Образование кластеров кварк-глюонных струн в pp-столкновениях при высоких энергиях

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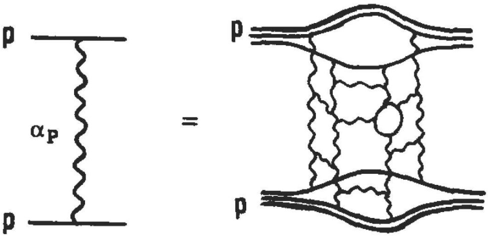
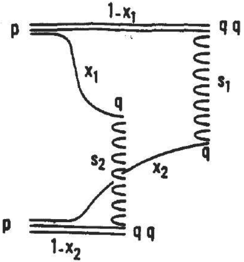
Президиум РАН, 17–21 февраля 2025

Strings as a cut pomeron

Pomeron as a cylindrical structure (in the large color number limit):
G. 't Hooft, Nucl. Phys. B **72** (1974) 461 - 't Hooft's 1/Nc expansion
G. Veneziano, Nucl. Phys. B **117** (1976) 519 - Veneziano's topol.expan.

Cut pomeron as two strings (color reconnection of dipoles):
A. Capella, U.P. Sukhatme, C.–I. Tan, J. Tran Thanh Van (DPM)
Phys. Lett. B **81**, 68 (1979); Phys. Rep. **236**, 225 (1994)
K. Werner (VENUS,EPOS), Phys. Rep. **232**, 87 (1993)

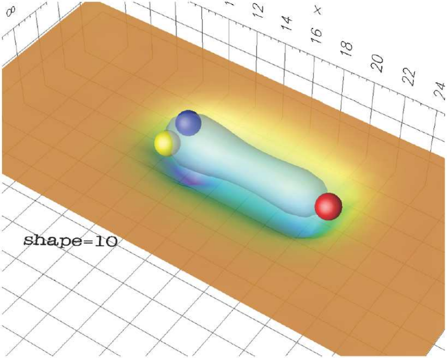




Strings as color flux tubes

Color flux-tubes (gluon, chromo-electric flux-tubes):
A.B. Kaidalov (QGSM), Phys. Lett. B **116**, 459 (1982)
A.B. Kaidalov, K.A. Ter-Martirosyan, Phys. Lett. B **117**,247 (1982)

Confirmed by lattice QCD simulations:
F. Bissey, A. I. Signal, D. B. Leinwebe, Phys. Rev. D **80**, 114506 (2009)



Fragmentation of strings

Schwinger mechanizm in QED:

J. Schwinger, Phys. Rev. 82, 664 (1951)

A.I. Nikshov, Nucl. Phys. B21, 346 (1970)

T.D. Cohen and D.A. McGady, Phys.Rev.D 78, 036008 (2008)

Schwinger based picture in QCD:

E.G. Gurvich, Phys.Lett. 87B (1979) 386

A. Casher, H. Neunberg and S. Nussinov, Phys. Rev. D20 (1979) 179

M. Gyulassy and A. Iwazaki, Phys. Lett. B165 (1985) 157

A. Bialas, Phys. Lett. B 466 (1999) 301

Geometrical approach to sting fragmentation:

X. Artru, *Phys. Rep.* **97** (1983) 147

K. Werner (VENUS,EPOS), *Phys. Rept.* **232** (1993) 87

V.V.V., *Proceedings of the Baldin ISHEPP XIX vol.1*, JINR, Dubna (2008) 276-281; arXiv:0812.0604.

Various versions of the string model are used in such Monte Carlo (MC) event generators as PYTHIA, VENUS, HIJNG, AMPT, EPOS, DIPSY etc., to describe soft processes in strong interactions when perturbation QCD does not work.

String fusion effects.

pp $\rightarrow$ pA $\rightarrow$ AA - the increase of the string density in transverse plain leads to the string fusion (color ropes or string cluster formation)

T.S. Biro, H.B. Nielsen, J. Knoll, Nucl. Phys. B **245**, 449 (1984)
A. Bialas, W. Czyz, Nucl. Phys. B **267**, 242 (1986)
M.A. Braun,C. Pajares, Phys.Lett. **B287**, 154 (1992);
Nucl. Phys. **B390**, 542 (1993)

Collective effects are observed also in high-multiplicity pp events at LHC

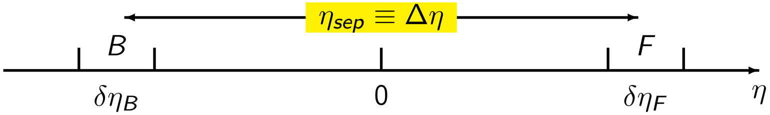
String fusion in pp collisions with increasing energy and centrality

$\Rightarrow$ Reduction of multiplicity, increase of transverse momenta.
$\Rightarrow$ The influence on the Long-Range FB Correlations (LRC).

N.S. Amelin, N. Armesto, M.A. Braun, E.G. Ferreiro, C. Pajares, Phys.Rev.Lett. **73**, 2813 (1994).

The same ideas in DIPSY:
C. Bierlich, G. Gustafson, L. Lonnblad, A. Tarasov JHEP **03** (2015) 148
Effects of Overlapping Strings in pp Collisions

Forward-Backward (FB) Rapidity Correlations



Forward-Backward (FB) Rapidity Correlations: $(k_z, \mathbf{k}_\perp) \Rightarrow (y, \mathbf{k}_\perp)$

$$y \equiv \frac{1}{2} \ln \frac{k_0 + k_z}{k_0 - k_z} \;, \qquad \eta \equiv \frac{1}{2} \ln \frac{|\mathbf{k}| + k_z}{|\mathbf{k}| - k_z} = -\ln \operatorname{tg}\left(\frac{\theta^*}{2}\right)$$

The correlation coefficient:

$$b_{BF} = \frac{\langle FB \rangle - \langle F \rangle \langle B \rangle}{\langle F^2 \rangle - \langle F \rangle^2} = \frac{cov(F,B)}{D_F} \tag{1}$$

Short- and long-range rapidity correlations

Traditional Observables

Traditional FB correlation:
B, F ⇒ n_B, n_F - the extensive variables ⇒ b_{nn}
A. Capella and A. Krzywicki, Phys.Rev.D**18**, 4120 (1978)
The locality of strong interaction in rapidity ⇒
Short-Range FB Correlations (SRC),
between particles from a same source (string).
$z - \eta$ correspondence, *X.Artru,Phys.Rept.**97**(1983)147,*
V.V.V.,arXiv:0812.0604
Event-by-event variance in the number of cut pomerons (strings) ⇒
Long-Range FB Correlations (LRC) at large η_{sep}
(the trivial "volume" fluctuations).

We'll look for observables, which is not sensitive to the fluctuation in the number of sources (strings), but is sensitive to the fluctuation in the quality of sources (e.g. to the formation of string clusters by string fusion).

The strongly intensive observable $\Sigma(n_F, n_B)$

The strongly intensive quantities [*M.I.Gorenstein,M.Gazdzicki,Phys.Rev.C84(2011)014904*].
We define the strongly intensive observable $\Sigma(n_F, n_B)$ between multiplicities in forward (n_F) and backward (n_B) windows [*E.V.Andronov,Theor.Math.Phys.185(2015)1383*] as

$$\Sigma(n_F, n_B) \equiv \frac{1}{\langle n_F \rangle + \langle n_B \rangle}[\langle n_F \rangle \, \omega_{n_B} + \langle n_B \rangle \, \omega_{n_F} - 2\,\mathrm{cov}(n_F \, n_B)] \ , \qquad (2)$$

where $\mathrm{cov}(n_F, n_B) \equiv \langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle$, and ω_{n_F} and ω_{n_B} are the corresponding scaled variances of the multiplicities:

$$\omega_n \equiv \frac{D_n}{\langle n \rangle} = \frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle} \ . \qquad (3)$$

Let us also introduce the so-called robust dispersion:

$$R_n \equiv (\omega_n - 1)/\langle n \rangle = C_2(0) \ . \qquad (4)$$

The model with independent identical strings

[*M.A. Braun, C. Pajares, V.V.V., Phys. Lett. B* ***493****, 54 (2000)*]

The number of strings, N, fluctuates event by event around some mean value, $\langle N\rangle$, with some scaled variance, $\omega_N = D_N/\langle N\rangle$.

Intensive observable does not depends on $\langle N\rangle$.

Strongly intensive observable does not depends on $\langle N\rangle$ and ω_N.

The fundamental characteristics of a string:

one- and two-particle rapidity distributions from a single string decay:

$$\lambda(\eta) = \mu_0, \qquad \lambda_2(\eta_1,\eta_2) = \lambda_2(\eta_1-\eta_2) = \lambda_2(\Delta\eta)$$

$\Lambda(\Delta\eta)$ - two-particle correlation function of a string (similar to $C_2(\Delta\eta)$, but for one string):

$$\Lambda(\eta_1,\eta_2) \equiv \frac{\lambda_2(\eta_1,\eta_2)}{\lambda(\eta_1)\lambda(\eta_2)} - 1 = \frac{\lambda_2(\Delta\eta)}{\mu_0^2} - 1 = \Lambda(\Delta\eta)\ .$$

Clear that in this model:

$$\langle n_F\rangle = \langle \mu_F\rangle\langle N\rangle = \mu_0\delta\eta\,\langle N\rangle, \qquad \omega_{n_F} = \omega_{\mu_F} + \langle\mu_F\rangle\omega_N, \qquad R_{n_F} = \frac{R_{\mu_F}+\omega_N}{\langle N\rangle}$$

and the same for n_B ($\delta\eta$ - the width of the observation window).

The model with independent identical strings

$$\omega_{\mu_F} = 1 + \mu_0\, \delta\eta\, J_{FF}$$

$$\omega_{n_F} = 1 + \mu_0\, \delta\eta\, [J_{FF} + \omega_N]$$

$$R_{\mu_F} = J_{FF}$$

$$R_{n_F} = [J_{FF} + \omega_N]/\langle N \rangle$$

$$\Sigma(n_F, n_B) = \Sigma(\mu_F, \mu_B) = 1 + \mu_0\, \delta\eta\, [J_{FF} - J_{FB}]$$

$$J_{FF} = \frac{1}{\delta\eta_F^2} \int_{\delta\eta_F} d\eta_1 \int_{\delta\eta_F} d\eta_2\, \Lambda(\eta_1 - \eta_2) \to \Lambda(0)$$

$$J_{FB} = \frac{1}{\delta\eta_F \delta\eta_B} \int_{\delta\eta_F} d\eta_1 \int_{\delta\eta_B} d\eta_2\, \Lambda(\eta_1 - \eta_2) \to \Lambda(\Delta\eta)$$

Vechernin V 2018 Eur.Phys.J.: Web of Conf. 191 04011
Andronov E, Vechernin V 2019 Eur.Phys.J. A 55 14
Vechernin V, Andronov E 2019 Universe 5 15

Note that Poisson fluctuations are excluded in any observation window even in the presence of only short-range rapidity correlations between particles.

Properties of Σ in model with independent identical strings

- We see that in the model with identical strings the $\Sigma(\Delta\eta)$ is a really strongly intensive quantity. It does not depend nor on the mean number of strings $\langle N\rangle$, nor on their event-by-event fluctuations $\omega_N \equiv D_N/\langle N\rangle$.
It depends ONLY on string parameters: μ_0 and $\Lambda(\Delta\eta)$.

$\Sigma(n_F, n_B) = \Sigma(\mu_F, \mu_B) = \Sigma(\Delta\eta)$ vs e.g. $C_2(\Delta\eta, \Delta\phi) = \frac{\Lambda(\Delta\eta,\Delta\phi)+\omega_N}{\langle N\rangle}$

V.V.,Nucl.Phys.A939(2015)21

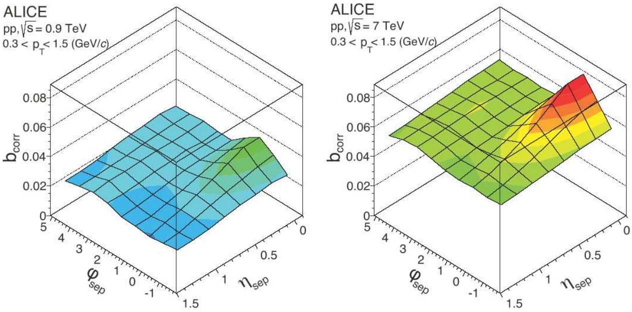
- The $\Sigma(0) = 1$ and increases with the gap between windows, $\Delta\eta$, as the $\Lambda(\Delta\eta)$ decrease to 0 with $\Delta\eta$, since the correlations in a string go off with increase of $\Delta\eta$.
- The rate of the $\Sigma(\Delta\eta)$ growth with $\Delta\eta$ is proportional to the width of the observation window $\delta\eta$ and μ_0 - the multiplicity produced from one string.
- The model predicts saturation of the $\Sigma(\Delta\eta)$ on the level

$$\Sigma(\Delta\eta) = 1 + \mu_0\delta\eta\,\Lambda(0) = \omega_\mu = D_\mu/\langle\mu\rangle$$ at large $\Delta\eta$,

since $\Lambda(\Delta\eta) \to 0$ at the $\Delta\eta \gg \eta_{corr}$,
where the η_{corr} is a string correlation length.

The ALICE data on b_{nn} in pp

ALICE collab., JHEP 05(2015)097



$$b_{nn} = \frac{\mu_0 \delta\eta\,[\omega_N + \Lambda(\Delta\eta,\Delta\phi)]}{1+\mu_0\delta\eta\,[\omega_N+\Lambda(0,0)]} \;\Rightarrow\; \omega_N,\, \Lambda(\Delta\eta,\Delta\phi) \;\Rightarrow\; \Lambda(\Delta\eta)$$

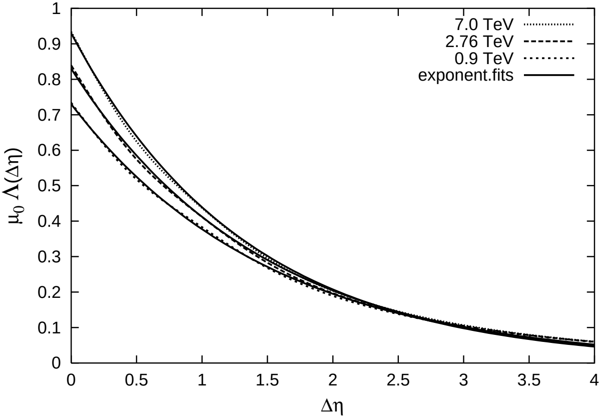
V.V.,Nucl.Phys.A939(2015)21; V.Vechernin, EPJ Web Conf. 191(2018)04011.

E. Andronov, V.Vechernin, Eur.Phys.J.A 55(2019)14,

The string correlation function $\Lambda(\Delta\eta)$

Then we find $\Lambda(\Delta\eta)$ integrating over azimuth:

$$\Lambda(\eta_{sep}) = \frac{1}{\pi}\int_0^{\pi}\Lambda(\eta_{sep},\phi_{sep})\,d\phi_{sep}\ .$$



The string correlation function $\Lambda(\Delta\eta)$

The obtained dependencies in this fugure for three initial energies are well approximated by the exponent:

$$\Lambda(\Delta\eta) = \Lambda_0 e^{-\frac{|\Delta\eta|}{\eta_{corr}}} , \tag{5}$$

with the parameters presented in the table:

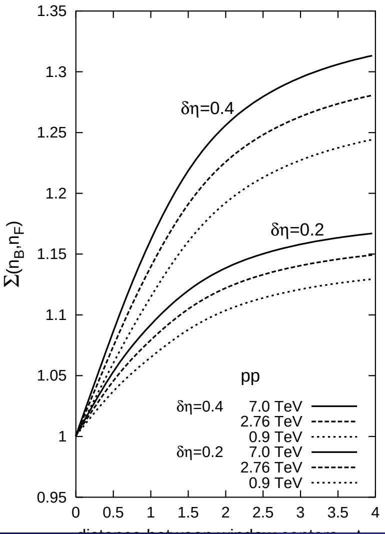
$\sqrt{s}$, TeV	0.9	2.76	7.0
$\mu_0\Lambda_0$	0.73	0.83	0.93
η_{corr}	1.52	1.43	1.33

[V.V., EPJ Web Conf. 191(2018)04011]

We see that the correlation length, η_{corr}, decreases with the increase of collision energy.

This can be interpreted as a signal of an increase with energy of the admixture of strings of a new type - the fused strings in pp collisions.

The predictions for the $\Sigma(n_F, n_B)$ in the model with independent identical strings



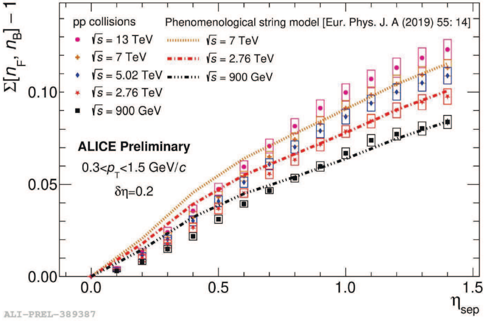
V.Vechernin, EPJ WoC 191 (2018) 04011, E.Andronov, V.Vechernin, Eur.Phys.J. A55 (2019) 14

Using the $\Lambda(\Delta\eta, \Delta\phi)$, extracted in V.Vechernin,Nucl.Phys.A939(2015)21 from the ALICE pp data on FB correlations in small acceptance windows, separated in azimuth and rapidity [ALICE collab., JHEP05(2015)097]

The string parameters occur dependent on initial energy (!)
The hint on the increase of the string cluster contribution to $\Sigma(n_F, n_B)$ with collision energy in pp collisions

Comparing the $\Sigma(n_F, n_B)$ with preliminary ALICE data

The comparison of the string model predictions with preliminary ALICE data for the $\Sigma(n_F, n_B)$ in pp collisions at energies 0.9 - 7 TeV [Andrey Erokhin (for the ALICE Collaboration) "Forward-backward multiplicity correlations with strongly intensive observables in pp collisions", The VI-th International Conference on the Initial Stages of High-Energy Nuclear Collisions (IS2021), 10-15 January 2021]:



"Phenomenological string model from [E. Andronov, V.Vechernin, Eur.Phys.J.A 55(2019)14] reproduces the quantitative behavior better than PYTHIA"

$\Sigma(n_F, n_B)$ in the model with string fusion

In the model with string fusion on transverse grid we find [*S.N. Belokurova, V.V.V., Theor.Math.Phys. 200(2019)1094*]:

$$\Sigma(n_F, n_B) = \sum_{k=1}^{\infty} \alpha_k \, \Sigma_k(\mu_F, \mu_B) , \qquad \alpha_k = \frac{\langle n^{(k)} \rangle}{\langle n \rangle} , \tag{6}$$

where k is a degree of string overlapping and $\langle n^{(k)} \rangle$ is a mean number of particles produced from areas with such overlapping. $\sum \alpha_k = 1$.
Here $\Sigma_k(\mu_F, \mu_B)$ is the variable Σ for the cluster formed by k strings:

$$\Sigma_k(\mu_F, \mu_B) = \Sigma_k(\Delta\eta) = 1 + \mu_0^{(k)} \delta\eta [\Lambda_k(0) - \Lambda_k(\Delta\eta)] ,$$

where $\mu_0^{(k)}$ and $\Lambda_k(\Delta\eta)$ are the corresponding parameters of the string cluster.

$$\Lambda_k(\Delta\eta) = \Lambda_0^{(k)} exp[-|\Delta\eta| / \eta_{corr}^{(k)}]$$

$\Sigma(n_F, n_B)$ in the model with string fusion

[*M.A.Braun, C.Pajares* Nucl.Phys.B 390 (1993) 542]

$$\mu_0^{(k)} = \mu_0^{(1)}\sqrt{k}\ , \qquad \Lambda_0^{(k)} = \Lambda_0^{(1)} = const\ , \qquad \eta_{corr}^{(k)} = \eta_{corr}^{(1)}/\sqrt{k}\ ,$$

which is instructive to compare with

$$\mu_0^{(k)} = \mu_0^{(1)}\, k\ , \qquad \Lambda_0^{(k)} = \Lambda_0^{(1)}/k\ , \qquad \eta_{corr}^{(k)} = \eta_{corr}^{(1)} = const\ .$$

for the case without string fusion in a given transverse cell.

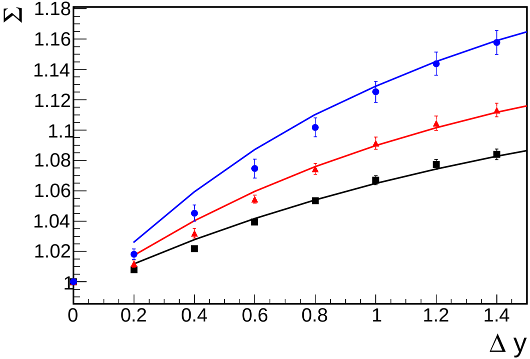
(In last case $\Sigma(n_F, n_B) = \Sigma_1(\mu_F, \mu_B)$ and does not depends on α_k.)

The values of the parameters $\Lambda_0^{(1)} = 0.8$ and $\eta_{corr}^{(1)} = 2.7$ were chosen to fit the ALICE experimental data.

[Andrey Erokhin (for the ALICE Collaboration), The VI-th International Conference on the Initial Stages of High-Energy Nuclear Collisions (IS2021), 10-15 January 2021]

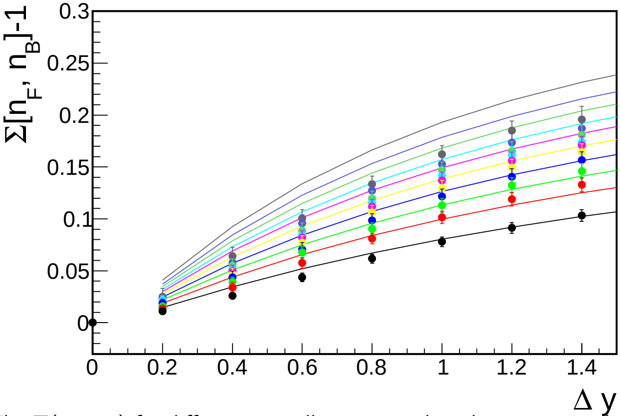
Comparing $\Sigma(n_F, n_B)$ with the ALICE experimental data

$$\Sigma(n_F, n_B) = \sum_{k=1}^{\infty} \alpha_k \, \Sigma_k(\mu_F, \mu_B)$$



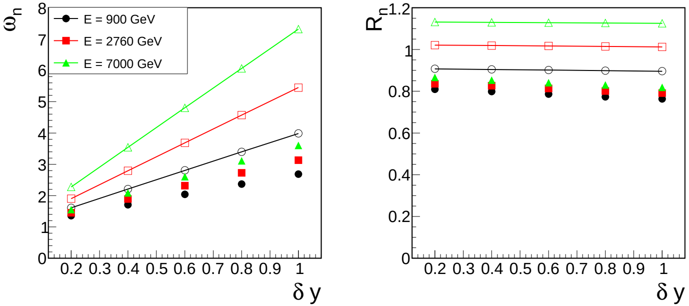
The $\Sigma(n_F, n_B)$ as a function of rapidity distance Δy between windows of δy =0.2 width for three initial energies 0.9 (black), 7 (red) and 13 (blue) TeV. Points - experimental values obtained at same energies in [A.Erokhin (for the ALICE Collaboration) reported at IS2021]

Comparing $\Sigma(n_F, n_B)$ with the ALICE experimental data



The $\Sigma(n_F, n_B)$ for different pp-collision centrality classes at initial energy 13 TeV. Experimental points from [A.Erokhin (for the ALICE Collaboration) reported at IS2021]. Curves - our results in the model with the formation of string clusters. The centrality classes defined as follows (top down): 0–1%, 1–5%, 5-10%, 10-15%, 15-20%, 20-30%, 30-40%, 40-50%, 50-70%, 70-100%.

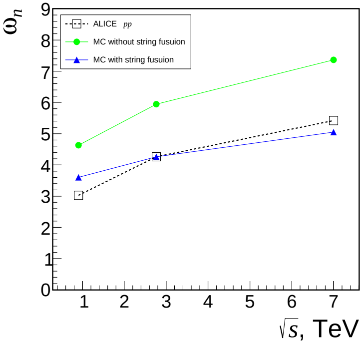
ω_n and R_n - with and without string fusion



Results of calculations of the scaled $\omega[n]$ (left panel) and robust $R[n]$ (right panel) variances as functions of the rapidity observation window width δy for pp collisions at three initial energies: $\sqrt{s}$ =900 (●), 2760 (■) and 7000(▲) GeV taking into account the formation of string clusters.

Open markers (○, □, △) represent the results obtained using the same MC algorithm of pp interaction, but without taking into account the process of string fusion into clusters (Lines are the results of analytical calculations in this case).

Comparison of the scaled variance $\omega[n]$ with ALICE data



Comparison is done for the rapidity window $y \in [-0.8, 0.8]$ and particles with transverse momenta $p_\perp \in (0.3, 1.5)$ GeV produced in pp interactions at three initial energies: $\sqrt{s}$ =0.9, 2.76 and 7 TeV.

□ – results of calculations in the model with formation of string clusters,

● – results of calculations in the model without string fusion (model parameters were fitted to reproduce experimental values of multipicity),

▲ – experimental values from *[ALICE collab., JHEP 05(2015)097]*.

Summary

It is shown that multiplicity fluctuations and especially the dependence of the strongly intense variable $\Sigma(n_F, n_B)$ on the initial energy and centrality of the pp collision can only be explained by the formation of string clusters in pp collisions at LHC energies, the characteristics of which differ from those of a single string.

The authors acknowledge Saint-Petersburg State University for a research project 103821868.

Backup

Backup slides

Various versions of string fusion

local fusion (overlaps)

M.A. Braun, C. Pajares Eur.Phys.J. **C16**, 349, (2000)

$\langle n\rangle_k = \mu_0\sqrt{k}\ S_k/\sigma_0$, $\langle p_t^2\rangle_k = p_0^2\sqrt{k}$, $k = 1, 2, 3, ...$

global fusion (clusters)

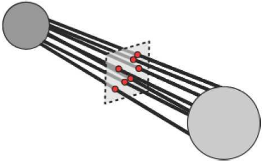
M.A. Braun, F. del Moral, C. Pajares, Phys.Rev. **C65**, 024907, (2002)

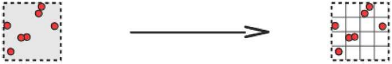
$\langle p_t^2\rangle_{cl} = p_0^2\sqrt{k_{cl}}$, $\langle n\rangle_{cl} = \mu_0\sqrt{k_{cl}}S_{cl}/\sigma_0$, $k_{cl} = k\sigma_0/S_{cl}$

the version of SFM with the finite lattice (grid) in transverse plane

Vechernin V.V., Kolevatov R.S., hep-ph/0304295; hep-ph/0305136

Braun M.A., Kolevatov R.S., Pajares C., Vechernin V.V., Eur.Phys.J. C32 (2004) 535





Domains in transverse area

The approach with string fusion on a transverse lattice (grid) was exploited later for a description of various phenomena (correlations, anisotropic azimuthal flows, the ridge) in high energy hadronic collisions in

ALICE collaboration et al., J. Phys. G **32** 1295 (2006), [Sect. 6.5.15]
V.V.V., Kolevatov R.S. Phys.of Atom.Nucl. **70** (2007) 1797; 1858
M.A. Braun, C. Pajares, Eur. Phys. J. C **71**, 1558 (2011)
M.A. Braun, C. Pajares, V.V.V., Nucl. Phys. A **906**, 14 (2013)
V.N. Kovalenko, Phys. Atom. Nucl. **76**, 1189 (2013)
M.A. Braun, C. Pajares, V.V.V., Eur. Phys. J. A **51**, 44 (2015)
V.V.V., Theor. Math. Phys. 184 (2015) 1271
V.V.V., Theor. Math. Phys. 190 (2017) 251

It leads to the splitting of the transverse area into domains with different, fluctuating values of color field within them.

What was also considered in the CGC approach
A.Kovner., M. Lublinsky, Phys.Rev. D **83**, 034017 (2011)

Distribution of strings in the transverse plane

pp interactions

[V.V. Vechernin, S.N. Belokurova, Theor.Math.Phys. 216(2023)1299]

$$w_{str}(\vec{s},\vec{b}) \sim T(\vec{s}-\vec{b}/2)\,T(\vec{s}+\vec{b}/2)/\sigma_{pp}(b) \qquad (7)$$

$\sigma_{pp} = \int \sigma_{pp}(b)\; d^2\vec{b}$ - non-diffractive pp cross section

$T(\vec{s}) = \int\limits_{-\infty}^{+\infty} \rho(\vec{s},z)\; dz$ - parton profile function of nucleon

$$\rho(r) = \frac{1}{\pi^{3/2}\alpha^3} e^{-r^2/\alpha^2}\,, \qquad T(s) = \frac{e^{-s^2/\alpha^2}}{\pi\alpha^2}\,, \qquad (8)$$

$$w_{str}(\vec{s},\vec{b}) \sim e^{-(\vec{s}+\vec{b}/2)^2/\alpha^2} e^{-(\vec{s}-\vec{b}/2)^2/\alpha^2}/\sigma_{pp}(b) = e^{-2s^2/\alpha^2} e^{-b^2/2\alpha^2}/\sigma_{pp}(b)$$

b–s factorization $\Rightarrow$

$$\langle N_{str}(b)\rangle \sim e^{-b^2/2\alpha^2}/\sigma_{pp}(b) \qquad (9)$$

Event-by-event fluctuations of the number of cut pomerons

$$P(N,b) = e^{-\overline{N}(b)}\overline{N}(b)^N/N!$$ -Poisson, $$P(0,b) = e^{-\overline{N}(b)}$$

$$\widetilde{P}(N,b) = P(N,b)/[1-P(0,b)]$$ -modified Poisson, $\sum_{N=1}\widetilde{P}(N,b) = 1$

$$\langle N(b)\rangle = \sum_{N=1} N\,\widetilde{P}(N,b) = \overline{N}(b)/[1-P(0,b)] \tag{10}$$

$$\sigma_{pp}^{ND}(b) = 1 - P(0,b) = 1 - e^{-\overline{N}(b)} \tag{11}$$

$$\langle N(b)\rangle = \overline{N}(b)/\sigma_{pp}^{ND}(b)$$

$$\overline{N}(b) = N_0 e^{-b^2/2\alpha^2}$$ $$\langle N(b)\rangle = \overline{N}(b)/[1-\exp(-\overline{N}(b))]$$

$N_{str} = 2N$, N - the number of cut pomerons in a given event

$$\langle N(b)\rangle = N_0 e^{-b^2/2\alpha^2}/\sigma_{pp}(b)$$

Probability to have N cut pomerons in a non-diffractive pp collision

Integration over the impact parameter b leads to

$$w_N = \frac{2\pi\alpha^2}{\sigma_{pp} N}\left[1 - e^{-N_0}\sum_{k=0}^{N-1} N_0^k/k!\right] = \frac{\sigma_N}{\sigma_{pp}^{ND}}$$

where we have introduced the σ_N by

$$\sigma_N \equiv \frac{2\pi\alpha^2}{N}\left[1 - e^{-N_0}\sum_{k=0}^{N-1} N_0^k/k!\right]$$

$$\sum_{N=1}^{\infty} \sigma_N = 2\pi\alpha^2[E_1(N_0) + \gamma + \ln N_0] = \sigma_{pp}^{ND}$$

where σ_{pp}^{ND} is the non-diffractive pp cross section.

$$E_1(z) = \int_1^{\infty} e^{-zt}\,\frac{dt}{t}\,, \qquad \gamma = 0.577...$$

Comparison with quasi-eikonal and Regge approaches

Now we see that our formula for the σ_N coincides with the well known result for the cross-section σ_N of N cut-pomeron exchange, obtained in the quasi-eikonal and Regge approaches:

$$\sigma_N = \frac{4\pi\lambda}{C\,N}\left[1 - e^{-z}\sum_{k=0}^{N-1} z^k/k!\right]$$

where

$$z = \frac{2\gamma C}{\lambda}\exp(\Delta\xi)\ , \qquad \lambda = R^2 + \alpha'\xi\ , \qquad \xi = \ln(s/1GeV^2)\ .$$

Here Δ and α' are the residue and the slope of the pomeron trajectory. The parameters γ and R characterize the coupling of the pomeron trajectory with initial hadrons. The quasi-eikonal parameter C is related to the small-mass diffraction dissociation of incoming hadrons.

K.A. Ter-Martirosyan Phys. Lett. B **44**, 377 (1973).
A.B. Kaidalov, K.A. Ter-Martirosyan Yad. Fiz. **39**, 1545 (1984); **40**, 211 (1984).
V.A. Abramovsky, V.N. Gribov, O.V. Kancheli Yad. Fiz. **18**, 595 (1973).

Comparison with the Regge approach

This enables to connect the parameters N_0 and α of our model with the parameters of the pomeron trajectory and its couplings to hadrons. Comparing we have

$$N_0 = z = \frac{2\gamma C}{\lambda} \exp(\Delta\xi) \ , \quad \alpha = \sqrt{\frac{2\lambda}{C}} \ , \quad \lambda = R^2 + \alpha'\xi \qquad (12)$$

The numerical values of the parameters in the paper:
G.H.Arakelyan,A.Capella,A.B.Kaidalov,Yu.M.Shabelski Eur.Phys.J.C**26**,81(2002)

$$\Delta = 0.139, \quad \alpha' = 0.21 \ GeV^{-2},$$
$$\gamma_{pp} = 1.77 \ GeV^{-2}, \quad R^2 = 3.18 \ GeV^{-2}, \quad C = 1.5 \ ,$$

Our values of the parameters:

$$\Delta = 0.2, \quad \alpha' = 0.05 \ GeV^{-2},$$
$$\gamma_{pp} = 1.035 \ GeV^{-2}, \quad R^2 = 3.3 \ GeV^{-2}, \quad C = 1.5 \ .$$

Soft and Hard Pomeron:
J. Bleibel, L.V. Bravina, E.E. Zabrodin. Phys. Rev. D 93, 114012 (2016)

Fitting the parameters of the initial string distribution in the impact parameter plane of pp collisions

Таблица: The non-diffractive cross section, the multiplicity density at mid-rapidity and the mean number of initial strings in *pp* collisions at different initial energies.

$\sqrt{s}(GeV)$	$\sigma_{th}^{ND}(mb)$	$\sigma_{MC}^{ND}(mb)$	dN^{ND}/dy	$\langle N_{str} \rangle$
60	24.9	24.9	2.44	4.2
900	39.9	39.9	3.76	7.8
7000	52.5	52.4	5.44	13.4
13000	56.5	56.6	6.03	16.0

$\sigma_{MC\ \ simulations}^{ND} = \frac{n_{sim}(N=0)}{n_{sim}(N\geq 0)} S_b$

$\mu_0^{(k)} = \mu_0^{(1)} \sqrt{k}$ with $\mu_0^1 = 0.7$

MC calculations of $\Sigma(n_F, n_B)$ in the model with string clusters formation

[V.V. Vechernin, S.N. Belokurova, J.Phys.:Conf.Ser. 1690(2020)012088, arXiv:2012.07682, S. Belokurova, Phys.Part.Nucl.53(2022)154, arXiv:2011.10434]

- Modelling the initial string distribution in the impact parameter plane of pp collisions for different initial energies to take into account string fusion processes. Like in [*V. Vechernin, I. Lakomov. Proceedings of Science (Baldin ISHEPP XXI) (2013) 072.*].
- Monte Carlo simulations of string configurations and calculation of weighting factors α_k as a function of centrality and initial energy of pp collision.

$$\alpha_k = \frac{\langle n^{(k)} \rangle}{\sum_{k=1}^{\infty} \langle n^{(k)} \rangle} = \frac{\langle m^{(k)} \rangle \mu_0^{(k)} \delta\eta}{\sum_{k=1}^{\infty} \langle m^{(k)} \rangle \mu_0^{(k)} \delta\eta} = \frac{\langle m^{(k)} \rangle \sqrt{k}}{\sum_{k=1}^{\infty} \langle m^{(k)} \rangle \sqrt{k}} \ ,$$

where the $\langle m^{(k)} \rangle$ is the mean number of clusters with k fused strings, which we take from our MC simulations of the string configurations.
- Calculation the $\Sigma(n_F, n_B)$ for different centralities of pp collision at few LHC energies using the relation (6).

C_2 through multiplicities in two small windows

For two small windows $\delta\eta_1$ and $\delta\eta_2$ around η_1 and η_2 we have

$$\rho(\eta) = \frac{\langle n \rangle}{\delta\eta} , \qquad \rho_2(\eta_1, \eta_2) = \frac{\langle n_1 n_2 \rangle}{\delta\eta_1\, \delta\eta_2} , \tag{13}$$

$$C_2(\eta_1, \eta_2) = \frac{\langle n_1 n_2 \rangle}{\langle n_1 \rangle \langle n_2 \rangle} - 1 , \tag{14}$$

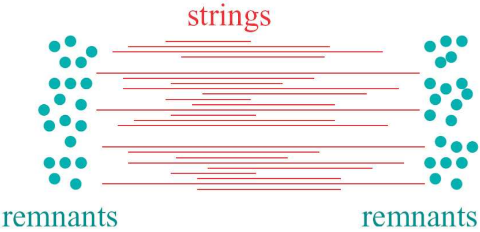
where n_1 and n_2 are the event multiplisities in these windows $\delta\eta_1$ and $\delta\eta_2$. Note that when $\eta_1 = \eta_2 = \eta$, $\eta_{sep} = 0$, we have to use

$$\rho_2(\eta, \eta) = \frac{\langle n(n-1) \rangle}{\delta\eta^2} , \qquad C_2(0) = \frac{\langle n(n-1) \rangle}{\langle n \rangle^2} - 1 = \frac{\omega_n - 1}{\langle n \rangle} = R_n , \tag{15}$$

where n is the number of particles in small window $\delta\eta$ around the point η. (see e.g. [C.Pruneau,S.Gavin,S.Voloshin,Phys.Rev.C66(2002)044904] or [V.V.,Nucl.Phys.A939(2015)21]).

Initial state in the EPOS event generator

K. Werner, Collective phenomena in AuAu@RHIC and pp@LHC, ALICE Club, 21 Nov 2008, CERN, 2008.



One flux tube is the result of merging many individual strings

Epos: initial energy density obtained from strings, not partons

The parametrization of the single correlation function

The parametrization for the pair correlation function $\Lambda(\eta, \phi)$ of a single string (reflecting the Schwinger mechanism of a string decay, was suggested in [V.V.,Nucl.Phys.A939(2015)21]:

$$\Lambda(\eta, \phi) = \Lambda_1 e^{-\frac{|\eta|}{\eta_1}} e^{-\frac{\varphi^2}{\varphi_1^2}} + \Lambda_2 \left(e^{-\frac{|\eta-\eta_0|}{\eta_2}} + e^{-\frac{|\eta+\eta_0|}{\eta_2}} \right) e^{-\frac{(|\varphi|-\pi)^2}{\varphi_2^2}} . \quad (16)$$

This formula has the nearside peak, characterizing by parameters Λ_1, η_1 and φ_1, and the awayside ridge-like structure, characterizing by parameters Λ_2, η_2, η_0 and φ_2 (two wide overlapping hills shifted by $\pm\eta_0$ in rapidity, η_0 - the mean length of a string decay segment). We imply that in formula (16)

$$|\varphi| \leq \pi . \quad (17)$$

If $|\varphi| > \pi$, then we use the replacement $\varphi \to \varphi + 2\pi k$, so that (17) was fulfilled. With such completions the $\Lambda(\eta, \phi)$ meets the following properties

$$\Lambda(-\eta, \phi) = \Lambda(\eta, \phi) , \quad \Lambda(\eta; -\phi) = \Lambda(\eta, \phi) , \quad \Lambda(\eta, \phi + 2\pi k) = \Lambda(\eta, \phi) \quad (18)$$

Fitting the model parameters by FBC in small windows

$\Lambda(\eta_{sep}, \phi_{sep})$ was fitted by the ALICE b_{nn} pp data with FB windows of small acceptance, $\delta\eta = 0.2, \delta\phi = \pi/4$, separated in azimuth and rapidity *[ALICE collab.,JHEP 05(2015)097]*. It gives for the parameters:

$\sqrt{s}$, TeV		0.9	2.76	7.0
LRC	$\mu_0\omega_N$	0.7	1.4	2.1
SRC	$\mu_0\Lambda_1$	1.5	1.9	2.3
	η_1	0.75	0.75	0.75
	ϕ_1	1.2	1.15	1.1
	$\mu_0\Lambda_2$	0.4	0.4	0.4
	η_2	2.0	2.0	2.0
	ϕ_2	1.7	1.7	1.7
	η_0	0.9	0.9	0.9

$\omega_N = \frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle}$ is the e-by-e scaled variance of the number of strings,
μ_0 is the average rapidity density of the charged particles from one string,
$i=1$ corresponds to the nearside and $i=2$ to the awayside contributions,
η_0 is the mean length of a string decay segment.

[V.V.,Nucl.Phys.A939(2015)21]

$\Sigma(n_F, n_B)$ in windows separated in azimuth and rapidity

For small observation windows:

$$\Sigma(\Delta\eta, \Delta\phi) = 1 + \frac{\delta\eta\,\delta\phi}{2\pi}\mu_0\,[\Lambda(0,0) - \Lambda(\Delta\eta, \Delta\phi)]$$

$$\Delta\eta \equiv \eta_{sep}, \qquad \Delta\phi \equiv \phi_{sep}$$

For observation windows of an arbitrary width $\delta\eta_F\delta\phi_F$ and $\delta\eta_B\delta\phi_B$:

$$\Lambda(\Delta\eta, \Delta\phi) \to J_{FB}(\Delta\eta, \Delta\phi) = \frac{1}{\delta\eta_F\delta\phi_F\delta\eta_B\delta\phi_B} \times$$

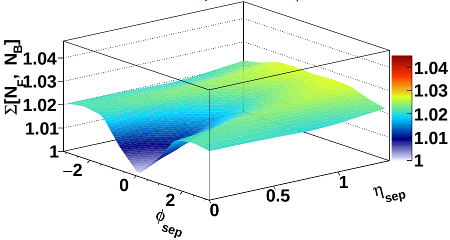
$$\times \int_{\delta\eta_F\delta\phi_F} d\eta_1 d\phi_1 \int_{\delta\eta_B\delta\phi_B} d\eta_2 d\phi_2\, \Lambda(\eta_1 - \eta_2; \phi_1 - \phi_2)\ ,$$

$$\Lambda(0,0) \to J_{FF} = \frac{1}{(\delta\eta_F\delta\phi_F)^2} \int_{\delta\eta_F\delta\phi_F} d\eta_1 d\phi_1 \int_{\delta\eta_F\delta\phi_F} d\eta_2 d\phi_2\, \Lambda(\eta_1 - \eta_2; \phi_1 - \phi_2)\ .$$

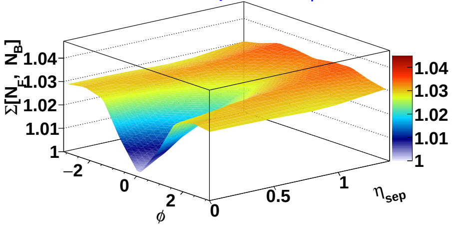
V.Vechernin, Nucl.Phys.A 939 (2015) 21

Σ for $\delta\eta$ $\delta\phi$ windows separated in azimuth and rapidity

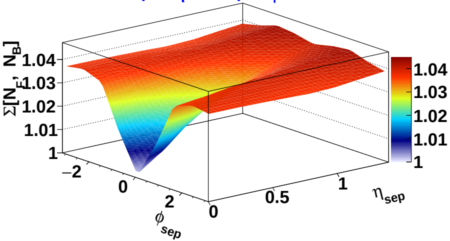
0.9 TeV, δη=0.2, δφ=0.25π



2.76 TeV, δη=0.2, δφ=0.25π



7 TeV, δη=0.2, δφ=0.25π



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