Absorption of twisted photon by an electron in a strong magnetic field

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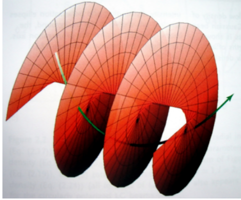
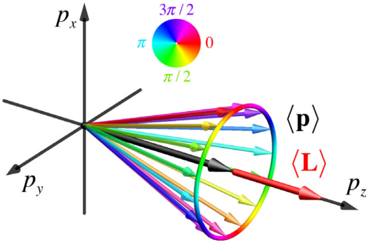
Twisted states

Twisted (vortex) states are **waves** with well-defined:

- longitudinal momentum, k_z;
- absolute transverse momentum, $\kappa = |\mathbf{k}_\perp|$;
- energy, $E = \frac{\kappa^2 + k_z^2}{2m}$;
- angular momentum projection, $l_z = m$.



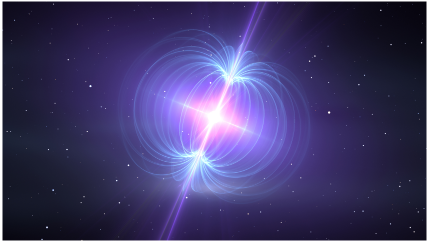


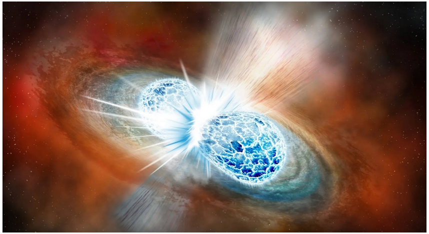
Generation of vortices in strong magnetic field

$\left(H_c = m_e^2 c^3 / e\hbar = 4.4 \times 10^{13}\,\mathrm{G}\right)$

Emission in strong magnetic field $(H \sim H_c)$ is mostly twisted **(Maruyama+'22, Pavlov&Karlovets'24)**.

Stellar nucleosynthesis. Radiative transfer in astrophysical environments such as neutron stars, kilonovae etc.





Gauge of the vector potential

Landau gauge

$$\mathcal{A}^\mu = \{0, 0, Hx, 0\}$$

$$\Phi(\boldsymbol{r}) \propto H_n(\tilde{x}) e^{-\tilde{x}^2/2} e^{i(p_z z + p_y y)}$$
$$\tilde{x} = (x - \rho_H^2 p_x)/\rho_H$$

Axial symmetry is not preserved.
No orbital angular momentum.

Symmetric gauge

$$\mathcal{A}^\mu = H/2 \times \{0, -y, x, 0\}$$

$$\Phi(\boldsymbol{r}) \propto \tilde{\rho}^l L_s^l(\tilde{\rho}) e^{-\tilde{\rho}^2/2} e^{ip_z z}$$
$$\tilde{\rho} = \rho/\rho_H$$

Axial symmetry is preserved.
Orbital angular momentum (l) arises.

Properties of Landau states in symmetric gauge

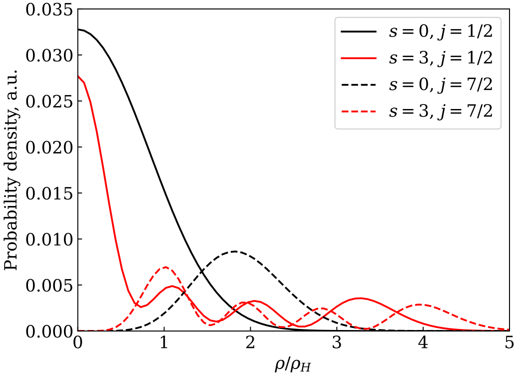
R.m.s. radius

$$\langle \rho \rangle = \rho_H \sqrt{2s + j + 1} \quad (1)$$

Energy

$$\varepsilon_{s,j}^2 = m_e^2 + p_z^2 + \frac{4}{\rho_H^2}\left(s + j + \frac{1}{2}\right), \quad (2)$$

$$s + j \geq -1/2$$

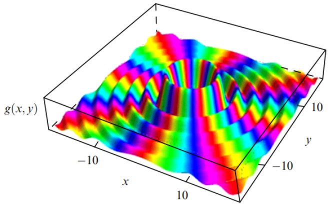


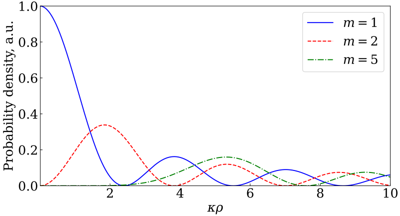
Twisted photon

Emitted radiation by an electron in strong magnetic field is mostly twisted **(Maruyama'22, Pavlov&Karlovets'24)**

The photon is described with the Bessel shape **(Knyazev&Serbo'18)**

$$\psi_{\kappa m k_z} = J_m(\kappa\rho)\exp[i(m\varphi_r + k_z z)] \quad (3)$$





Kinematics

Conservation laws

$$\begin{aligned} \varepsilon_f &= \varepsilon_i + \omega, \\ p_{zf} &= p_{zi} + k_z, \\ j_f &= j_i + m. \end{aligned} \tag{4}$$

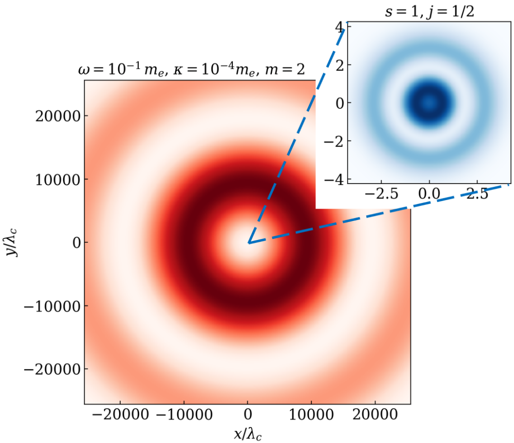
Energy of the absorbed photon

$$\omega = (s_f - s_i + m)\omega_c - \frac{\kappa^2}{2\varepsilon_{s_i,j_i}}, \quad \omega_c = \frac{eH}{\varepsilon_{s_i,j_i}} \tag{5}$$

First order matrix element

$$S_{fi}^{(1)} = \langle e_f | \hat{S} | e_i, \gamma \rangle =$$
$$= -ie \int d^4x \, j_{fi}^{\mu}(x) A_{\mu}(x). \quad (6)$$

$$j_{fi}^{\mu}(x) = \overline{\Psi}_f(x) \gamma^{\mu} \Psi_i(x). \quad (7)$$



Generalized cross section

Cross section

$$d\sigma = \frac{dW}{\mathcal{L}} \tag{8}$$

Absorption probability

$$dW = \left| S_{fi}^{(1)} \right|^2 \frac{L}{2\pi} dp_{zf} \Delta s_f \Delta j_f, \tag{9}$$

Dependence on κ

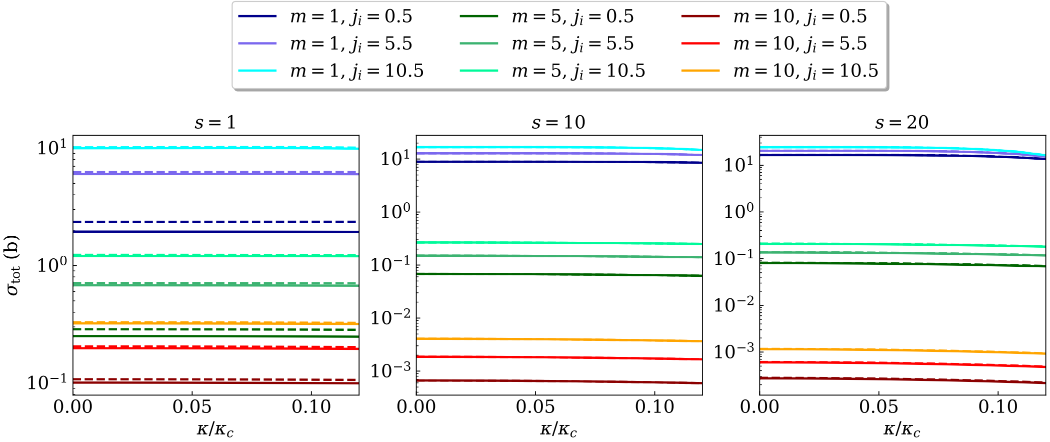
$$\left| S_{fi}^{(1)} \right|^2 \propto \exp\left(-\frac{(\kappa/\kappa_c)^2}{2} \right) \tag{10}$$

$$\kappa_c = m_e \sqrt{H/H_c} \tag{11}$$

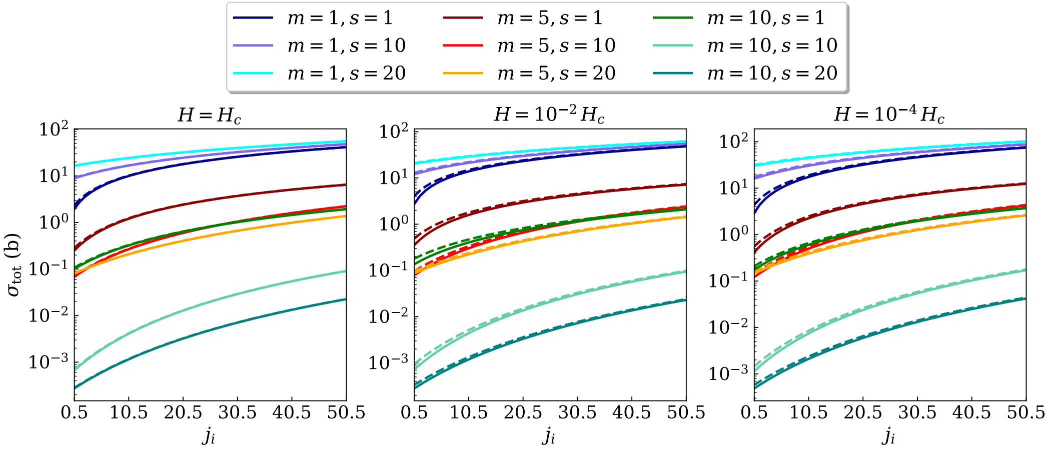
Luminosity

$$\mathcal{L} = \int d^4x \frac{d^3p}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} v(\boldsymbol{p}, \boldsymbol{k}) w^{(e)}(\boldsymbol{r}, \boldsymbol{p}) w^{(\gamma)}(\boldsymbol{r}, \boldsymbol{k}), \tag{12}$$

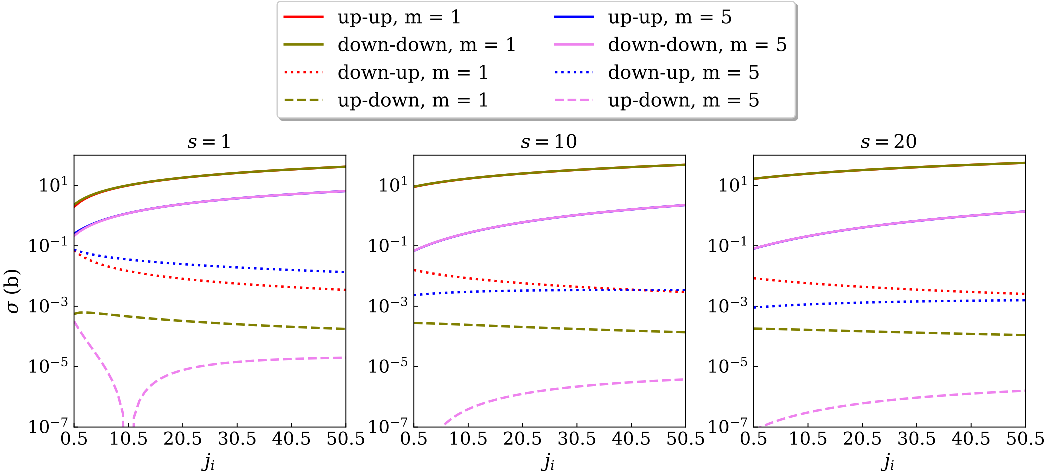
Total absorption cross-section, κ



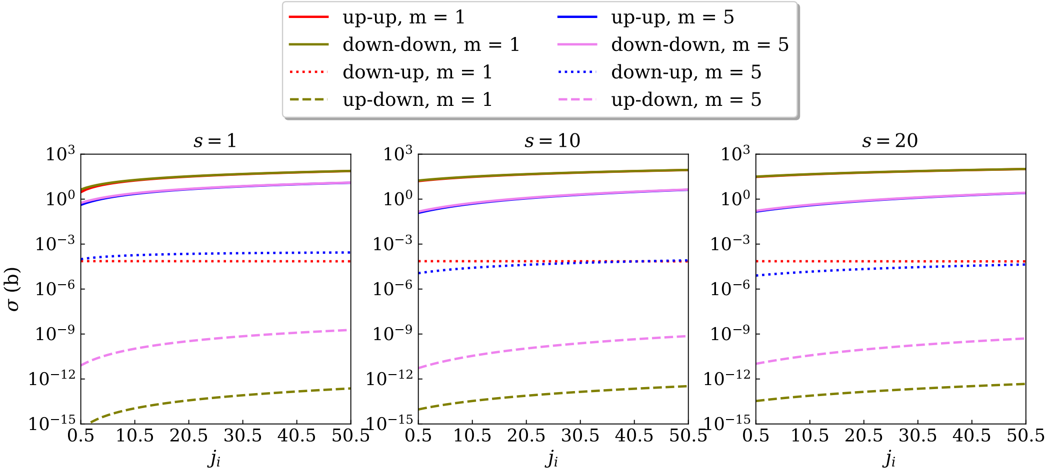
Total absorption cross-section, j_i



Cross sections for spin transitions, $H = H_c$



Cross sections for spin transitions, $H = 10^{-4} H_c$



Key results

Twisted photon absorption cross sections are exponentially suppressed when transverse photon momentum $\kappa > m_e\sqrt{H/H_c}$.

Absorption cross sections **grow** with **increasing** TAM of the initial electron and **decrease** with **increasing** TAM of the incident photon

Cross sections for "**down** $\rightarrow$ **up**" spin flip are always **larger** than for "**up** $\rightarrow$ **down**" flip. It can be the reminiscent of the Sokolov-Ternov effect.

Acknowledgments

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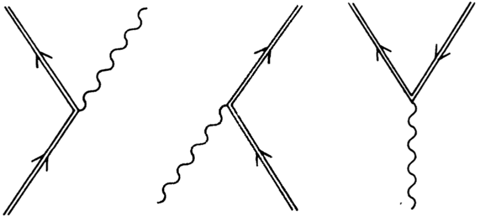
Thank you for your attention!

Reference to our publication:

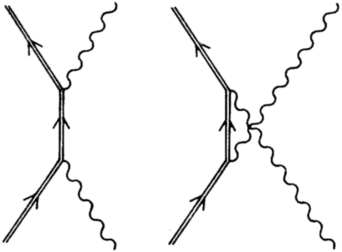
https://link.springer.com/article/10.1140/epjc/s10052-024-13697-3

QED processes in magnetic field

First order



Second order



Landau states

Squared Dirac equation

$$\left[(i\partial_\mu - e\mathcal{A}_\mu)^2 - m_e^2 + eH\Sigma_z\right]\Phi(x) = 0 \tag{13}$$

Spin operator

$$\Sigma_z = \begin{pmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{pmatrix} = \mathrm{diag}\{1, -1, 1, -1\}. \tag{14}$$

Solution to the squared Dirac equation

$$\Phi(x) = \begin{pmatrix} c_1\Phi_+(x) \\ c_2\Phi_-(x) \\ c_3\Phi_+(x) \\ c_4\Phi_-(x) \end{pmatrix} \quad (15)$$

$$\Phi_\pm(x) = N_{s,j}\psi_{s,j\mp 1/2}(\tilde{\rho})e^{-it\varepsilon_{s,j}+i(j\mp 1/2)\varphi+ip_z z} \quad (16)$$

$$\psi_{s,j\mp 1/2}(\tilde{\rho}) = \tilde{\rho}^{|j\mp 1/2|}L_s^{|j\mp 1/2|}(\tilde{\rho}^2)e^{-\tilde{\rho}^2/2} \quad (17)$$

$s = 0, 1, 2, \ldots$
$j = \pm 1/2, \pm 3/2, \ldots$

$\tilde{\rho} = \rho/\rho_H$
$\rho_H = \sqrt{2/|e|H}.$

Solution basis

$$\Phi_\uparrow(x) = \begin{pmatrix} \Phi_+(x) \\ 0 \\ 0 \\ 0 \end{pmatrix}, \ \sigma_z = \frac{1}{2}, \qquad \Phi_\downarrow(x) = \begin{pmatrix} 0 \\ \Phi_-(x) \\ 0 \\ 0 \end{pmatrix}, \ \sigma_z = -\frac{1}{2} \tag{18}$$

Bispinor solution

$$\Psi(x) = \left[\gamma^\mu (i\partial_\mu - e\mathcal{A}_\mu) + m_e\right] \Phi(x). \tag{19}$$

Solution to the Dirac equation

$$\Psi_{\uparrow}(x) = N_{s,j} \begin{pmatrix} (\varepsilon + m_e)\psi_{s,j-1/2}(\tilde{\rho})e^{-i\varphi/2} \\ 0 \\ p_z\psi_{s,j-1/2}(\tilde{\rho})e^{-i\varphi/2} \\ 2\rho_H^{-1} i\psi_{s,j+1/2}(\tilde{\rho})e^{i\varphi/2} \end{pmatrix} e^{i(p_z z + j\varphi - \varepsilon t)}, \tag{20}$$

$$\Psi_{\downarrow}(x) = N_{s,j} \begin{pmatrix} 0 \\ (\varepsilon + m_e)\psi_{s,j+1/2}(\tilde{\rho})e^{i\varphi/2} \\ -2\rho_H^{-1} i\left(s + j + 1/2\right)\psi_{s,j-1/2}(\tilde{\rho})e^{-i\varphi/2} \\ -p_z\psi_{s,j+1/2}(\tilde{\rho})e^{i\varphi/2} \end{pmatrix} e^{i(p_z z + j\varphi - \varepsilon t)}. \tag{21}$$

Vector potential for twisted photon

Vector potential

$$\boldsymbol{A}_{\kappa m k_z \Lambda}(\boldsymbol{r}) = \int a_{\kappa m k_z}(\boldsymbol{q}) \boldsymbol{e}_{\boldsymbol{q}\Lambda} e^{i\boldsymbol{q}\cdot\boldsymbol{r} - i\omega t} \frac{d^3 q}{(2\pi)^3}, \tag{22}$$

$$a_{\kappa m k_z}(\boldsymbol{q}) = \frac{(2\pi)^2}{\kappa} \delta(q_\perp - \kappa)\delta(q_z - k_z) e^{im\varphi_q}. \tag{23}$$

Normalization

$$\int_V \frac{\boldsymbol{E}^2 + \boldsymbol{H}^2}{8\pi} d^3\boldsymbol{r} = \omega \quad \Longleftrightarrow \quad \int_V |\boldsymbol{A}|^2 d^3\boldsymbol{r} = \frac{2\pi}{\omega} \tag{24}$$

Vector potential of the twisted photon

$$\boldsymbol{A}_{\kappa m k_z \Lambda}(\boldsymbol{r}) = \sqrt{\frac{\pi\kappa}{3\omega L R}} e^{i k_z z - i\omega t} \sum_{\sigma=0,\pm1} d^{(1)}_{\sigma,\lambda}(\theta_k) e^{i(m-\sigma)\varphi_r} J_{m-\sigma}(\kappa\rho) \boldsymbol{\chi}_\sigma \quad (25)$$

Wigner matrices

$$d^{(1)}_{\sigma,\lambda}(\theta_k) = \frac{1}{2}(1 + \sigma\lambda\cos\theta_k), \quad \sigma \neq 0$$

$$d^{(1)}_{0,\lambda}(\theta_k) = \frac{\lambda}{2}\sin\theta_k$$

Spin operator eigenvectors

$$\boldsymbol{\chi}_\pm = \mp\frac{1}{\sqrt{2}}\{1, \pm i, 0\}$$

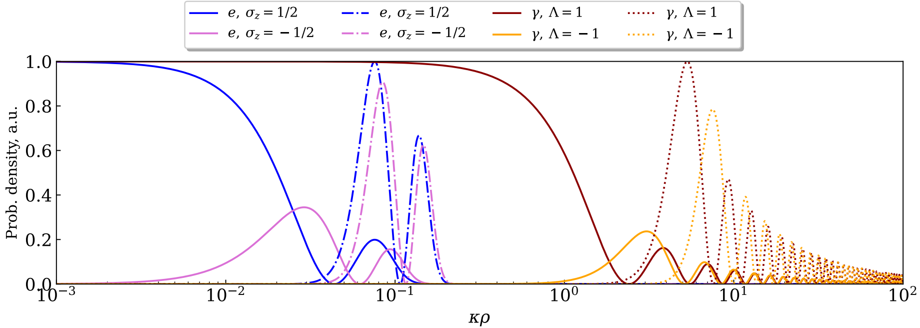
$$\boldsymbol{\chi}_0 = \{0, 0, 1\}$$

Vector potential in the coordinate space

$$\boldsymbol{A}_{\kappa m k_z \Lambda}(\boldsymbol{r}) = \sqrt{\frac{\pi\kappa}{3\omega L R}} e^{ik_z z + im\varphi_r - i\omega t} \times$$
$$\times \begin{pmatrix} i d^{(1)}_{-1\Lambda}(\theta_k) J_{m+1}(\kappa\rho) e^{i\varphi_r} + i d^{(1)}_{1\Lambda}(\theta_k) J_{m-1}(\kappa\rho) e^{-i\varphi_r} \\ d^{(1)}_{-1\Lambda}(\theta_k) J_{m+1}(\kappa\rho) e^{i\varphi_r} - d^{(1)}_{1\Lambda}(\theta_k) J_{m-1}(\kappa\rho) e^{-i\varphi_r} \\ \sqrt{2} d^{(1)}_{0\Lambda}(\theta_k) J_m(\kappa\rho) \end{pmatrix}. \quad (26)$$

Photon and electron density profiles

$s_i = s_f = 1$, $H = H_c$. Solid lines - $j = 1/2$, $m = 1$; dashed lines - $j = 11/2$, $m = 5$



S-matrix for "up-up" transition

$$S^{(1)}_{\uparrow\uparrow} = \sqrt{\frac{2\pi\kappa}{3\omega LR}} ieN_iN_f(2\pi)^3\rho_H\delta(\varepsilon_i - \varepsilon_f + \omega)\delta(p_{zi} - p_{zf} + k_z)\delta_{j_f,j_i+m}\Bigg[2\sqrt{2}\times$$
$$\times\Big((\varepsilon_i+m_e)d^{(1)}_{-1\Lambda}(\theta_k)\mathcal{F}^{j_f+1/2,j_i-1/2}_{s_f,s_i}(\kappa\rho_H)-(\varepsilon_f+m_e)d^{(1)}_{1\Lambda}(\theta_k)\mathcal{F}^{j_f-1/2,j_i+1/2}_{s_f,s_i}(\kappa\rho_H)\Big)$$
$$+\rho_H d^{(1)}_{0\Lambda}(\theta_k)\left(m_e(p_{zi}+p_{zf})+p_{zf}\varepsilon_i+p_{zi}\varepsilon_f\right)\mathcal{F}^{j_f-1/2,j_i-1/2}_{s_f,s_i}(\kappa\rho_H)\Bigg], \quad (27)$$

$$\mathcal{F}^{l_f,l_i}_{s_f,s_i}(\kappa\rho_H) \propto \exp\left(-(\kappa/\kappa_c)^2/2\right), \quad \kappa_c = \sqrt{2}/\rho_H = m_e\sqrt{H/H_c}$$

Special function $\mathcal{F}$

$$\mathcal{F}_{s,s'}^{\ell,\ell'}(y) = \int_0^\infty dx x^{\ell+\ell'+1} L_s^{\ell}(x^2) L_{s'}^{\ell'}(x^2) J_{\ell-\ell'}(yx) e^{-x^2} =$$

$$\frac{(s'+\ell')!}{s!} \frac{1}{2^{2(s-s')+\ell-\ell'+1}} y^{2(s-s')+\ell-\ell'} L_{s'+\ell'}^{s-s'+\ell-\ell'}(y^2/4) L_{s'}^{s-s'}(y^2/4) e^{-y^2/4}, \quad (28)$$

(Gradshtein & Ryzhik'63)

Next values are investigated

$$\sigma_{\text{tot}} = \frac{1}{2} \sum_{\Lambda=\pm 1} \sum_{\sigma_{zf}=\pm 1/2} \int d\sigma, \tag{29}$$

$$\sigma_{\text{spin}}^{fi} = \frac{1}{2} \sum_{\Lambda=\pm 1} \int d\sigma. \tag{30}$$

Luminosity

$$\mathcal{L} = \int d^4x \frac{d^3p}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} v(\boldsymbol{p}, \boldsymbol{k}) w^{(e)}(\boldsymbol{r}, \boldsymbol{p}) w^{(\gamma)}(\boldsymbol{r}, \boldsymbol{k}) \tag{31}$$

$$v(\boldsymbol{p}, \boldsymbol{k}) = \frac{p_\mu k^\mu}{\varepsilon(\boldsymbol{p})\omega(\boldsymbol{k})} = 1 - \frac{\boldsymbol{p} \cdot \boldsymbol{k}}{\varepsilon(\boldsymbol{p})\omega(\boldsymbol{k})} \tag{32}$$

Wigner functions

$$w^{(e)}_{s,j}(\boldsymbol{r},\boldsymbol{p},t)=\frac{1}{(2\pi)^3}\int\Psi^{\dagger}_{s,j}(\boldsymbol{r}-\boldsymbol{y}/2,t)\Psi_{s,j}(\boldsymbol{r}+\boldsymbol{y}/2,t)e^{i\boldsymbol{p}\cdot\boldsymbol{r}}d^3\boldsymbol{y}, \tag{33}$$

$$w^{(\gamma)}(\boldsymbol{r},\boldsymbol{p},t)=\frac{1}{(2\pi)^3\omega}\int\boldsymbol{E}^{*}_{\kappa m k_z\Lambda}(\boldsymbol{r}-\boldsymbol{y}/2,t)\boldsymbol{E}_{\kappa m k_z\Lambda}(\boldsymbol{r}+\boldsymbol{y}/2,t)e^{i\boldsymbol{k}\cdot\boldsymbol{r}}d^3\boldsymbol{y} \tag{34}$$

$$\int w^{(e)}_{s,j}(\boldsymbol{r},\boldsymbol{p},t)\frac{d^3\boldsymbol{p}}{(2\pi)^3}=\Psi^{\dagger}_{s,j}\Psi_{s,j} \tag{35}$$

$$\int w^{(\gamma)}(\boldsymbol{r},\boldsymbol{p},t)\frac{d^3\boldsymbol{k}}{(2\pi)^3}=\frac{1}{\omega}\boldsymbol{E}^{*}_{\kappa m k_z\Lambda}\boldsymbol{E}_{\kappa m k_z\Lambda}=\omega\boldsymbol{A}^{*}_{\kappa m k_z\Lambda}\boldsymbol{A}_{\kappa m k_z\Lambda} \tag{36}$$

Sharp minimum of $\sigma_{\downarrow\uparrow}$ and $\sigma_{\uparrow\downarrow}$

$$\omega_{\downarrow\uparrow,\min} \approx m_e \frac{H}{H_c}\frac{\Lambda}{\Lambda-1}\frac{2(m+1)}{(1+\varepsilon/m_e)}, \quad \omega_{\uparrow\downarrow,\min} \approx \frac{\kappa^2}{m_e}\frac{\Lambda}{\Lambda+1}\frac{(s_i+j_i+1/2)}{m(1+\varepsilon_i/m_e)} \tag{37}$$

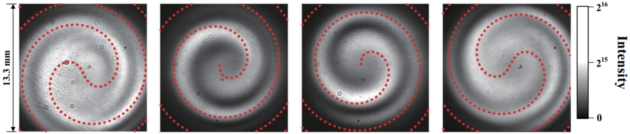
$$j_{\min} \approx \frac{1}{2}\frac{H_c}{H}(m^2-1) - s - \frac{1}{2} \tag{38}$$

Generation of vortices

Electron can emit a vortex photon being

- in an external magnetic field **(Katoh+'23)**
- in a helical undulator **(Katoh+'23)**
- in a circularly polarised laser wave **(Epp+'23, Zhang+'21)**

by helical undulator:



by circularly-polarised laserwave: