamma \ll \gamma_{crit} \quad \rightarrow \quad N_* = 0, \qquad P \sim e^{-\frac{\pi m^2}{eE}},$$

$$\gamma \gg \gamma_{crit} \quad \rightarrow \quad N_* = \frac{4m}{\omega}\sqrt{\frac{3m\omega/eE}{|\log \varepsilon|}}, \qquad P \sim e^{-8\frac{m^2}{eE}\sqrt{\frac{|\log \varepsilon|}{3m\omega/eE}}}$$

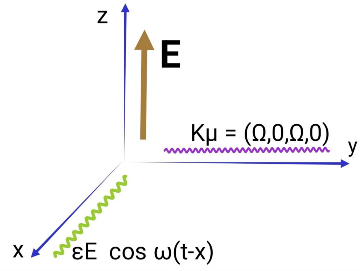


Double assistance by a plane wave and photon

$$\vec{E} = E_0 \vec{e}_z + \varepsilon E_0 \cos\omega(t-x)\vec{e}_z$$

$$\vec{K} = (\Omega, 0, \Omega, 0)$$

$$\omega \ll m, \quad \Omega \sim m$$



$$P = \sum_{N=1}^{\infty} \varepsilon^{2N} P_N, \qquad P_N : \omega_\Sigma = N\omega + \Omega, \quad k_\Sigma = \sqrt{N^2\omega^2 + \Omega^2}$$

$$\varepsilon^{2N} P_N = \exp\left(2N\log\varepsilon - \frac{m^2}{eE} S_N\right)$$

$$S_N = \left[\left(2 + \frac{k_\Sigma^2}{2m^2}\right)\arctan\frac{\sqrt{4m^2 - \omega_\Sigma^2 + k_\Sigma^2}}{\omega_\Sigma} - \frac{\omega\sqrt{4m^2 - \omega_\Sigma^2 + k_\Sigma^2}}{2m^2}\right]$$

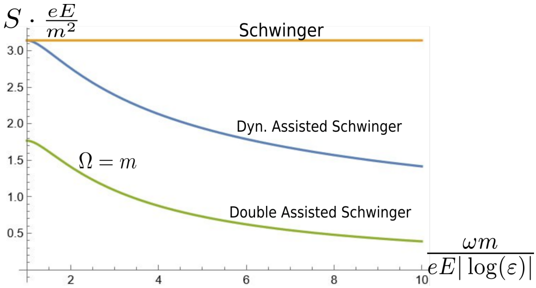
Double assistance by a plane wave and photon

saddle point: $$-\frac{m}{2\omega}\cdot\frac{dS_N}{dN}\bigg|_{N=N_*}=\frac{|\log\varepsilon|}{m\omega/eE}\equiv\frac{\gamma_{crit}}{\gamma}$$

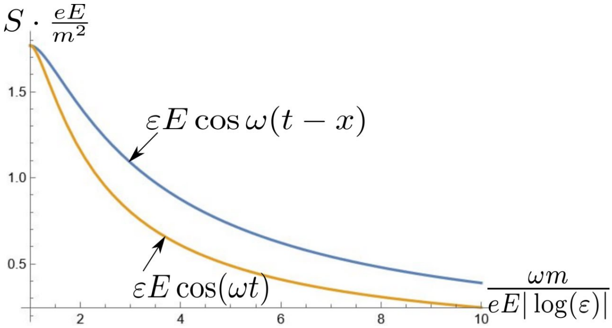
$$\sqrt{1-\frac{N_*\omega\Omega}{2m^2}}-\frac{N_*\omega}{2m}\arctan\frac{\sqrt{4m^2-2N_*\omega\Omega}}{N_*\omega+\Omega}=\frac{\gamma_{crit}}{\gamma}$$

$\gamma\ll\gamma_{crit}\quad\rightarrow\quad N_*=0,\qquad P\sim e^{-\frac{m^2}{eE}\left[\left(2+\frac{\Omega^2}{2m^2}\right)\arctan\left(\frac{2m}{\Omega}\right)-\frac{\Omega}{m}\right]}$

$\gamma\gg\gamma_{crit}\quad\rightarrow\quad N_*=\frac{2m^2}{\omega\Omega}\left(1-\frac{\gamma_{crit}^2}{\gamma^2}\frac{(2m^2+\Omega^2)^2}{\Omega^4}\right)\qquad P\sim e^{-\frac{4m^2}{eE}\frac{\gamma_{crit}}{\gamma}}$



Plane wave vs time-dependent electric field



Double assistance with plane wave is less effective

Conclusions

- 3 components of EM field increase the pair production probability compared to 2
- Intermediate plane wave - worse than time-dependent field / standing wave
- Experimental convenience for plane//standing wave for X-ray laser?
- other approaches - comparison

Mahlin Villalba-Chavez Mueller 2023

Thank you for your attention!