Quantum corrections to the Schwinger's particle creation

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The talk is based on

One loop corrections for the constant electric field have been found in:
https://doi.org/10.1007/JHEP09(2014)071 (arXiv:1405.5285);
https://doi.org/10.1007/JHEP09(2015)085 (arXiv:1412.1554);

One loop corrections for the electric pulse have been found in:
https://doi.org/10.1103/PhysRevD.107.125006 (arXiv:2303.08624);

Other papers: ArXiv:2012.00399, ArXiv:0901.0424;

The work on two-loop corrections is ArXiv:2409.00684
The work on resummation is in preparation.

Setup of the problem

To simplify the situation without loosing the physical content we consider scalar electrodynamics:

$$S[\phi, \phi^\dagger; A^\mu] = \int d^4x \left[\left| \partial_\mu \phi + ieA_\mu \phi \right|^2 - m^2 \left| \phi \right|^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - j_\mu^{\text{cl}} A^\mu \right], \tag{1}$$

and divide the vector potential A^μ into the classical and quantum parts, $A^\mu = A^\mu_{\text{cl}} + a^\mu$, where

$$A^\mu_{\text{cl}} = \Big(0, A_1(t), 0, 0\Big), \quad A_1(t) = ET \tanh \frac{t}{T}, \tag{2}$$

is due to j_μ^{cl}. We consider the lengthy pulse: $eET^2 \gg 1$. EM field is quantized in the standard way in the Lorentz gauge. At the same time, the scalar field operator:

$$\widehat{\phi}(t, \mathbf{x}) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left(\widehat{a}_{\mathbf{p}} e^{i\mathbf{p}\mathbf{x}} f_{\mathbf{p}}(t) + \widehat{b}^\dagger_{\mathbf{p}} e^{-i\mathbf{p}\mathbf{x}} f^*_{-\mathbf{p}}(t) \right); \tag{3}$$

Quantization

Where

$$\left(\partial_t^2 + \Big[\mathbf{p} + e\mathbf{A}(t)\Big]^2 + m^2\right) f_{\mathbf{p}}(t) = 0. \tag{4}$$

We take the modes to be single waves at past infinity (in-modes):

$$f_{\mathbf{p}}^{\text{in}}(t/T \to -\infty) \simeq \frac{e^{-i\omega_- t}}{\sqrt{2\omega_-}}, \quad \omega_\pm = \sqrt{[\mathbf{p} + \mathbf{A}(\pm\infty)]^2 + m^2} \tag{5}$$

The asymptotic form of these modes in the future infinity is:

$$f_{\mathbf{p}}^{\text{in}}(t/T \to +\infty) \simeq C_+(\mathbf{p}) \frac{e^{i\omega_+ t}}{\sqrt{2\omega_+}} + C_-(\mathbf{p}) \frac{e^{-i\omega_+ t}}{\sqrt{2\omega_+}}, \tag{6}$$

There is a clear physical explanation of such a behaviour of the modes due to QM scattering. In a week field $m^2/e \gg E$ one can use the semiclassical approximation for the modes. But we can calculate even for strong field.

The tree-level current

For the Fock space in-ground state, $\widehat{a}_{\mathbf{p}} \left| in \right\rangle = 0$, the tree-level current is:

$$j_1^{\text{tree}}(t) \simeq \frac{E^2 e^3 T}{2\pi^3 \mu} \cdot \exp\left[-\frac{\pi m^2}{eE}\right], \tag{7}$$

where $\mu = m\sqrt{\pi}/\sqrt{eE}$.
This expression has a clear physical explanation: during the pulse the pairs of charged particles are created (when $eEL = 2m$) with the Schwinger's probability rate per unit four-volume

$$w_s \sim E^2 e^2 \, \exp\left[-\pi m^2/eE\right].$$

Hence, by the end of the pulse duration one obtains the current density of the form $j_1^{\text{tree}} \sim e\, w\, T$. Such a current is totally due to the amplification of the zero point fluctuations.

Photon propagator for a generic state

For spatially homogeneous state the Keldysh propagator can be represented as follows:

$$G^K_{\mu\nu}\left(\Big|\mathbf{x}-\mathbf{y}\Big|;\tau_1,\tau_2\right) \equiv \frac{1}{2}\left\langle\left\{a_\mu(\mathbf{x},\tau_1), a_\nu(\mathbf{y},\tau_2)\right\}\right\rangle = \int \frac{d^3\mathbf{q}}{(2\pi)^3}\, e^{i\mathbf{q}\,(\mathbf{x}-\mathbf{y})}\, G^K_{\mu\nu}\Big(\mathbf{q};\tau_1,\tau_2\Big), \quad (8)$$

For a generic (not necessary spatially homogeneous) state:

$$G^K_{\mu\nu} = \left(\frac{1}{2}\eta_{\mu\nu}\,\delta\Big(\mathbf{p}-\mathbf{q}\Big) + \left\langle \alpha^+_{\mu,\mathbf{p}}\,\alpha_{\nu,\mathbf{q}}\right\rangle\right)\frac{e^{-i|\mathbf{q}|(\tau_1-\tau_2)}}{2|\mathbf{q}|} + \left\langle \alpha_{\mu,\mathbf{p}}\,\alpha_{\nu,\mathbf{q}}\right\rangle \frac{e^{-i|\mathbf{q}|(\tau_1+\tau_2)}}{2|\mathbf{q}|} + \text{h.c.}., \quad (9)$$

where $\alpha^+_{\mu,\mathbf{p}}$ and $\alpha_{\nu,\mathbf{q}}$ are creation and annihilation operators for photons.

Red - is due to zero point fluctuations;

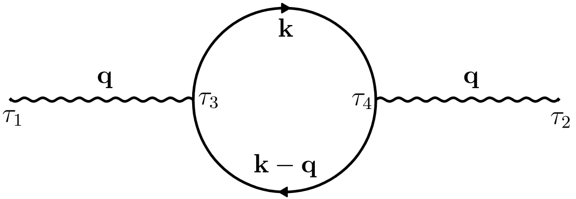
Brown - is due to higher energy levels and anomalous averages.

Loop corrections (story of photons)

For a spatially homogeneous state loop diagrams in the limit $|\tau_1 - \tau_2| \ll \mathcal{T} = (\tau_1 + \tau_2)/2$ correct the propagator as

$$\Delta_{loops} G^K_{\mu\nu}\Big(\mathbf{q}; \tau_1, \tau_2\Big) \simeq n_{\mu\nu}(\mathbf{q}, \mathcal{T}) \frac{e^{-i|\mathbf{q}|(\tau_1 - \tau_2)}}{2|\mathbf{q}|} + \kappa_{\mu\nu}(\mathbf{q}, \mathcal{T}) \frac{e^{-i2|\mathbf{q}|\mathcal{T}}}{2|\mathbf{q}|} + \text{h.c.}, \tag{10}$$

where $\left\langle \alpha^+_{\mu,\mathbf{p}} \, \alpha_{\nu,\mathbf{q}} \right\rangle = n_{\mu\nu}(\mathbf{q}, \mathcal{T}) \delta\Big(\mathbf{p} - \mathbf{q}\Big)$ and $\left\langle \alpha_{\mu,\mathbf{p}} \, \alpha_{\nu,\mathbf{q}} \right\rangle = \kappa_{\mu\nu}(\mathbf{q}, \mathcal{T}) \delta\Big(\mathbf{p} + \mathbf{q}\Big)$.
In the Schwinger-Keldysh technique one has a sum of such diagrams in the first loop correction to the photon propagator:



From one loop when $\mathcal{T} < T$ the photon level-population, $n_{\mu\nu}(\mathbf{q}, \mathcal{T})$, receives the secular correction:

$$n_{\mu\nu}(\mathbf{q}, \mathcal{T}) = 2e^2(\mathcal{T} + T) \int_{-\infty}^{+\infty} d\tau \frac{e^{-2i|\mathbf{q}|\tau}}{2|\mathbf{q}|} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \times \\ \times f_{\mathbf{k}_\perp}(k_1 + eE\tau) \overleftrightarrow{D}_\mu f_{\mathbf{k}_\perp - \mathbf{q}_\perp}(k_1 - q_1 + eE\tau) \times \\ \times f^*_{\mathbf{k}_\perp}(k_1 - eE\tau) \overleftrightarrow{D}^\dagger_\nu f^*_{\mathbf{k}_\perp - \mathbf{q}_\perp}(k_1 - q_1 - eE\tau), \tag{11}$$

where $\overleftrightarrow{D}_\mu = \overleftarrow{D}_\mu - \overrightarrow{D}^\dagger_\mu$, while anomalous averages, $\kappa_{\mu\nu}(\mathbf{q}, \mathcal{T})$, do not receive secularly growing corrections. In the absence of an external electric field $n_{\mu\nu}(\mathbf{q}, \mathcal{T})$ is also vanishing.

This has a clear physical meaning: RHS of the last equation is the collision integral describing creation of photons together with charged particles under the action of the background strong electric field, while the ground state for the photon field does not change. Thus, under the action of the strong electric field there is not only the electrical breakdown of the vacuum, but also this vacuum acts as a kind of medium for a laser that radiates photons.

Loop corrections (story of changed particles)

Somewhat similar story there is a for charged scalars, but with some peculiarities. For a generic spatially homogeneous state the Fourier transformed Keldysh propagator can be represented as follows:

$$D^K\Big(\mathbf{p};\tau_1,\tau_2\Big) \equiv \frac{1}{2}\Big\langle\Big\{\phi^+(\mathbf{p},\tau_1),\phi(-\mathbf{p},\tau_2)\Big\}\Big\rangle = \qquad (12)$$

$$= \left(\frac{1}{2} + n_p\right) f_{\mathbf{p}}(\tau_1) f^*_{\mathbf{p}}(\tau_2) + \kappa_p\, f_{\mathbf{p}}(\tau_1) f_{\mathbf{p}}(\tau_2) + \text{h.c.},$$

where $\Big\langle \widehat{a}^+_{\mathbf{p}}\,\widehat{a}_{\mathbf{q}}\Big\rangle = n_{\mathbf{q}}\delta\Big(\mathbf{p}-\mathbf{q}\Big)$, $\Big\langle \widehat{a}_{\mathbf{p}}\,\widehat{b}_{\mathbf{q}}\Big\rangle = \kappa_{\mathbf{q}}\,\delta\Big(\mathbf{p}+\mathbf{q}\Big)$ and $\Big\langle \widehat{b}^+_{\mathbf{p}}\,\widehat{b}_{\mathbf{q}}\Big\rangle = n^*_{\mathbf{q}}\delta\Big(\mathbf{p}-\mathbf{q}\Big)$.

Here $1/2$ - is due to zero point fluctuations;
While $n_{\mathbf{q}}$ and $\kappa_{\mathbf{q}}$ - are due to higher energy levels and anomalous averages.

Tree-level vs. loop corrections to the current of created particles

The current is calculated with the use of the Keldysh function:

$$j_1 \equiv \left\langle :\phi^+(\underline{x}) \overleftrightarrow{D}_\mu \phi(\underline{x}): \right\rangle = \overleftrightarrow{D}_\mu \, D^K\left(\underline{x}, \underline{y}\right)\Big|_{x=y}$$

The calculated above tree-level current $j_1^{\text{tree}} \sim e\,w\,T$ is totally due to zero point fluctuations. For a very long electric pulse the loop corrections to the current gained during the pulse can be encompassed into the expression

$$\Delta j_1^{loops}(t) \approx 2e \int \frac{d^3\mathbf{p}}{(2\pi)^3} (p_1 + eET) \left[|f_\mathbf{p}(t)|^2 \, n_\mathbf{p}(t) + f_\mathbf{p}^2(t) \kappa_\mathbf{p}(t) + h.c. \right], \quad (13)$$

If $n_\mathbf{p}$ and $\kappa_\mathbf{p}$ are generated in the loops, then the current under consideration receives corrections. Meanwhile if they grow with time, then the corrections to the current also grow with time.

Loop corrections to the current

Where from the first two loops we have that

$$n_{\mathbf{p}}(t) \simeq 2e^2 \int_{t_0}^{t} d\tau_1 \int_{t_0}^{t} d\tau_2 \int \frac{d^3\mathbf{q}}{(2\pi)^3} G^K_{\mu\nu}(\mathbf{q};\tau_1,\tau_2) \times \\ \times f^*_{\mathbf{p}}(\tau_1) \overleftrightarrow{D}^{\mu\dagger} f^*_{\mathbf{p}-\mathbf{q}}(\tau_1) \cdot f_{\mathbf{p}}(\tau_2) \overleftrightarrow{D}^{\nu} f_{\mathbf{p}-\mathbf{q}}(\tau_2) \tag{14}$$

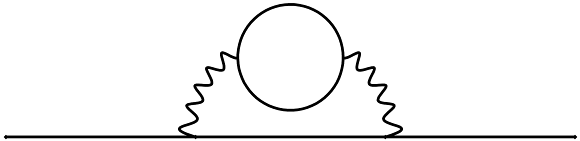
and the anomalous average

$$\kappa_{\mathbf{p}}(t) \simeq -2e^2 \int_{t_0}^{t} d\tau_1 \int_{t_0}^{\tau_1} d\tau_2 \int \frac{d^3\mathbf{q}}{(2\pi)^3} G^K_{\mu\nu}(\mathbf{q};\tau_1,\tau_2) \times \\ \times f_{\mathbf{p}}(\tau_1) \overleftrightarrow{D}^{\mu\dagger} f^*_{\mathbf{p}-\mathbf{q}}(\tau_1) \cdot f_{\mathbf{p}}(\tau_2) \overleftrightarrow{D}^{\nu} f_{\mathbf{p}-\mathbf{q}}(\tau_2). \tag{15}$$

First vs. second loop correction

Interestingly enough, the first loop correction to the current Δj_1^{loops} does not grow with time faster than the tree-level contribution.

But the second loop correction due to the Schwinger-Keldysh diagrams of the form:



does grow much faster than the first loop. That is due to the growth of $n_{\mu\nu}$ in $G^K_{\mu\nu}$ in the first loop:

$$\Delta j_1(T) \propto -\left(\frac{w_s}{\mu}\right)^2 \frac{e^4\,T^5}{\mu^2} \ln\left(\frac{T}{\mu}\right) \cos\left[\frac{m^2}{eE}\right] \tag{16}$$

where $\mu = m\sqrt{\pi}/\sqrt{eE}$ and $w_s \sim E^2 e^2 \, \exp\left[-\mu^2\right]$ – Schwinger's probability rate.

Discussion

Thus, for a long enough pulse, when duration of the pulse T compensates the power of the electric charge e and the Schwinger's factor $\exp\left[-\pi m^2/eE\right]$, loop corrections become of the same order as the tree-level contribution to the current. For strong field E that should happen even faster. Hence, the Schwinger's answer for the current gets drastically modified:

Other non-stationary backgrounds

We observe similar secular corrections to the stress-energy flux and electric currents in other strong background fields. E.g.:

- FRW expanding and contracting universes, including de Sitter space-time;
- Black hole collapse;
- Strong constant electric field;
- Dynamical Casimir effect due to the moving mirrors;
- Strong scalar field background

Strong background fields are more similar to the condensed matter physics rather than to high energy particle physics.

Conclusions

In accelerators one pumps out vacuum and scatters single particles. That is the reason to consider stationary Poincare invariant correlation functions to construct amplitudes and cross-sections. In strong background fields, in the very early universe and in the background of collapsing microscopic black holes there is no reason to assume the state to be vacuum or even thermal.

THANKS!