NEUTRON SKIN THICKNESS AND SYMMETRY ENERGY

Arsenyev Nikolay

Bogoliubov Laboratory of Theoretical Physics,
Joint Institute for Nuclear Research
In collaboration with
A. P. Severyukhin — BLTP, JINR, Dubna

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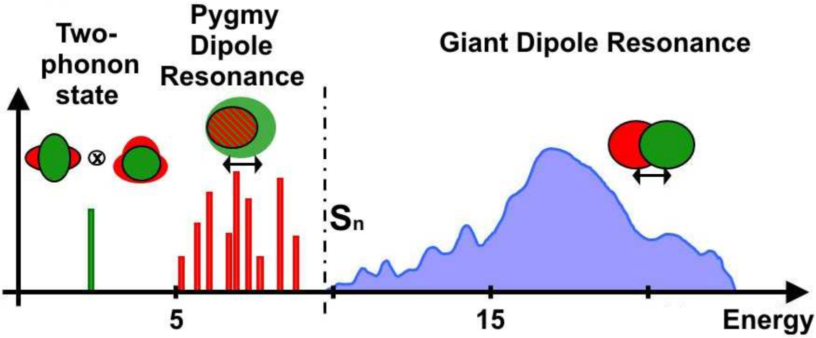
Outline

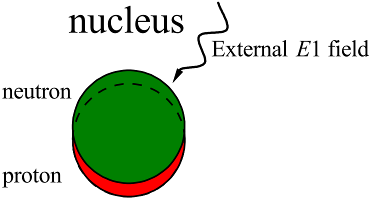
Introduction

$E1$ strength in (spherical) atomic nuclei

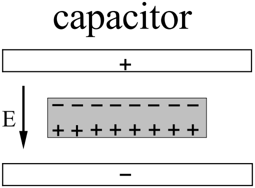


Courtesy: N. Pietralla

Introduction: Electric dipole polarizability



Electric dipole polarizability



Electric dipole polarization

External field
↕ Balanced ⟹ Displacement dipole polarizability
Restoring force

$$\alpha_D = \frac{8\pi}{9} \sum_{\nu} \frac{B(E1; 0^+_{\text{g.st.}} \to 1^-_{\nu})}{E_{1^-_{\nu}}}$$

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MAIN INGREDIENTS OF THE MODEL

Realization of QRPA

The starting point of the method is the HF-BCS calculations of the ground state, where spherical symmetry is assumed for the ground states. The continuous part of the single-particle spectrum is discretized by diagonalizing the HF Hamiltonian on a harmonic oscillator basis.

J. P. Blaizot and D. Gogny, **Nucl. Phys. A284, 429 (1977).**

We employ the effective Skyrme interaction in the particle-hole channel

$$
\begin{aligned}
V(\vec{r}_1,\vec{r}_2) = {} & t_0\left(1+x_0\hat{P}_\sigma\right)\delta(\vec{r}_1-\vec{r}_2)+\frac{t_1}{2}\left(1+x_1\hat{P}_\sigma\right)\left[\delta(\vec{r}_1-\vec{r}_2)\vec{k}^2+\vec{k}'^2\delta(\vec{r}_1-\vec{r}_2)\right] \\
& +t_2\left(1+x_2\hat{P}_\sigma\right)\vec{k}'\cdot\delta(\vec{r}_1-\vec{r}_2)\vec{k}+\frac{t_3}{6}\left(1+x_3\hat{P}_\sigma\right)\delta(\vec{r}_1-\vec{r}_2)\rho^\alpha\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right) \\
& +iW_0\left(\vec{\sigma}_1+\vec{\sigma}_2\right)\cdot\left[\vec{k}'\times\delta(\vec{r}_1-\vec{r}_2)\vec{k}\right].
\end{aligned}
$$

T. H. R. Skyrme, **Nucl. Phys. 9, 615 (1959).**

D. Vautherin and D. M. Brink, **Phys. Rev. C5, 626 (1972).**

Realization of QRPA

The Hamiltonian includes the pairing correlations are generated by the density-dependent zero-range force in the particle-particle channel

$$V_{pair}(\vec{r}_1, \vec{r}_2) = V_0 \left(1 - \eta \frac{\rho(r_1)}{\rho_{sat}}\right) \delta(\vec{r}_1 - \vec{r}_2)\,,$$

where ρ_{sat} is the nuclear saturation density; η and V_0 are model parameters. For example, $\eta=0$ and $\eta=1$ are the case of a volume interaction and a surface-peaked interaction, respectively.

A. P. Severyukhin, V. V. Voronov, N. V. Giai, Phys. Rev. C77, 024322 (2008).

The residual interaction in the particle-hole channel $V_{res}^{\rm ph}$ and in the particle-particle channel $V_{res}^{\rm pp}$ can be obtained as the second derivative of the energy density functional $\mathcal{H}$ with respect to the particle density ρ and the pair density $\tilde{\rho}$, respectively.

$$V_{res}^{\rm ph} \sim \frac{\delta^2 \mathcal{H}}{\delta\rho_1 \delta\rho_2} \qquad V_{res}^{\rm pp} \sim \frac{\delta^2 \mathcal{H}}{\delta\tilde{\rho}_1 \delta\tilde{\rho}_2}\,.$$

G. T. Bertsch and S. F. Tsai, Phys. Rep. 18, 125 (1975).

Realization of QRPA

We introduce the phonon creation operators

$$Q^{+}_{\lambda\mu i}=\frac{1}{2}\sum_{jj'}\left[X^{\lambda i}_{jj'}\,A^{+}(jj';\lambda\mu)-(-1)^{\lambda-\mu}Y^{\lambda i}_{jj'}\,A(jj';\lambda-\mu)\right],$$

$$A^{+}(jj';\lambda\mu)=\sum_{mm'}C^{\lambda\mu}_{jm\,j'm'}\alpha^{+}_{jm}\alpha^{+}_{j'm'}\ .$$

The index λ denotes total angular momentum and μ is its z-projection in the laboratory system. One assumes that the ground state is the QRPA phonon vacuum $|0\rangle$ and one-phonon excited states are $Q^{+}_{\lambda\mu i}|0\rangle$ with the normalization condition

$$\langle 0|[Q_{\lambda\mu i},Q^{+}_{\lambda\mu i'}]|0\rangle=\delta_{ii'}\ .$$

Making use of the linearized equation-of-motion approach one can get the QRPA equations

$$\begin{pmatrix}\mathcal{A} & \mathcal{B}\\ -\mathcal{B} & -\mathcal{A}\end{pmatrix}\begin{pmatrix}X\\ Y\end{pmatrix}=\omega\begin{pmatrix}X\\ Y\end{pmatrix}.$$

Solutions of this set of linear equations yield the one-phonon energies ω and the amplitudes X, Y of the excited states.

D. J. Rowe, **Nuclear Collective Motion (Methuen, London 1970).**

Phonon-phonon coupling (PPC)

To take into account the effects of the phonon-phonon coupling (PPC) in the simplest case one can write the wave functions of excited states as a linear combination of one- and two-phonon configurations

$$\Psi_\nu(JM) = \left[\sum_i R_i(J\nu) Q^+_{JMi} + \sum_{\lambda_1 i_1 \lambda_2 i_2} P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) \left[Q^+_{\lambda_1 \mu_1 i_1} Q^+_{\lambda_2 \mu_2 i_2}\right]_{JM}\right] |0\rangle$$

with the normalization condition

$$\sum_i R_i^2(J\nu) + 2 \sum_{\lambda_1 i_1 \lambda_2 i_2} \left[P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)\right]^2 = 1\,.$$

V. G. Soloviev, **Theory of Atomic Nuclei: Quasiparticles and Phonons (Inst. of Phys., Bristol 1992).**

Phonon-phonon coupling (**PPC**)

Using the variational principle in the form

$$\delta\Big(\langle\Psi_\nu(JM)|\mathcal{H}|\Psi_\nu(JM)\rangle - E_\nu\left[\langle\Psi_\nu(JM)|\Psi_\nu(JM)\rangle - 1\right]\Big) = 0\,,$$

one obtains a set of linear equations for the unknown amplitudes $R_i(J\nu)$ and $P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)$:

$$(\omega_{Ji} - E_\nu)R_i(J\nu) + \sum_{\lambda_1 i_1 \lambda_2 i_2} U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) = 0;$$

$$\sum_i U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)R_i(J\nu) + 2(\omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} - E_\nu)P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) = 0\,.$$

$U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)$ is the matrix element coupling one- and two-phonon configurations:

$$U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) = \langle 0|Q_{Ji}\mathcal{H}\left[Q^+_{\lambda_1 i_1}Q^+_{\lambda_2 i_2}\right]_J|0\rangle\,.$$

These equations have the same form as the QPM equations, but the single-particle spectrum and the parameters of the residual interaction are calculated with the Skyrme forces.

A. P. Severyukhin, V. V. Voronov, N. V. Giai, Eur. Phys. J. A22, 397 (2004).

A. P. Severyukhin, N. N. Arsenyev, N. Pietralla, Phys. Rev. C86, 024311 (2012).

$E1$ STRENGTH DISTRIBUTIONS

Details of calculations

The dipole transition probabilities can be expressed as

$$B(E1;0^+_{gs}\rightarrow 1^-_i) = \left| e^{(\mathrm{n})}_{eff}\langle i|\hat{M}^{(\mathrm{n})}|0\rangle + e^{(\mathrm{p})}_{eff}\langle i|\hat{M}^{(\mathrm{p})}|0\rangle \right|^2 ,$$

where $\hat{M}^{(\mathrm{p})} = \sum_i^Z r_i Y_{1\mu}(\hat{r}_i)$ and $\hat{M}^{(\mathrm{n})} = \sum_i^N r_i Y_{1\mu}(\hat{r}_i)$. The spurious 1^- state is excluded from the excitation spectra by introduction of the effective neutron $e^{(\mathrm{n})}_{eff} = -Z/A\,e$ and proton $e^{(\mathrm{p})}_{eff} = N/A\,e$ charges.

A. Bohr and B. Mottelson, **Nuclear Structure Vol. II (Benjamin, New York 1975).**

The $E1$ strength function $b(E1;\omega)$ which is determined as follows:

$$b(E1;\omega) = \sum_i B(E1;0^+_{gs}\rightarrow 1^-_i)\rho(\omega - E_{1^-_i}),$$

where is the Lorentz weight (the Lorentz averaging parameter is Δ=1 MeV)

$$\rho(\omega - E_{1^-_i}) = \frac{1}{2\pi}\frac{\Delta}{\left(\omega - E_{1^-_i}\right)^2 + \Delta^2/4}.$$

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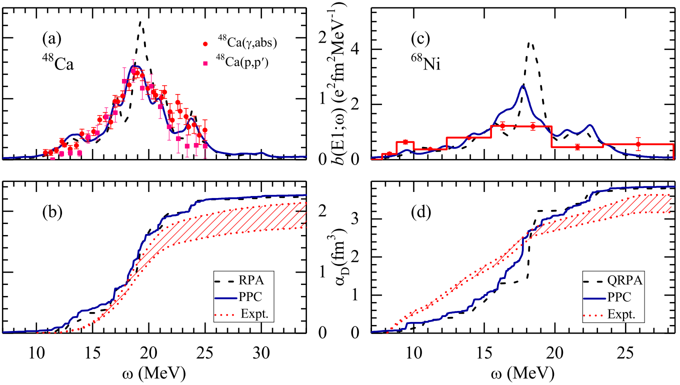
$E1$ strength distributions of ^{48}Ca and ^{68}Ni SLy5



$$\alpha_{\mathrm{D}} = \frac{8\pi}{9} \sum_i B(E1; 0^+_{gs} \to 1^-_i)/E_{1^-_i} \ \ (\mathrm{fm}^3)$$

N. N. Arsenyev, A. P. Severyukhin, V. V. Voronov, N. V. Giai, Phys. Part. Nucl. 50, 528 (2019).

J. Birkhan et al., PRL 118, 252501 (2017).

D. M. Rossi et al., PRL 111, 242503 (2013).

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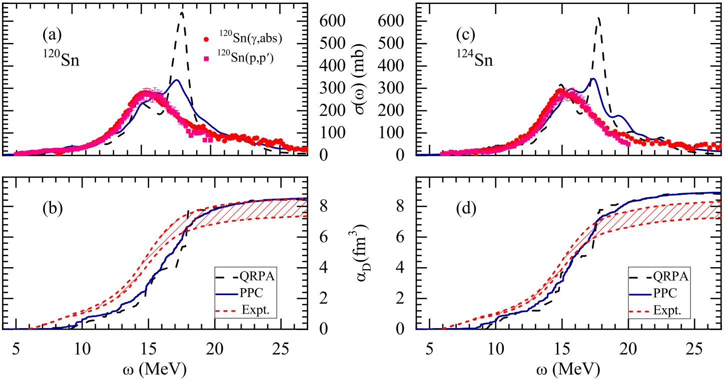
Photoabsorption cross sections on ^{120,124}Sn SLy5



$$\alpha_D = \frac{8\pi}{9} \sum_i B(E1; 0^+_{gs} \to 1^-_i)/E_{1^-_i} \quad (\mathrm{fm}^3)$$

***N. N. Arsenyev, A. P. Severyukhin*, in preparation.**

***S. C. Fultz et al.*, Phys. Rev. 186, 1255 (1969).**

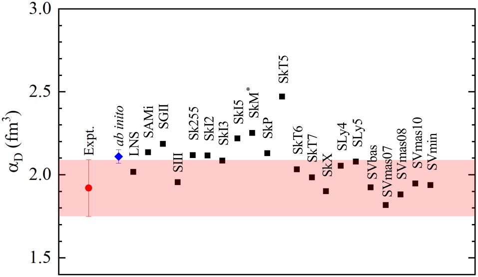
***S. Bassauer et al.*, Phys. Rev. C102, 034327 (2020).**

ESTIMATION OF THE SYMMETRY ENERGY AND ITS SLOPE

Details of calculations

For our systematic analysis, we employ 21 Skyrme interactions: LNS, SAMi, SGII, SIII, SK255, SkI2, SkI3, SkI5, SkM*, SkP, SkT5, SkT6, SkT7, SkX, SLy4, SLy5, SVbas, SVmas07, SVmas08, SVmas10, and SVmin. The choice of these parameterizations is due to the large range of values for the effective nucleon mass $m^* = 0.58 - 1.00$ and the symmetry energy at saturation density $J = 26.8 - 37.4$ MeV.

Electric dipole polarizability α_D of ^{40}Ca



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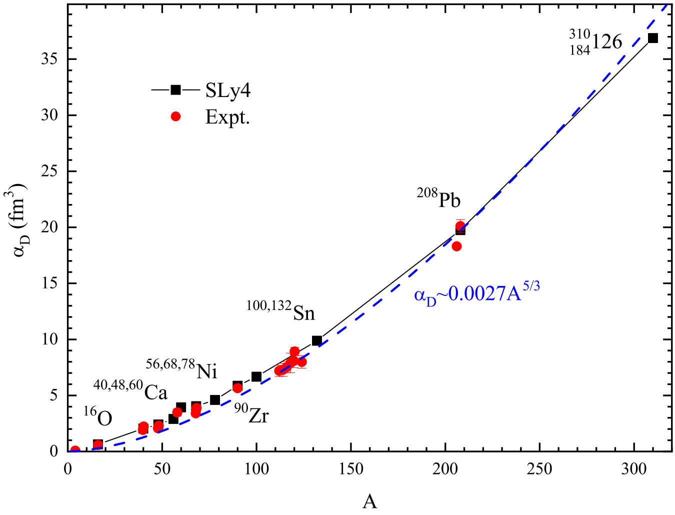
R. W. Fearick et al., Phys. Rev. Res. 5, L022044 (2023).

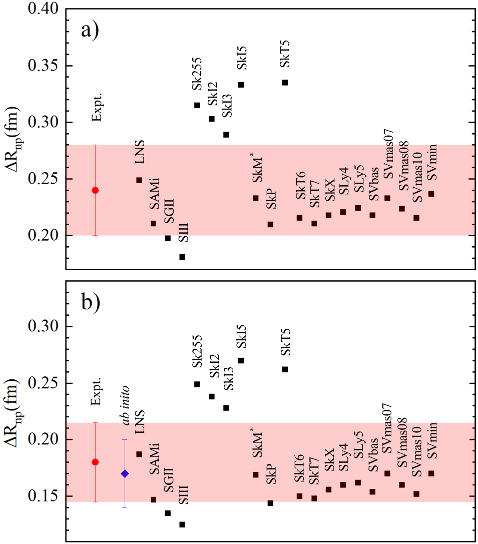
G. Hagen et al., Nature Phys. 12, 186 (2016).

Electric dipole polarizability: Expt. vs Theory



N. N. Arsenyev, A. P. Severyukhin, in preparation.

Neutron skin thickness ΔR_{np} in ^{132}Sn and ^{208}Pb

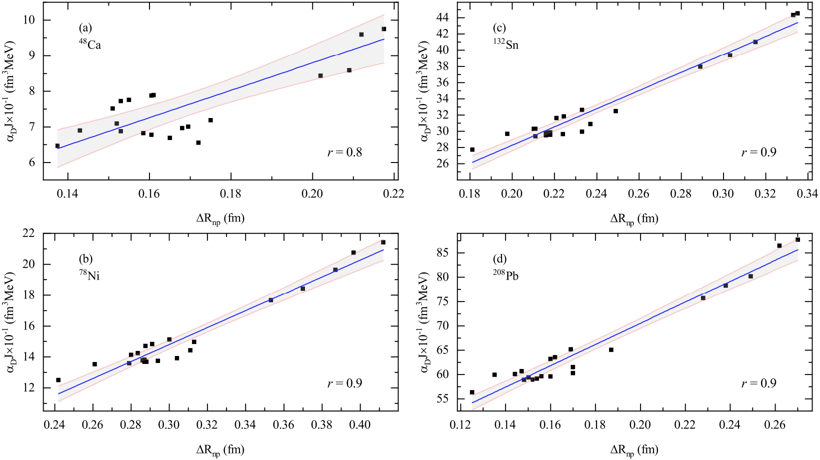


N. N. Arsenyev and A. P. Severyukhin, Moscow Univ. Phys. Bull. 79, 200 (2024).

A. Klimkiewicz et al., Phys. Rev. C76, 051603 (2007).

B. Hu et al., Nature Phys. 18, 1196 (2022).

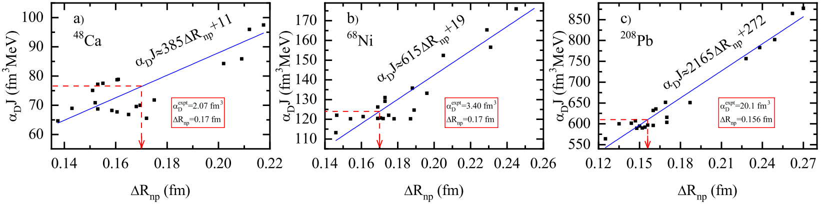
Correlations: α_D vs ΔR_{np} and J



N. N. Arsenyev and A. P. Severyukhin, Moscow Univ. Phys. Bull. 79, 200 (2024).

Estimation of the symmetry energy J

We carried out a theoretical analysis of the recently measured α_D and ΔR_{np} in ^{48}Ca, ^{68}Ni, and ^{208}Pb to extract information about the symmetry energy, by using a strong correlation between $\alpha_D J$ and ΔR_{np}. Combining the experimental data and the RPA theory constraints yields the interval of $J = 30 - 37$ MeV.



N. N. Arsenyev and A. P. Severyukhin, **Moscow Univ. Phys. Bull. 79, 200 (2024).**

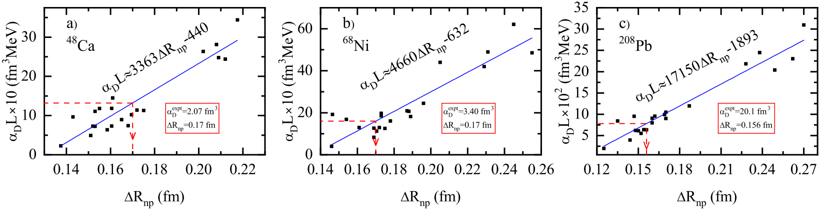
A. Tamii et al., **PRL 107, 062502 (2011).**

D. M. Rossi et al., **PRL 111, 242503 (2013).**

J. Birkhan et al., **Phys. Rev. Lett. 118, 252501 (2017).**

Estimation of the slope parameter L

We carried out a theoretical analysis of the recently measured α_{D} and ΔR_{np} in ^{48}Ca, ^{68}Ni, and ^{208}Pb to extract information about the slope parameter, by using a strong correlation between $\alpha_{\mathrm{D}} L$ and ΔR_{np}. Combining the experimental data and the RPA theory constraints yields the interval of $L = 39 - 64$ MeV.



N. N. Arsenyev and A. P. Severyukhin, in preparation.

A. Tamii et al., PRL 107, 062502 (2011).

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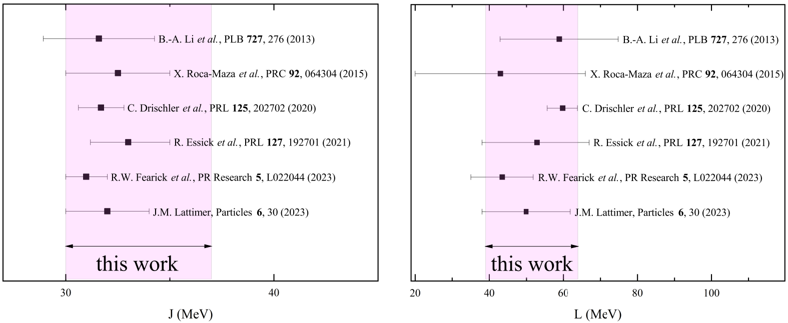
Constraints of the symmetry energy J and its slope L



***N. N. Arsenyev and A. P. Severyukhin*, in preparation.**

Conclusions

Starting from the Skyrme mean-field calculations, the properties of the electric dipole strength in neutron-rich Ca, Ni and Sn isotopes are studied by taking into account the coupling between one- and two-phonons terms in the wave functions of excited states. It is shown that the PPC have small influence on the dipole polarizability.

We have computed the nuclear dipole polarizability ($\alpha_{\rm D}$) and neutron skin thickness ($\Delta R_{\rm np}$) of the magic nuclei using a broad set of Skyrme functionals. It is shown that the neutron skin thickness is correlated the product of the electric dipole polarizability and the symmetry energy (J) and its slope (L) at saturation density.

We carried out a theoretical analysis of the recently measured $\alpha_{\rm D}$ and $\Delta R_{\rm np}$ in ^{48}Ca, ^{68}Ni, and ^{208}Pb to extract information about the symmetry energy and its slope parameter L, by using a strong correlation between $\alpha_{\rm D}$ and $\Delta R_{\rm np}$. Combining the experimental data and the RPA theory constraints yields the interval of J=30$-$37 MeV and L=39$-$64 MeV.

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THE END