Infinite (continuous) spin particle on curve background

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based on

I.L. Buchbinder, SF, A.P. Isaev, V.A. Krykhtin , Phys. Lett. B853 (2024) 138689;
I.L. Buchbinder, SF, A.P. Isaev, M.A. Podoinitsyn , Phys. Lett. B861 (2025) 139226

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Our results on describing the infinite (continuous) particle
on curve space-time will be presented.

In $4D$ space-time infinite spin representations are massless ($P^m P_m = 0$) unitary irreps of the Poincaré group which obey [E.Wigner, 1939; V.Bargmann, E.Wigner, 1948]:

$$W^m W_m = -\mu^2 \neq 0\,, \qquad W_m = \tfrac{1}{2}\,\varepsilon_{mnkl} P^n M^{kl}\,.$$

Each of these representations contains infinite tower of states with all (integer or half-integer) helicities.

The same spectrum of states is used in higher spin theory that considers infinite tower of the helicity states ($W^m W_m = 0$, $W_m = \lambda P_m$).

There are some similarities higher spin theory and infinite spin theory.

But there are a number of differences:

- in highest spin theory all spins unite with each other through interaction, whereas in infinite spin theory all spins are combined together initially;
- in infinite spin theory there is dimensionfull parameter μ.

Infinite spin reps were researched in the 2000s in many studies. But, in almost all paper, infinite spin particles were considered in flat space.

There are very few papers on infinite spin particle in curved space.

• In [Metsaev, 2017], the Lagrangian infinite spin formulation was constructed by specific deformation of the Fronsdal Lagrangian on AdS. This result was later confirmed in [Metsaev, 2019] within light-cone approach.

• In [Khabarov, Zinoviev, 2017], the Lagrangian construction for an infinite spin field in AdS space was realized in the frame-like formulation.

• Infinite spin fields in AdS space were discussed also in the review [Bekaert, Skvortsov, 2017].

• In these papers, the Lagrangian formulation for infinite spin fields was constructed not in an arbitrary curved space-time, but only in AdS space.

This talk will present $1d$ infinite spin particle model and define the representation of the $\mathrm{SO}(2,3)$ group realized on wave function of the infinite spin particle in the AdS_4 space.

$4D$ infinite (continuous) spin fields

Since infinite spin representations are infinite-dimensional (at a fixed space-time coordinate), it is necessary

- to consider an infinite tower of space-time fields
- or consider generalized fields that depend on an additional coordinate.

To take into account symmetries, the second choice is preferred.

One of the possible choices is to use commuting vector y^m as additional coordinate (see, for example, [E.Wigner, V.Bargmann], as well as most articles on infinite spin fields).

In such formulation, infinite spin field $\Phi(x^m, y^m)$ depends on the position vector x^m and vector coordinate y^m.

To describe irreducible infinite spin representations, the field $\Phi(x^m, y^m)$ obeys the Wigner-Bargmann equations of motion.

We use different formulation in describing infinite (continuous) spin representations.

$4D$ infinite spin fields with additional spinor coordinate

In our formulation, the additional variable is

commuting Weyl spinor ξ^α, $\bar\xi^{\dot\alpha} = (\xi^\alpha)^*$, $\alpha = 1, 2$.

The field $\Phi(x;\xi,\bar\xi)$ describing infinite integer-spin representation obeys the following equations of motion [I.Buchbinder, SF, A.Isaev, V.Krykhtin, 2018]

$$
\begin{aligned}
\partial^m\partial_m\,\Phi(x;\xi,\bar\xi) &= 0\,,\\
\Big[i\,(\xi\sigma^m\bar\xi)\,\partial_m \;+\; \mu\Big]\,\Phi(x;\xi,\bar\xi) &= 0\,,\\
\Big[i\left(\frac{\partial}{\partial\xi}\sigma^m\frac{\partial}{\partial\bar\xi}\right)\partial_m \;-\; \mu\Big]\,\Phi(x;\xi,\bar\xi) &= 0\,,\\
\Big[\xi\frac{\partial}{\partial\xi} \;-\; \bar\xi\frac{\partial}{\partial\bar\xi}\Big]\,\Phi(x;\xi,\bar\xi) &= 0\,.
\end{aligned}
$$

Due to the first three equations, the field $\Phi(x;\xi,\bar\xi)$ satisfies

$$
W^2\Phi(x;\xi,\bar\xi) \;=\; -\mu^2\Phi(x;\xi,\bar\xi)\,.
$$

and describes irreducible massless infinite spin representation.

$4D$ infinite spin particle in flat space

In $1d$ description, the dynamics of infinite spin particle is determined by the set of the first class constraints: [I.Buchbinder, SF, A.Isaev, V.Krykhtin, 2018]

$$
\begin{aligned}
p^m p_m &\approx 0\,, \\
\left(\xi\sigma^m\bar{\xi}\right) p_m - \mu &\approx 0\,, \\
(\bar{\pi}\tilde{\sigma}^m\pi)\, p_m - \mu &\approx 0\,, \\
\xi\pi - \bar{\pi}\bar{\xi} &\approx 0
\end{aligned}
$$

in the phase space with canonical Poisson brackets

$$
\{x^m, p_n\}_{PB} = \delta^m_n\,, \qquad \{\xi^\alpha, \pi_\beta\}_{PB} = \delta^\alpha_\beta\,, \qquad \left\{\bar{\xi}^{\dot{\alpha}}, \bar{\pi}_{\dot{\beta}}\right\}_{PB} = \delta^{\dot{\alpha}}_{\dot{\beta}}\,.
$$

It is flat-space formulation of infinite spin particle.

The main task is to generalize this model to the case of curved space-time.

The basic requirements for such a construction are:

- General covariant generalization of constraints with a given flat limit.
- Closure of the algebra of new constraints.

These conditions lead restrictions on space-time geometry.

Generalization to curved space-time

Now $x^\mu(\tau)$ are local coordinates in curved space, ξ^α, $\bar\xi^{\dot\alpha}$, π_α, $\bar\pi^{\dot\alpha}$ are two-component spinors in tangent space. Their Poisson brackets are

$$\{x^\mu, p_\nu\}_{PB} = \delta^\mu_\nu\,, \qquad \{\xi^\alpha, \pi_\beta\}_{PB} = \delta^\alpha_\beta\,, \qquad \{\bar\xi^{\dot\alpha}, \bar\pi_{\dot\beta}\}_{PB} = \delta^{\dot\alpha}_{\dot\beta}\,.$$

Space-time geometry is described by metric $g_{\mu\nu}(x)$, vierbein $e^m_\mu(x)$ and spin connection $\omega_\mu{}^{mn}(x)$.

Transition from flat space-time to curved space-time involves the replacement

$$p_\mu \quad \to \quad \mathcal{P}_\mu = p_\mu + \frac{1}{2}\,\omega_\mu{}^{mn} M_{mn}\,, \qquad \mathcal{P}_m = e^\mu_m \mathcal{P}_\mu\,,$$

where $M_{mn} = \xi\sigma_{mn}\pi - \bar\pi\tilde\sigma_{mn}\bar\xi$ – generators of tangent Lorentz transformations

Components of "covariant momentum" satisfy the Poisson brackets algebra

$$\{\mathcal{P}_\mu, \mathcal{P}_\nu\}_{PB} = -\frac{1}{2}\,R_{\mu\nu}{}^{mn} M_{mn}\,,$$

where $R_{\mu\nu}{}^m{}_n$ is the curvature tensor.

Restriction on space-time geometry

Covariant generalization of the flat space constraints should be of the form:

$$\mathcal{P}^m\mathcal{P}_m + \cdots \approx 0\,, \qquad \left(\xi\sigma^m\bar{\xi}\right)\mathcal{P}_m - \boldsymbol{\mu} + \cdots \approx 0\,, \qquad \left(\bar{\pi}\tilde{\sigma}^m\pi\right)\mathcal{P}_m - \boldsymbol{\mu} + \cdots \approx 0$$

were the dots mean terms that disapper at vanishing the gravity background.

Closing the algebra of constraints leads to the following important consequences:

- Curved space must be the Riemannian space with zero torsion: $T_{\mu\nu}{}^m = 0$.
- Curvature tensor of the background geometry should be of the form:

$$R_{mn}{}^{k\ell} \;=\; \kappa\left(\delta^k_m\delta^\ell_n - \delta^\ell_m\delta^k_n\right),$$

  where κ is a constant. That is, only spaces of constant curvature are allowed as background: Minkowski space ($\kappa = 0$), de Sitter space ($\kappa > 0$) and anti de Sitter space ($\kappa < 0$).
- All additional terms (the dots in the formulas above) in the constraints are fully defined. They include special non-minimal terms proportional to curvature.

Curve space-time constraints

In the case of AdS$_4$ space the final constraints have the form

$$
\begin{aligned}
\mathcal{F}_0 &= \mathcal{P}^m\mathcal{P}_m - \frac{1}{4}R_{mn}{}^{k\ell}M^{mn}M_{k\ell} - \frac{1}{2}\kappa\mathcal{K}^2 - 2\boldsymbol{\mu}|\kappa|^{1/2} \approx 0\,, \\
\mathcal{F} &= \left(\xi\sigma^m\bar{\xi}\right)\mathcal{P}_m - \boldsymbol{\mu} + \frac{1}{4}|\kappa|^{1/2}\mathcal{K}^2 \approx 0\,, \\
\tilde{\mathcal{F}} &= \left(\bar{\pi}\tilde{\sigma}^m\pi\right)\mathcal{P}_m - \boldsymbol{\mu} + \frac{1}{4}|\kappa|^{1/2}\mathcal{K}^2 \approx 0\,, \\
\mathcal{U} &= N - \bar{N} \approx 0\,,
\end{aligned}
$$

where $\mathcal{K} = N + \bar{N}$, $N = \xi^\alpha\pi_\alpha$, $\bar{N} = \bar{\pi}_{\dot{\alpha}}\bar{\xi}^{\dot{\alpha}}$.

The only non-zero Poisson bracket of constraints has the form

$$
\left\{\mathcal{F},\tilde{\mathcal{F}}\right\}_{_{PB}} = -K\mathcal{F}_0 + |\kappa|^{1/2}K\left(\mathcal{F}+\tilde{\mathcal{F}}\right)\,.
$$

Replacing phase space coordinates with operators (taking into account their ordering) and considering constraints as equations of motion (first operator quantization), we obtain field theory of infinite spin particle on AdS$_4$ space.

Infinite spin field in AdS$_4$ space

The infinite spin states in AdS$_4$ are described by the fields $\Phi = \Phi(x, \xi, \bar{\xi})$, subject to the conditions

$$L_0 \Phi = 0\,, \qquad L\Phi = 0\,, \qquad L^+\Phi = 0\,, \qquad U\Phi = 0\,,$$

with the constraint operators

$$\begin{aligned}
L_0 &= D^2 + \kappa\left(N\bar{N} + N + \bar{N}\right) + 2\boldsymbol{\mu}\,|\kappa|^{1/2} - \frac{1}{2}\,\kappa K^2\,,\\
L &= i(\xi\sigma^m\bar{\xi})\,e^\mu_m D_\mu - \boldsymbol{\mu} - \frac{1}{4}\,|\kappa|^{1/2}K^2,\\
L^+ &= i\left(\frac{\partial}{\partial\xi}\sigma^m\frac{\partial}{\partial\bar{\xi}}\right)e^\mu_m D_\mu - \boldsymbol{\mu} - \frac{1}{4}\,|\kappa|^{1/2}K^2,\\
U &= N - \bar{N}\,.
\end{aligned}$$

Here the D_μ are the covariant derivative operators defined by

$$D_\mu \;=\; \partial_\mu \;+\; \frac{1}{2}\,\omega_\mu{}^{mn}\,\mathcal{M}_{mn}\,, \qquad \mathcal{M}_{mn} \;=\; \xi^\alpha\,(\sigma_{mn})_\alpha{}^\beta\,\frac{\partial}{\partial\xi^\beta} \;+\; \bar{\xi}_{\dot{\alpha}}\,(\tilde{\sigma}_{mn})^{\dot{\alpha}}{}_{\dot{\beta}}\,\frac{\partial}{\partial\xi^{\dot{\beta}}}$$

L_0, L, L^+, N, $\bar{N}$, K are operators corresponding to $\mathcal{F}_0$, $\mathcal{F}$, $\tilde{\mathcal{F}}$, $\mathcal{N}$, $\bar{\mathcal{N}}$, $\mathcal{K}$.

For example, $N = \xi^\alpha\,\dfrac{\partial}{\partial\xi^\alpha}$, $\bar{N} = \bar{\xi}_{\dot{\alpha}}\,\dfrac{\partial}{\partial\bar{\xi}^{\dot{\alpha}}}$

SO(2,3) symmetry

On considered space it is realized the AdS_4 algebra $\mathfrak{so}(2,3)$:

$$[P_m, P_n] = R^{-2} J_{mn} \,,$$

$$[J_{mn}, P_l] = \eta_{nl} P_m - \eta_{ml} P_n \,, \qquad [J_{mn}, J_{kl}] = \eta_{nk} J_{ml} + \eta_{ml} J_{nk} - \eta_{nl} J_{mk} - \eta_{mk} J_{nl} \,,$$

where $R = |\kappa|^{-1/2}$ is the curvature radius of AdS_4 space.

The generators of the $\mathfrak{so}(2,3)$ algebra have the form:

$$P_m = e^{\mu}_{m} \left(\partial_\mu - \frac{1}{2} \omega_\mu{}^{kl} J_{kl} \right),$$

$$J_{mn} = (\eta_{m\mu} \delta^{\nu}_{n} - \eta_{n\mu} \delta^{\nu}_{m}) \, x^\mu \frac{\partial}{\partial x^\nu} + \mathcal{M}_{mn} \,.$$

We obtain a very strong conclusion: the zero commutators

$$[P_m, L_0] = [P_m, L] = [P_m, L^+] = [P_m, U] = 0 \,,$$

$$[J_{mn}, L_0] = [J_{mn}, L] = [J_{mn}, L^+] = [J_{mn}, U] = 0$$

of all constraints operators and all $\mathfrak{so}(2,3)$ generators.

Thus, the infinite spin field in AdS_4 space are $SO(2,3)$-invariant.

Irreducible unitary SO(2, 3) representations

SO(2, 3) irreducible representations are defined by two Casimir operators:

- the second order Casimir operator

$$\mathcal{C}_2 = R^2 \, P^m P_m - \frac{1}{2} \, J^{mn} J_{mn} \,,$$

- the fourth order Casimir operator

$$\mathcal{C}_4 = -\frac{1}{2} \, J^{mn} J_{mn} \, \mathcal{C}_2 + R^2 P_m J^{mn} P^k J_{kn} + \frac{1}{4} \, J^{mn} J_{mn} - \frac{1}{8} \, (J^{mn} J_{mn})^2 - \frac{1}{4} \, J_{mn} J^{nk} J_{kl} J^{lm}$$

Infinite spin field in AdS$_4$ space is the eigenvector of Casimir operators:

$\mathcal{C}_2 \Phi = c_2 \Phi \,, \quad \mathcal{C}_4 \Phi = c_4 \Phi \,,$ where $c_2 = -2(1 + \mu R), \; c_4 = \mu R (1 + \mu R).$

Casimir operators are not independent on the fields Φ: $\mathcal{C}_4 = \frac{1}{2} \, \mathcal{C}_2 \left(\frac{1}{2} \, \mathcal{C}_2 + 1 \right).$

Since there is only one independent Casimir operator in the given irrep, this irrep exactly corresponds to the most degenerate representation for the SO(2, 3) group [N.Limić, J.Niederle, R.Raczka, 1966].

These obtained results correspond to classification of classically unitary infinite spin fields in AdS$_4$ obtained in [R.Metsaev, 2019] with using the light-cone formalism.

Conclusion

We obtained the following results:

- We present a new particle model that generalizes for curved space-time an infinite spin particle in flat space. The model is described by commuting Weyl spinor additional coordinates.
- It is proved that this model is consistent only in an external gravitational field corresponding to constant curvature spaces.
- It is shown that in our model, infinite spin fields in AdS_4 space are described by the most degenerate representations of the $\mathsf{SO}(2,3)$ group.

<u>Further research</u>

- Formulation of fermionic infinite spin field theory in curve space.
- Derivation of cubic interaction vertex for infinite spin fields on AdS_4 space among themselves and for their interactions with finite spin fields.
- Construction of supersymmetric infinite spin filed theory in AdS_4 space

In next talk I.L. Buchbinder will present the BRST-like Lagrangian formulation for infinite spin fields in AdS_4 space.

Thank you very much for your attention !