Нарушение симметрии и геометрические переходы в эффекте Казимира
Symmetry breaking and geometric transitions in the Casimir effect

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In this talk

1. Solution of diffraction grating problem. The Casimir energy in the system of two gratings. Scattering approach.
2. Lateral Casimir force experiment in the system of two diffraction gratings.
3. Torque in the system of two rotated infinite gratings. Breaking of translational symmetry. Geometric transition.
4. Giant torque in the system of two rotated finite gratings.

The ground state energy of the bosonic system:

$$E = \sum_i \frac{\omega_i}{2}, \tag{1}$$

the sum is over all eigenfrequencies of the system.
To evaluate (1) the argument principle can be used:

$$\frac{1}{2\pi i} \oint \phi(\omega) \frac{d}{d\omega} \ln f(\omega) d\omega = \sum \phi(\omega_0) - \sum \phi(\omega_\infty), \tag{2}$$

where

$$\phi(\omega) = \omega/2$$

and

$$f(\omega) = \det(I - R_{2up}(\omega) R_{1down}(\omega)). \tag{3}$$

Scattering approach: curved boundaries

- T. Emig, R.L. Jaffe, M. Kardar and A. Scardicchio, Casimir interaction between a plate and a cylinder, *Phys. Rev. Lett.* **96**, 080403 (2006).
- A. Lambrecht and V.N. Marachevsky, Casimir interaction of dielectric gratings, *Phys. Rev. Lett.* **101**, 160403 (2008).
- S.J. Rahi, T. Emig, N. Graham, R.L. Jaffe and M. Kardar, Scattering theory approach to electromagnetic Casimir forces, *Phys. Rev. D* **80**, 085021 (2009).
- A. Canaguier-Durand, P.A. Maia Neto, I. Cavero-Pelaez, A. Lambrecht and S. Reynaud, Casimir interaction between plane and spherical metallic surfaces, *Phys. Rev. Lett.* **102**, 230404 (2009).
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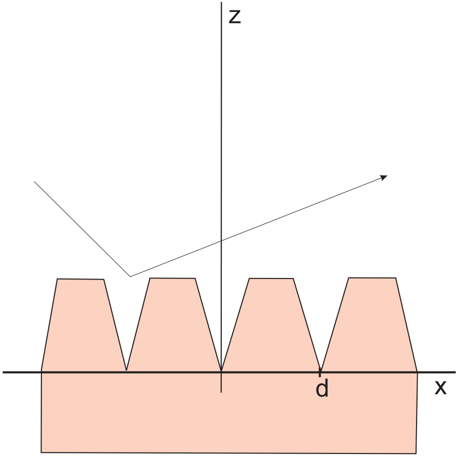
Diffraction grating



O.M. Rayleigh, On the dynamical theory of gratings, *Proc. Roy. Soc. A* **79**, 399–415 (1907).

Rayleigh decomposition for 1D gratings.

Rayleigh expansion for an incident electromagnetic wave on a single grating

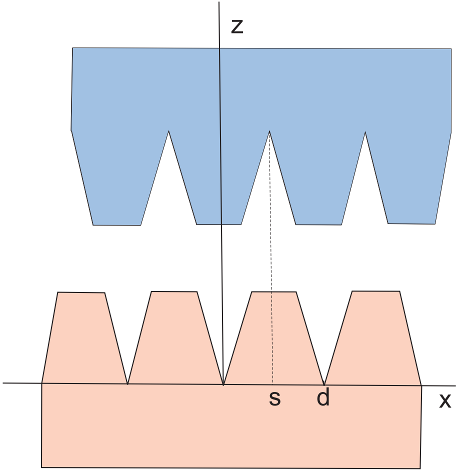
$$
\begin{aligned}
E_y(x,z) &= I_p^{(e)} \exp(i\alpha_p x - i\beta_p^{(1)} z) + \sum_{n=-\infty}^{+\infty} R_{np}^{(e)} \exp(i\alpha_n x + i\beta_n^{(1)} z), \\
H_y(x,z) &= I_p^{(h)} \exp(i\alpha_p x - i\beta_p^{(1)} z) + \sum_{n=-\infty}^{+\infty} R_{np}^{(h)} \exp(i\alpha_n x + i\beta_n^{(1)} z).
\end{aligned}
$$

Here $\alpha_p = k_x + 2\pi p/d$ and $\beta_p^{(1)2} = \omega^2 - k_y^2 - \alpha_p^2$.
The reflection matrix is constructed as follows:

$$
R_1(\omega) = \begin{pmatrix} R_{n_1 l_1}^{(e)}(I_p^{(e)} = \delta_{pl_1}, I_p^{(h)} = 0) & R_{n_2 l_2}^{(e)}(I_p^{(e)} = 0, I_p^{(h)} = \delta_{pl_2}) \\ R_{n_3 l_3}^{(h)}(I_p^{(e)} = \delta_{pl_3}, I_p^{(h)} = 0) & R_{n_4 l_4}^{(h)}(I_p^{(e)} = 0, I_p^{(h)} = \delta_{pl_4}) \end{pmatrix}.
$$

Rayleigh expansion is exact outside gratings. The unknown coefficients can be determined from the exact solution of Maxwell equations.

Two diffraction gratings



Casimir energy of two gratings

$$E = \frac{1}{(2\pi)^3} \int_0^{+\infty} d\omega \int_{-\infty}^{+\infty} dk_y \int_{-\frac{\pi}{d}}^{\frac{\pi}{d}} dk_x \quad \ln \det \Big(I - R_{2up} R_{1down} \Big)$$

$$R_{2up}(i\omega, kx, ky) = Q^* K(i\omega) R_{2down}(i\omega, kx, -ky) K(i\omega) Q, \qquad (4)$$

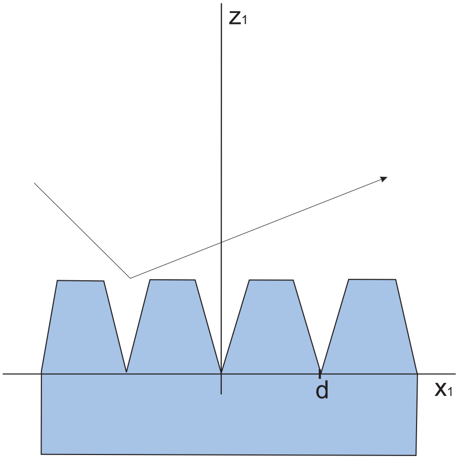
$$K(i\omega) = \begin{pmatrix} G_1 & 0 \\ 0 & G_1 \end{pmatrix}, \qquad (5)$$

with matrix elements $e^{-L\sqrt{\omega^2 + k_y^2 + (k_x + \frac{2\pi p}{d})^2}}, p = -N \dots N$ on the main diagonal of a matrix G_1,

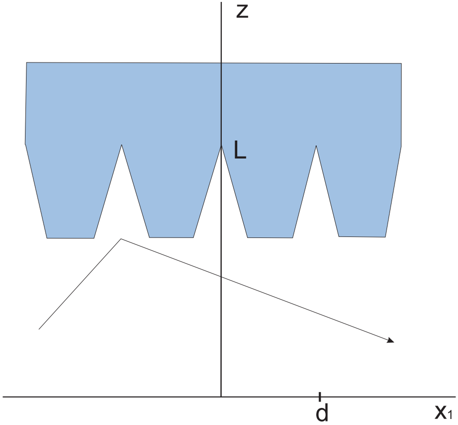
$$Q = \begin{pmatrix} G_2 & 0 \\ 0 & G_2 \end{pmatrix}, \qquad (6)$$

with matrix elements $e^{2\pi i m s/d}, p = -N \dots N$ on the main diagonal of a matrix G_2. [A.Lambrecht and V.N.Marachevsky, *Phys.Rev.Lett.* **101**, 160403 (2008); *Int. J. Mod. Phys. A* **24**, 1789–1795 (2009).].

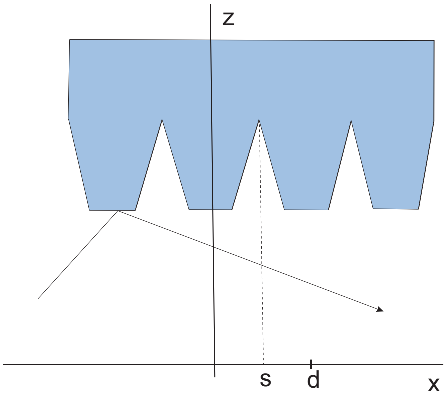
Reflection R_{2down}.



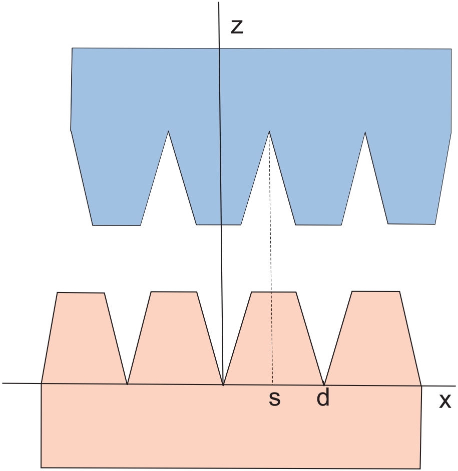
Change of coordinates $z = -z_1 + L$, $y = -y_1$ *in the solution.*



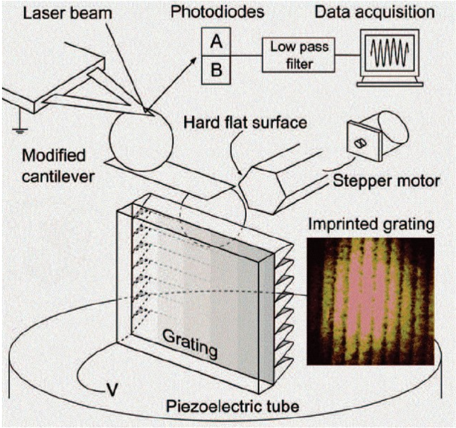
Lateral change of coordinates $x = x_1 - s$ in the solution. Reflection from the upper grating constituting R_{2up}.



$R_{2up}R_{1down}$



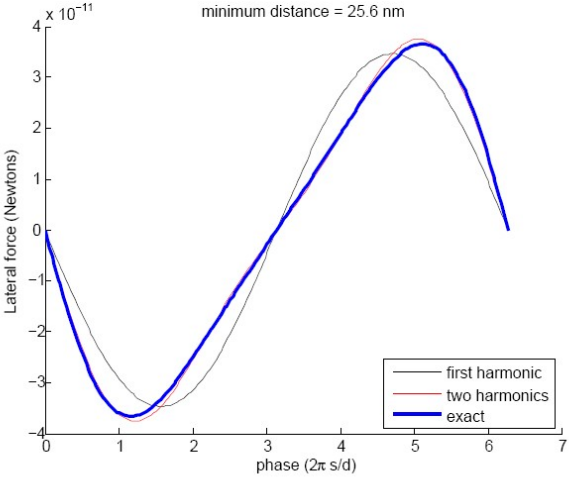
Lateral Casimir force experiment



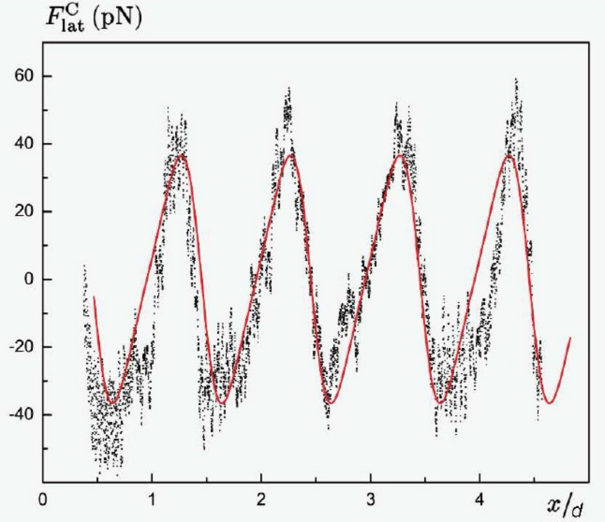
H.-C.Chiu, G.L.Klimchitskaya, V.N.Marachevsky, V.M.Mostepanenko and U.Mohideen, *Phys.Rev.B* **80**, 121402(R) (2009); *Phys.Rev.B* **81**, 115417 (2010).

Lateral Casimir force experiment

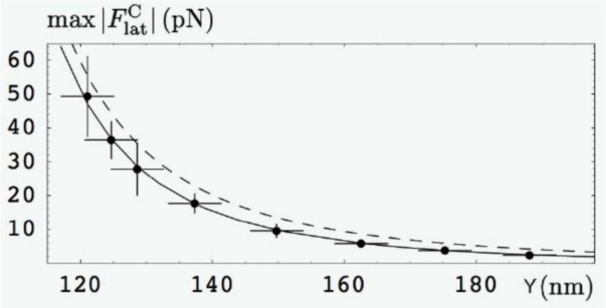
Consider gold sinusoidal corrugations with amplitudes A1= 85.4 nm, A2=13.7 nm, diameter of the sphere 2R=194.8 micrometers.



Lateral Casimir force experiment



Lateral Casimir force experiment



Maximum values of the measured lateral Casimir force are shown as crosses. Solid and dashed lines are predictions of the exact theory and the Proximity Force Approximation based on Lifshitz theory for two dielectric half-spaces.

Torque in the Casimir effect

- E. I. Kats, Van der Waals forces in non-isotropic systems, *Sov. Phys. JETP* **33**, 634 (1971).
- V. A. Parsegian and G. H.Weiss, Nonretarded van derWaals interaction between anisotropic long thin rods at all angles, *J. Chem. Phys.* **56**, 4393 (1972).
- Y. S. Barash, Moment of van der Waals forces between anisotropic bodies, *Radiophys. Quantum Electron.* **21**, 1138 (1978).
- S. J. van Enk, Casimir torque between dielectrics, *Phys. Rev. A* **52**, 2569 (1995).
- J.N. Munday, D. Iannuzzi, Y. Barash and F. Capasso, Torque on birefringent plates induced by quantum fluctuations, *Phys. Rev. A* **71**, 042102 (2005).

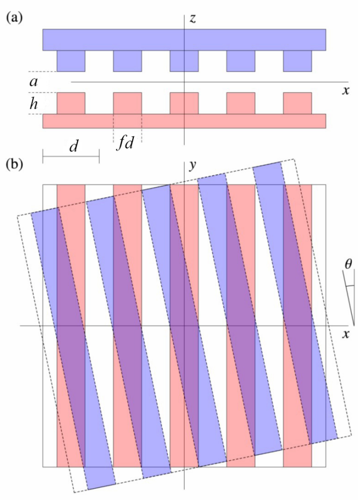
Torque in the Casimir effect

- R.B. Rodrigues, P.A. Maia Neto, A. Lambrecht and S. Reynaud, Vacuum-induced torque between corrugated metallic plates, *Europhys. Lett.* **76**, 822 (2006).
- T.G. Philbin and U. Leonhardt, Alternative calculation of the Casimir forces between birefringent plates, *Phys. Rev. A* **78**, 042107 (2008).
- X. Chen and J. C. H. Spence, On the measurement of the Casimir torque, *Phys. Status Solidi B* **248**, 2064 (2011).
- R. Guérout, C. Genet, A. Lambrecht and S. Reynaud, Casimir torque between nanostructured plates, *Europhys. Lett.* **111**, 44001 (2015).
- D. A. T. Somers and J. N. Munday, Casimir-Lifshitz Torque Enhancement by Retardation and Intervening Dielectrics, *Phys. Rev. Lett.* **119**, 183001 (2017).

Torque in the Casimir effect

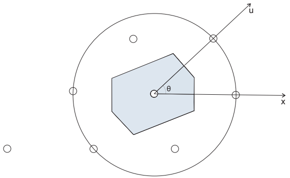
- D.A.T.Somers, J.L.Garrett, K.J.Palm, and J.N.Munday, Measurement of the Casimir torque, *Nature* **564**, 386 (2018).
- M.Antezza, H.B.Chan, B.Guizal, V.N.Marachevsky, R.Messina and M.Wang, Giant Casimir torque between rotated gratings and the $\theta = 0$ anomaly, *Phys.Rev.Lett.* **124**, 013903 (2020).
- B.Spreng, T.Gong, and J.N.Munday, Recent developments on the Casimir torque, *Int.J.Mod.Phys.A* **37**, 2241011 (2022).

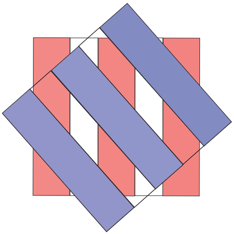
Rotated gratings



Consider the system of two Au rectangular gratings with parameters $d = 400$ nm, $f = 0.5$, $h = 200$ nm, $a = 100$ nm; the angle of rotation is θ.

Rotated gratings. Reciprocal lattice space.

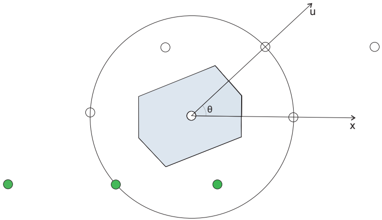


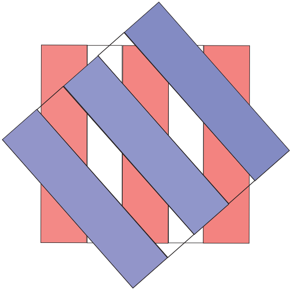


The vectors which are coupled by diffraction can be written as $\mathbf{k}_{nm} = \mathbf{k} + \frac{2\pi}{d}(n\mathbf{e}_x + m\mathbf{e}_u)$, where the vector $\mathbf{k}$ belongs to the first Brillouin zone.

Reflections from the lower grating constituting R_{1down}.

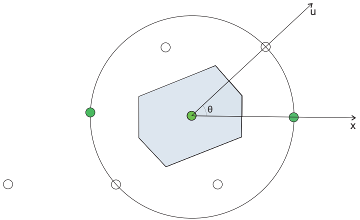
Reciprocal lattice vectors with $m = -1$ are highlighted green.

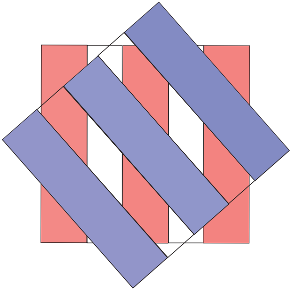




Reflections from the lower grating constituting R_{1down}

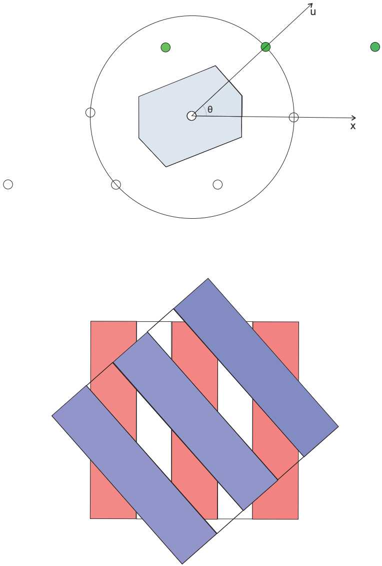
Reciprocal lattice vectors with $m = 0$ are highlighted green.





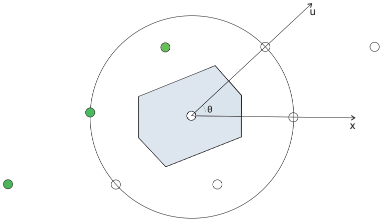
Reflections from the lower grating constituting R_{1down}

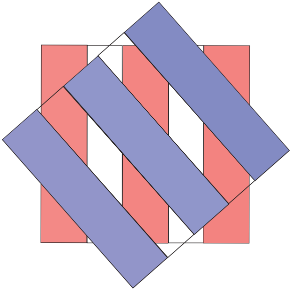
Reciprocal lattice vectors with $m = 1$ are highlighted green.



Reflections from the upper grating constituting R_{2up}

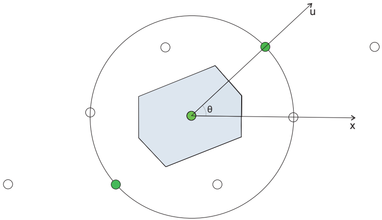
Reciprocal lattice vectors with $n = -1$ are highlighted green.

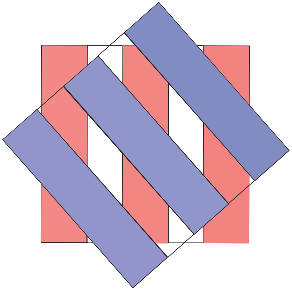




Reflections from the upper grating constituting R_{2up}

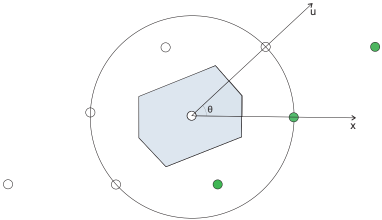
Reciprocal lattice vectors with $n = 0$ are highlighted green.

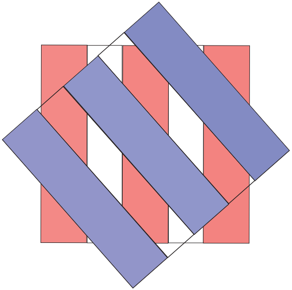




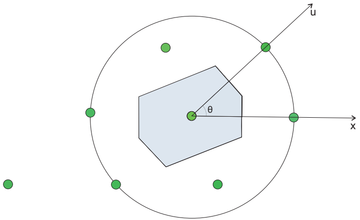
Reflections from the upper grating constituting R_{2up}

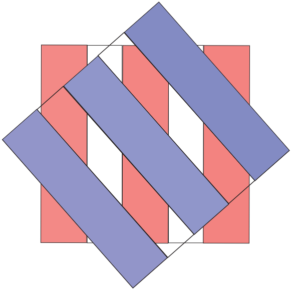
Reciprocal lattice vectors with $n = 1$ are highlighted green.





Casimir energy of two gratings depends on $R_{2up}R_{1down}$





Casimir energy and torque for two infinite rotated gratings

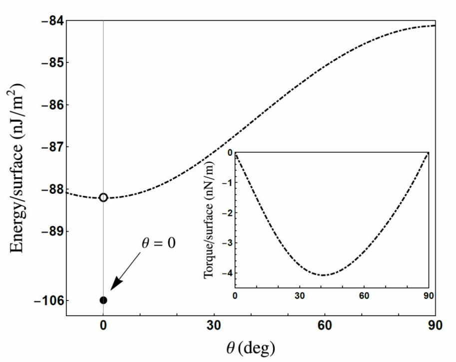
The Casimir energy of two infinite rotated gratings is defined by Rayleigh reflection coefficients contained in matrices R_{1down}, R_{2up} of the order $2(2N+1)^2$:

$$E(z,\theta) = \frac{1}{(2\pi)^3} \int_0^{+\infty} d\omega \int_{BZ} dk_x dk_y \\ \ln \det \Big(I - R_{2up}(i\omega, k_x, k_y) R_{1down}(i\omega, k_x, k_y) \Big). \quad (7)$$

The Casimir torque:

$$\tau = -\frac{\partial E(z,\theta)}{\partial \theta}. \quad (8)$$

Torque for infinite rotated gratings. 1D-2D geometric transition.



M.Antezza, H.B.Chan, B.Guizal, V.N.Marachevsky, R.Messina and M.Wang, Giant Casimir torque between rotated gratings and the $\theta = 0$ anomaly, *Phys.Rev.Lett.* **124**, 013903 (2020).

Energy discontinuity at rotation angle $\theta = 0$

Consider wave vectors coupled by diffraction in reciprocal lattice space in 1D system (strictly for $\theta = 0$):

$$\mathbf{k}_n = \mathbf{k} + \frac{2\pi n}{d}\mathbf{e}_x, \tag{9}$$

the first Brillouin zone is $-\pi/d < k_x < \pi/d$, while k_y takes all real values.

Consider wave vectors coupled by diffraction in reciprocal lattice space in 2D system (for any finite θ):

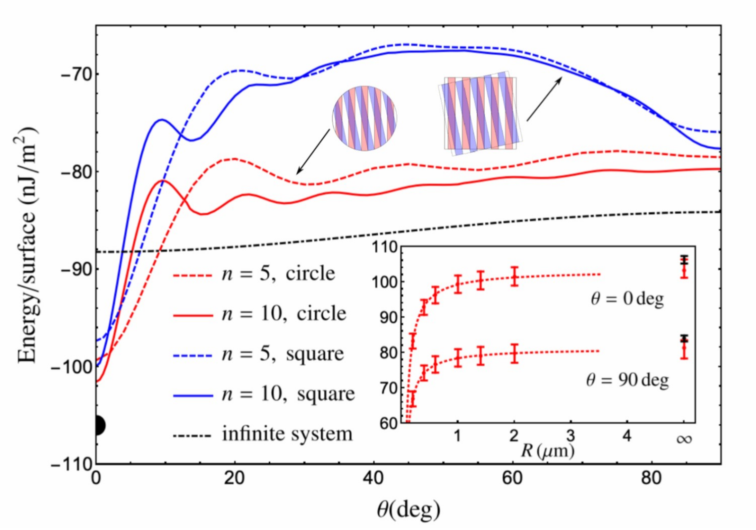
$$\mathbf{k}_{nm} = \mathbf{k} + \frac{2\pi}{d}(n\mathbf{e}_x + m\mathbf{e}_u) \tag{10}$$

While for two aligned gratings the y component of the total wave vector is strictly conserved in any scattering process, this conservation law is lost even for a small non-vanishing value of the rotation angle θ, since (see Eq.(10)) changing the value of the diffraction order m modifies the values of both x and y components of the wave vector.

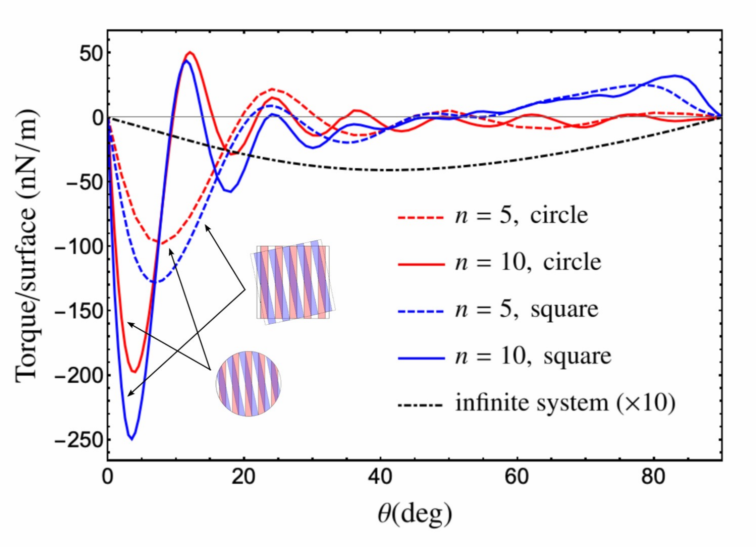
Energy discontinuity at rotation angle $\theta = 0$

The reason for appearance of energy discontinuity at rotation angle $\theta = 0$ is breaking of conservation of the k_y component of the wave vector in reciprocal lattice space due to rotation of the system and, as a result, the fundamental change of structure of reciprocal lattice space.

Casimir energy for finite rotated gratings



Casimir torque for finite rotated gratings



Conclusions

1. 1D-2D geometric transition (energy discontinuity at rotation angle $\theta = 0$) is found in the system of two infinite gratings with coinciding periods.
2. There is a conservation of k_y momentum in 1D system at rotation angle $\theta = 0$, breaking of k_y momentum conservation takes place in 2D system at any finite rotation angle θ.
   The reason for appearance of energy discontinuity at rotation angle $\theta = 0$ is breaking of conservation of k_y component of the wave vector in reciprocal lattice space due to rotation of the system and, as a result, the fundamental change of structure of reciprocal lattice space.
3. Giant torque is found in the system of two finite rotated gratings. Torque is growing without bounds when the size of gratings increases.

Conclusions

4. The effect should be of strong interest due to a novel mechanism of symmetry breaking which may be used to find analogous effects in various physical systems with spatial periodicity.