Константа связи в изотропных голографических моделях легких и тяжелых кварков

На основе совместной работы с И.Я. Арефьевой, А. Хаджилу, П.С. Слеповым
Phys.Rev.D 110 (2024) 12, 126009

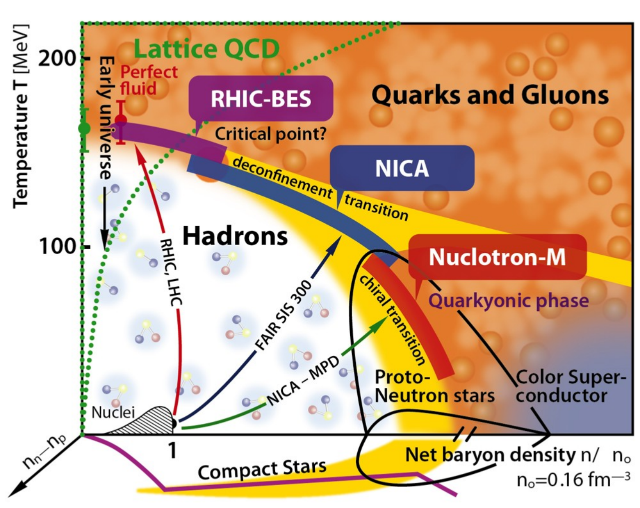
Марина Усова

Математический институт им. В.А. Стеклова РАН

Сессия-конференция «Физика фундаментальных взаимодействий»,
посвященная 70-летию со дня рождения академика РАН
Валерия Анатольевича Рубакова

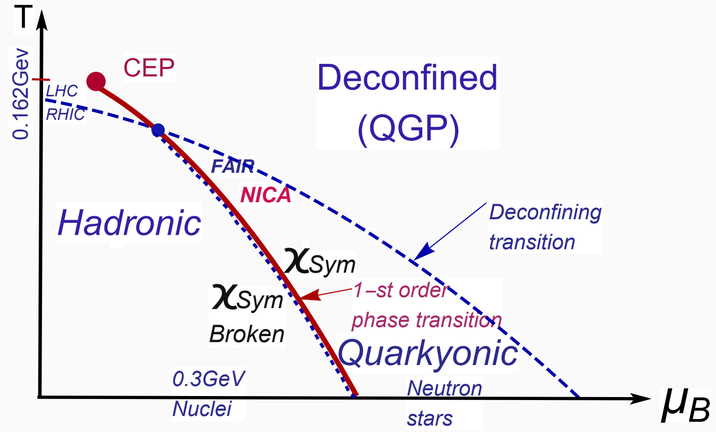
20 февраля 2025

Studies of QCD Phase Diagram is the main goal of new facilities

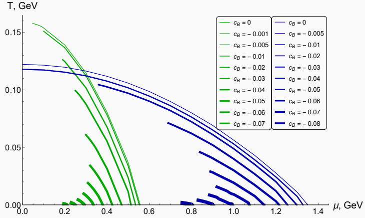


From: https://nica.jinr.ru/physics.php

Holographic QCD phase diagram for light quarks

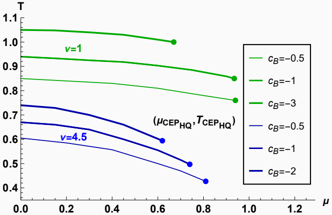


Light quarks



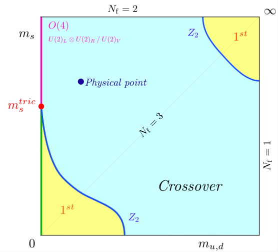
I.A, Ermakov, Rannu, Slepov,EPJC'23

Heavy quarks



I.A, A. Hajilou, K.R., P.S.EPJC'23

- QCD Phase Diagram from Lattice Columbia plot (Brown et al.'90 Philipsen, Pinke'16)
- Main problem on Lattice: $\mu \neq 0$



The main question to discuss today is: what directly measurable quantities indicate the presence of 1-st order phase transitions?

- Jet Quenching – I. Ya. Aref'eva's talk
- Direct photons – Ref.: I. Ya. Aref'eva, A. Ermakov and P. Slepov, "Direct photons emission rate ... with first-order phase transition," EPJC 82 (2022) 85
- Energy lost – P.Slepov's talk
- Cross-sections – this talk and A.Nikolaev's talks

- Details of the CEP locations K.Rannu's talk

Holographic isotropic models for light and heavy quarks

The 5-dimensional Einstein-Maxwell-scalar action

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-\mathfrak{g}}\left[R - \frac{\mathfrak{f}_0(\varphi)}{4}F^2 - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \mathcal{V}(\varphi)\right],$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Anzatz for the metric:

$$ds^2 = B^2(z)\left[-g(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{g(z)}\right],$$
$$B(z) = \frac{Le^{A(z)}}{z}\ (L=1), \quad \varphi = \varphi(z), \quad A_\mu = \left(A_t(z)\ , \vec{0}, 0\right)$$

Light quarks (d, u)

$$A_{LQ}(z) = -a\log\left(bz^2+1\right)$$
$$f_{LQ}(z) = e^{-c\,z^2 - A(z)}$$

fitting with experimental data: ↘

$a = 4.046, b = 0.01613 GeV^2, c = 0.227 GeV^2$

(Li et.al.'17; Aref'eva et. al.'21, '22)

Heavy quarks (b, c)

$$A_{HQ}(z) = -\frac{\mathrm{s}}{3}z^2 - p\,z^4$$
$$f_{HQ}(z) = e^{-\mathrm{s}\,z^2 - A(z)}$$

↘

$\mathrm{s} = 1.16 GeV^2, p = 0.273 GeV^4$

(Yang & Yuan, '16; IA et. al.'20, '23)

Applied boundary conditions

$$A_t(0) = \mu, \ \ A_t(z_h) = 0, \qquad g(0) = 1, \ \ g(z_h) = 0,$$

$$\boxed{\varphi(z)\Big|_{z=z_0} = 0, \quad \text{where} \quad z_0 = \mathfrak{z}(z_h)} \begin{cases} \mathfrak{z}_{LQ}(z_h) = 10\exp[-z_h/4] + 0.1 \\ \mathfrak{z}_{HQ}(z_h) = \exp[-z_h/4] + 0.1 \end{cases}$$

(I. Aref'eva, K.Rannu, P.Slepov'21)

The choice of $z_0 = \mathfrak{z}(z_h)$ allows to fit known lattice data for the string tension temperature dependence (Cordaso, Bicudo 1111.1317) and reproduces the correct behavior of the coupling.

Dilaton solutions of EOM

$$\varphi'(z) = \sqrt{-6\left(A'' - A'^2 + \frac{2}{z}A'\right)}$$

$$\varphi_{LQ}(z,\varphi_0) = \varphi_0 + 2\sqrt{3a}\left[\sqrt{2a+1}\,\text{arcsinh}\left(\sqrt{\frac{b(2a+1)}{3}}\,z\right) - \sqrt{2(a-1)}\,\text{arctanh}\left(\sqrt{\frac{2b(a-1)}{(2a+1)bz^2+3}}\,z\right)\right]$$

$$\varphi_{HQ}(z,\varphi_0) = \varphi_0 + \sqrt{6}\int_0^z d\xi\sqrt{\left(-4\,p\,\xi^3 - \frac{2\,s\,\xi}{3}\right)^2 + 2\left(4\,p\,\xi^2 + \frac{2\,s}{3}\right) + 12\,p\,\xi^2 + \frac{2\,s}{3}}$$

Running coupling in holographic prescription

The holographic dictionary

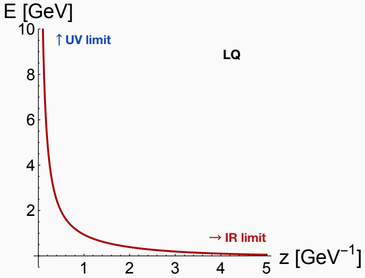
- $\boxed{\alpha = e^{\varphi(z)}}$ is identified as running coupling of the field theory
- the energy scale E of the dual field theory (Galow et. al. '09)

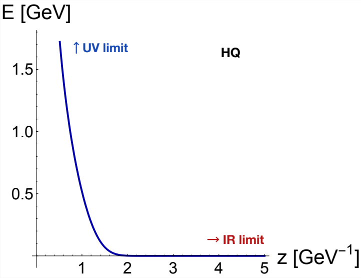
$$\boxed{E = B(z)} \qquad E_{LQ} = \frac{1}{z\,(1+bz^2)^a}, \qquad E_{HQ} = \frac{1}{z\,e^{\frac{c}{3}z^2+p\,z^4}}$$

- β-function (DeWolfe et. al. '14, Kiritsis et.al.'14)

$$\beta = \frac{d\alpha}{d\log E}\Big|_{QFT} = \alpha\frac{d\varphi}{d\log B}\Big|_{Holo}$$

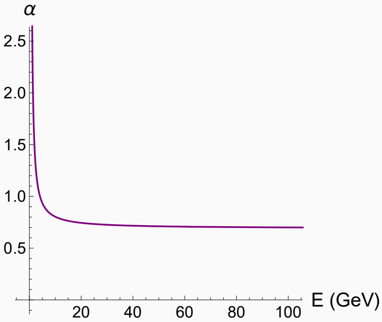
I.Ya. Aref'eva, A.Hajilou, P.S. Slepov, MU, TMF (2024), PRD (2025)



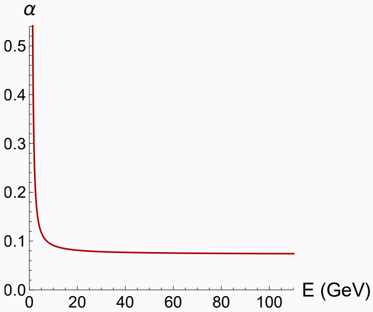


On the relevance of the holographic isotropic models

Light quarks



Heavy quarks

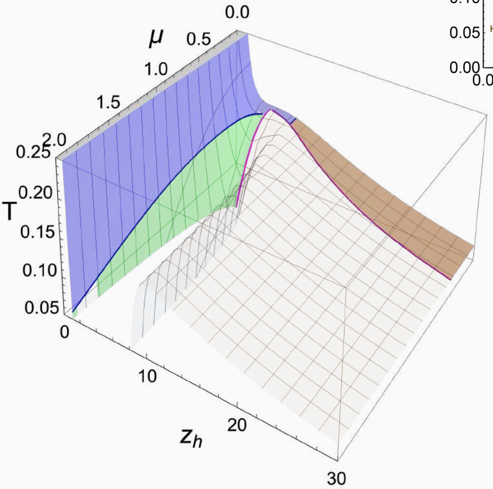


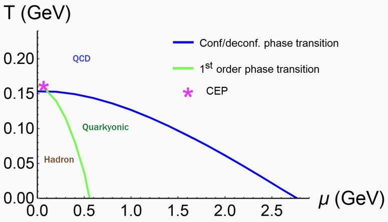
ultra-UV regime cannot be reached

`BUT strongly coupled and`
`a small part of weakly coupled regimes are covered`

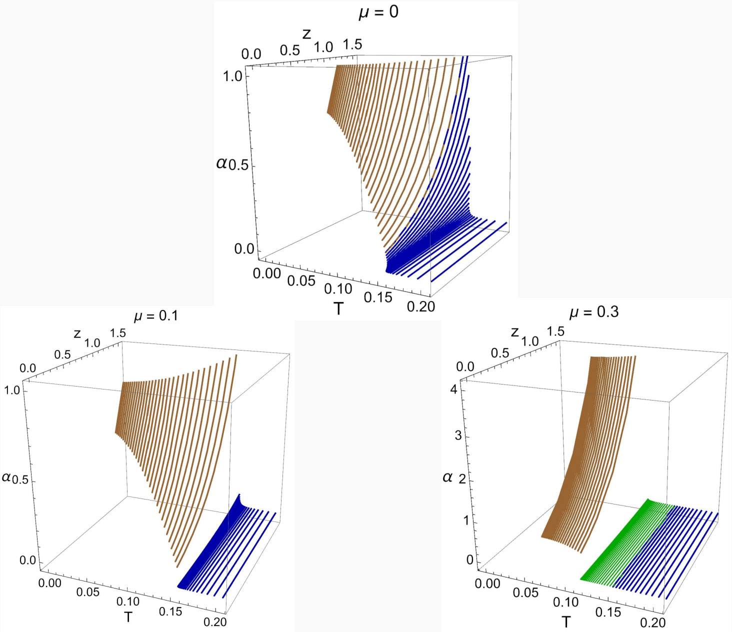
hQCD phase structure for light quarks model

$$T(z_h) = \frac{|g'|}{4\pi}\bigg|_{z=z_h}$$

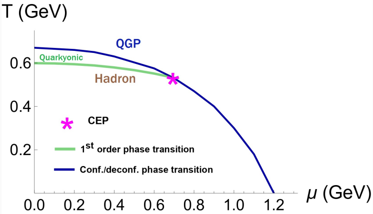




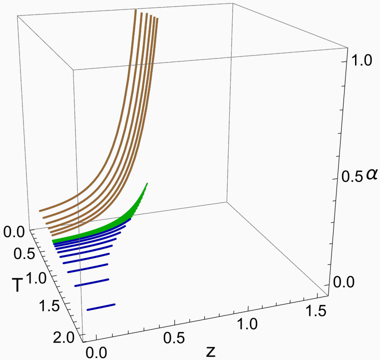
Light quarks running coupling dependence



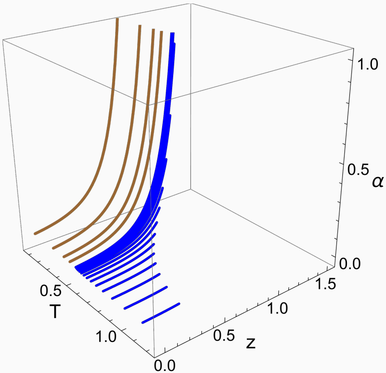
Heavy quarks running coupling dependence



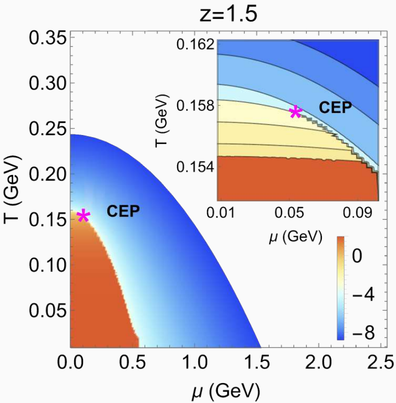
$\mu = 0.3$

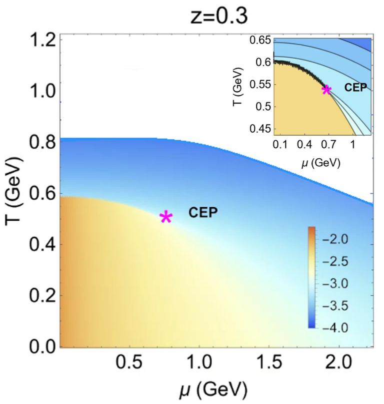


$\mu = 0.8$



Light quarks and heavy quarks $\log \alpha(z; \mu, T)$





- The holographic isotropic models of light quarks and heavy quarks under consideration reproduces a proper behavior of the running coupling in the sense of the energy scales
- The 1-st order phase transition effects on the running coupling and leads to the jumps of its values
- In isotropic case, the magnitude of these jumps is defined by the chemical potential and temperature

- Exact correspondence of the coupling values with experimental data is in progress
- The model needs modifications to the more realistic cases taking into account, e.g., anisotropic effects and an inclusion of different types of quarks

Thank you for your attention!