Muon and Electron Signatures in Super–Kamiokande and Sensitivity to Proton Decay

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1 Energy Deposition Differences

Electrons ($m_e c^2 = 0.511$ MeV/c^2) and heavy charged particles (for example, muon $m_\mu c^2 = 105.658$ MeV/c^2) are able to lose energy in matter through the following processes:

- ionization,
- radiation (bremsstrahlung),
- pair production.

In the region of low electron energies ($E < 10$ MeV), ionization processes of interaction with atomic electrons, including the ionization of atoms, provide the determining contribution to energy losses. Specific ionization energy losses of electrons[1]:

$$\left(\frac{dE}{dx}\right)_{\text{e ion}} = -\frac{2\pi}{\beta^2} n_e r_e^2 m_e c^2 \left[\ln\left(\frac{m_e c^2 T_e}{\bar{I}^2} \cdot \frac{\beta^2}{2(1-\beta^2)}\right) - \left(2\sqrt{1-\beta^2} - 1 + \beta^2\right)\ln 2 + 1 - \beta^2\right], \tag{1}$$

where m_e – mass of the electron; T_e – kinetic energy of the electron; $\beta = \frac{v}{c}$, v – speed of the particle, c – speed of light; $n_e = N_A \left(\frac{Z}{A}\right)\rho$ – electron density of the medium, N_A – Avogadro's number, Z – medium atomic number, A – medium atomic mass, ρ – medium density; $\bar{I}$ – mean ionization potential of the atoms in the medium: $\bar{I} = 13.5\,Z\,\text{eV}$, $r_e = \frac{e^2}{m_e c^2} = 2.818 \cdot 10^{-13}\,\text{cm}$ – classical electron radius.

Hard collision (Moller scattering) is elastic scattering of an electron on an atomic electron. A charged particle (electron) transfers part of its energy to the atomic electron. As a result, the atomic electron is knocked out of the atom and becomes a δ -electron that is also able to produce ionization. This process occurs at $E < 10$ MeV.

For electrons, it is also significant that at energies of a few MeV, radiative losses become noticeable. At higher energies, radiation losses dominate over ionization losses ($E > 10$ MeV). Specific radiative energy loss of electrons:

$$\left(\frac{dE}{dx}\right)_{\text{e rad}} = -E_e \cdot \frac{1}{X_0}, \tag{2}$$

where E_e – energy of the electron; X_0 – radiation length of the material, a characteristic length over which an electron loses all but $1/e$ of its energy due to bremsstrahlung.

A high-energy electron in the field of the nucleus (or atomic electron) produces an electron-positron pair. Pair production becomes significant at $E > 10$ GeV and competes with bremsstrahlung at $E > 100$ GeV. Energy losses of electrons for pair production:

$$\left(\frac{dE}{dx}\right)_{\text{e pair}} = -\alpha r_e^2 \frac{Z^2}{A} E_e \rho \cdot \Phi(E, Z), \tag{3}$$

where α – fine structure constant; Z – medium atomic number; A – medium atomic mass; ρ – medium density; $\Phi(E, Z)$ – function that takes into account the kinematics of the process.

Muons are heavy charged particles. The ionization losses for them dominate in the whole energy range (especially at $E < 100$ GeV). Specific ionization energy losses of muons:

$$\left(\frac{dE}{dx}\right)_{\mu\text{ ion}} = -Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2}\ln\left(\frac{2m_e c^2 \beta^2 \gamma^2 T_{\max}}{\bar{I}^2}\right) - \beta^2 - \frac{\delta}{2}\right], \tag{4}$$

where K – simplifying constant, $K = 0.307\,\text{MeV} \cdot \text{cm}^2/\text{g}$; z – charge of the muon; γ – relativistic factor, $\gamma = \frac{E}{m_\mu c^2}$; T_{max} – maximum kinetic energy transfer in a single collision; δ – density correction.

Hard collision (Moller scattering) also occurs with a muon on an atomic electron, but it is negligible. The cross section of the process is inversely proportional to the muon mass. When a heavy particle scatters on a light particle, the momentum/energy transfer is small (as when a billiard ball hits a grain of sand).

The braking radiation for muons is negligible at $E < 100$ GeV (due to their mass as well). Specific radiative energy loss of muons (significant at $E > 1$TeV):

$$\left(\frac{dE}{dx}\right)_{\mu\text{ rad}} = -\left(\frac{m_e}{m_\mu}\right)^2 E_\mu \cdot \frac{1}{X_0}, \tag{5}$$

[1] All following formulas are taken from [1].

where E_μ – energy of the muon; X_0 – radiation length of the material, a characteristic length over which an electron loses all but $1/e$ of its energy due to bremsstrahlung.

A muon in the nucleus field emits a virtual photon, which transforms into an electron-positron pair, but this process is rare and only becomes important at ultra-relativistic energies($E > 1$ TeV), due to suppression in $\left(\frac{m_e}{m_\mu}\right)^2 \approx 10^{-4}$. Energy losses for pair birth:

$$\left(\frac{dE}{dx}\right)_{\mu\ \text{pair}} = -\alpha r_e^2 \frac{Z^2}{A} E_\mu \rho \left(\frac{m_e}{m_\mu}\right)^2 \cdot \Phi(E, Z). \quad (6)$$

In brief:

- Ionization losses are the main loss mechanism for electrons at $E < 10$ MeV and dominate the entire energy range for muons (especially at $E < 100$ GeV). Electrons have a shorter range due to their mass and strong scattering, while muons have a longer range (200 times longer e at the same energy).
- Hard collision (Moller scattering) is critical for electrons at $E < 1$ MeV, forms δ-electrons, and is negligible for muons at $E < 1$ TeV. Moller scattering for muons at $E > 1$ TeV is formally possible, but still uncompetitive with radiative losses.
- Braking radiation (bremsstrahlung) is dominant for electrons at $E > 10$ MeV and negligible for muons at $E < 100$ GeV. Braking radiation becomes significant for muons at $E > 100$ GeV and dominates at $E > 1$ TeV.
- Pair production becomes significant for electrons at $E > 10$ GeV and competes with bremsstrahlung at $E > 100$ GeV. For muons, the process is suppressed and is only relevant at ultra-high energies $E > 1$ TeV.

Super–Kamiokande is able to detect neutrinos in the energy range from 4.5 MeV to 1 TeV. As a result of quasi-elastic scattering of neutrinos through a charged current, negatively charged particles such as electrons and muons appear:

$$\nu_e + n \to e^- + p,$$

$$\nu_\mu + n \to \mu^- + p.$$

The identification of events in the detector is based on the shape and size of the Cherenkov rings.

- For electrons produced by electron neutrinos, radiation losses and electromagnetic showers dominate, forming a fuzzy cone of Cherenkov light. At energies below $E < 100$ MeV, showers do not develop, so such rings are less fuzzy.
- Muons produced by the muon neutrino in the process of their passage through the detector form a clear cone of Cherenkov radiation due to the main process of ionization losses. Cherenkov light concentrates along the track. For muons with $E > 10$ GeV, radiation losses are possible, but they are rare in SK.

A comparison of the described Cherenkov rings is shown in Fig. 1.

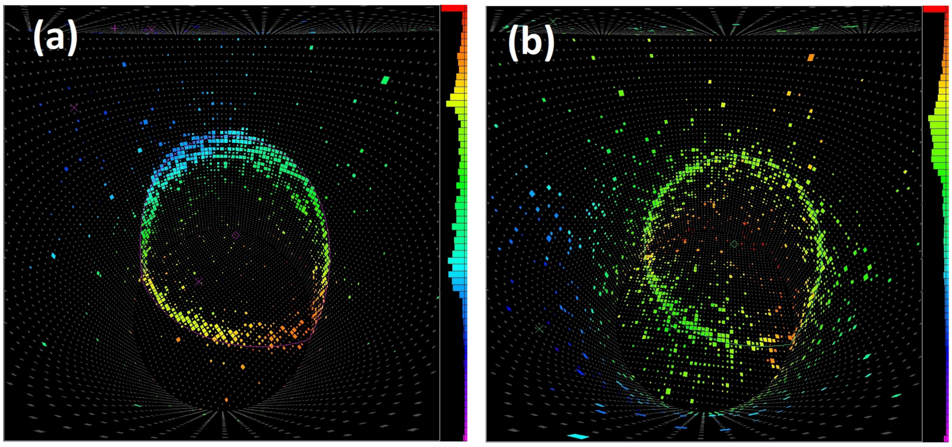

Fig 1. Cherenkov rings detected by SK in 1998: (a) from muons with a momentum of 604 MeV, (b) from electrons with a momentum of 492 MeV. This figure of a simulation in the SK detector is a courtesy of the SK collaboration.

2 Range Estimation in Matter

Electrons lose energy via radiation. The range R_e can be estimated as:

$$R_e \approx \frac{X_0}{\rho} \cdot \ln\left(\frac{E_e}{E_c}\right), \tag{7}$$

where X_0 – radiation length, ρ – medium density, E_c – critical energy, $E_c \approx 80$ MeV, E_e – initial energy.

Material	X_0 $\frac{\text{g}}{\text{cm}^2}$	ρ $\frac{\text{g}}{\text{cm}^3}$	E_c MeV
Water	36.1	1.0	78.0
Lead	6.4	11.3	7.8

1: Parameters' values for water and lead [1].

Rough numerical estimations of R_e:

- Water:

$$R_e \approx \frac{36.1}{1} \cdot \ln\left(\frac{1000}{78}\right) \approx 90 \text{ cm},$$

- Lead:

$$R_e \approx \frac{6.4}{11.3} \cdot \ln\left(\frac{1000}{7.8}\right) \approx 2.7 \text{ cm}.$$

Muons lose energy via ionization. The range R_μ can be estimated as:

$$R_\mu \approx \frac{E_\mu}{\rho_w \left\langle \frac{dE}{dx} \right\rangle}, \tag{8}$$

where $\left\langle \frac{dE}{dx} \right\rangle$ – average energy loss per unit length (about $2\ \text{MeV} \cdot \text{g}^{-1}\text{cm}^2$ for water and $1.3\ \text{MeV} \cdot \text{g}^{-1}\text{cm}^2$ for lead). Rough numerical estimations of R_μ:

- Water:

$$R_\mu \approx \frac{1000}{1 \cdot 2} \approx 500 \text{ cm},$$

- Lead:

$$R_\mu \approx \frac{1000}{11.3 \cdot 1.3} \approx 70 \text{ cm}.$$

3 Containment in SK

The geometric size of the SK Inner Detector (ID) [2]:

- Total height: 36.2 m,
- Diameter: 33.8 m.

The fiducial volume (for neutrino interactions) is the virtual cylinder inscribed within the ID at a distance of 200 cm from its walls:

- Height (H_{fid}): ~ 32 m,
- Diameter (D_{fid}): ~ 30 m,

Maximum contained track length (diagonal to the cylinder):

$$L_{\text{max}} = \sqrt{H_{\text{fid}}^2 + D_{\text{fid}}^2} = \sqrt{32^2 + 30^2} \approx 44 \text{ m}.$$

Electrons lose energy via radiation. Their energy is calculated using equation (7):

$$(E_e)_{max} \approx E_c \cdot e^{\frac{L_{max} \cdot \rho_w}{X_0}} \approx 80 \cdot e^{\frac{44 \cdot 100}{36.1}} \approx 2.6 \cdot 10^{52} \text{ GeV}.$$

Electrons with any energy will remain contained.

Muons lose energy via ionization (Bethe-Bloch). Their energy is calculated using equation (8):

$$(E_\mu)_{max} \approx L_{max} \cdot \rho_w \cdot 2\,\text{MeV} \cdot \text{g}^{-1}\text{cm}^2 \approx 44 \cdot 100 \cdot 2 \approx 8.8 \text{ GeV}.$$

Multi-GeV muons will exit if they pass in the detector less than the specified length L_{max}. Muons with energy $E_\mu >$ 8.8 GeV will escape the SK limit.

4 Proton Decay Signal

The expected signal for proton decay:

$$p \to e^+ + \pi^0.$$

This is one of the channels for proton decay searches in water Cherenkov detectors such as Super-Kamiokande. The positron produces a single Cherenkov ring (electron-like). The neutral pion decays almost immediately ($\tau \sim 10^{-17}$ s):

$$\pi^0 \to \gamma + \gamma.$$

Each photon from produces an electromagnetic shower (via pair conversion), resulting in two more electron-like rings. A maximum total of three Cherenkov rings are expected from proton decay.

A free (from H, are available in Super-K) proton is at rest, all of its energy is shared between the positron and neutral pion. They go in opposite directions to compensate for the momentum. Kinematic features of the event for a free proton:

$$\vec{p_p} = \vec{p_{\pi^0}} + \vec{p_{e^+}} = \vec{0} \Rightarrow p_{\pi^0} = p_{e^+}, \tag{9}$$

$$p = \sqrt{E^2 - m^2} = \sqrt{m^2 + 2mT + T^2 - m^2} = \sqrt{2mT + T^2}. \tag{10}$$

Combining equations [9] and [10], and energy conservation law:

$$\begin{cases} m_p = T_e + T_{\pi^0} + m_e + m_{\pi^0}, \\ 2m_eT_e + T_e^2 = 2m_{\pi^0}T_{\pi^0} + T_{\pi^0}^2. \end{cases} \tag{11}$$

From [11] can get that:

$$T_{e(\pi^0)} = \frac{(m_p - m_e - m_{\pi^0}) \cdot (m_p - (+)m_e + (-)m_{\pi^0})}{2m_p}. \tag{12}$$

$$\text{Particle masses}: m_p = 938 \text{ MeV}, m_{\pi^0} = 135 \text{ MeV}, m_{e^+} \approx 0.5 \text{ MeV (negligible)}.$$

So far kinetic energy for the positron $T_e \approx 460$ MeV, for neutral pion $T_{\pi^0} = 340$ MeV. A positron with such energy will lose it via radiation. Thus, a ring from the positron will be electron-like.

Photons from a neutral pion usually have a small angle between them because of the limited momentum. So, the two rings can overlap or be very close. The type of rings from gammas will be electron-like.

The signature of a free proton decay will be characteristic rings on opposite sides of the detector (back-to-back). If we consider a proton inside the oxygen nucleus, decay products can undergo nuclear effects [3]. Due to nuclear effects, observable particles can differ from a "free proton"particles. For example, a neutral pion interaction inside a nucleus would result in positive pion production. This positive pion would give (depending on the energy) a a muon-like ring or no ring at all. Nuclear effects spoil the signal and make it inseparable from backgrounds.

Main Backgrounds:

- Atmospheric neutrino interactions through a charged-current:

$$\nu_e + n \to e^- + p,$$

$$\bar{\nu_e} + p \to e^+ + n.$$

- Interactions of muon neutrinos from the beam.
- Neutrino interactions through a neutral-current:

$$\nu + p \to \nu + p + \pi^0.$$

- Cosmic muons decays:

$$\mu^- \to e^- + \bar{\nu_e} + \nu_\mu,$$

$$\mu^+ \to e^+ + \nu_e + \bar{\nu_\mu}.$$

The following cuts can be used to suppress backgrounds:

- Interaction vertex inside Super-Kamiokande FV.
- No delayed signal (from charged pions decay).
- Total collected energy ≈ 940 MeV.
- No tagged neutron: to tag neutrons and thus differentiate the signal from some backgrounds' channels, Gd (0.01%) was added to Super-Kamiokande.
- Back-to-back topology for e-like rings.

5 Proton Lifetime Sensitivity

Proton Lifetime Lower Bound Estimation for Super-Kamiokande

Experimental parameters:

- Fiducial mass: $M_{\text{fid}} = 22.5\,\text{kt} = 2.25 \times 10^{10}$ g of water,
- Proton count per gram: $N_p/\text{gram} = 6.022 \cdot 10^{23} \cdot \frac{10}{18} \approx 3.35 \times 10^{23}$,
- Total protons: $N_p = 2.25 \cdot 10^{10} \times 3.35 \cdot 10^{23} \approx 7.5 \times 10^{33}$,
- Exposure time: $T = 10\,\text{year} \approx 3.15 \times 10^{8}\,\text{s}$.

Statistical limit calculation

For zero observed decays with perfect efficiency and no background, the 90% confidence level lower limit on the proton lifetime τ_p follows from Poisson statistics:

$$P(0) = e^{-\mu} \geq 0.1 \quad \Rightarrow \quad \mu \leq \ln(10) \approx 2.3026,$$

where μ is the expected number of decays:

$$\mu = N_p - N_p(T) = N_p - N_p \cdot \exp(-T/\tau_p) \approx N_p \cdot (1 - (1 - T/\tau_p)) = N_p \cdot T/\tau_p, \tag{13}$$

$$\mu = \frac{N_p \cdot T}{\tau_p}.$$

Combining these gives:

$$\tau_p \geq \frac{N_p \cdot T}{\ln(10)} \approx N_p \cdot T \cdot 0.434 \geq 7.5 \cdot 10^{33} \cdot 3.15 \cdot 10^{8} \cdot 0.434 \geq \boxed{1.03 \times 10^{42}\,\text{s}} \approx 3.26 \times 10^{34}\,\text{years}.$$

Comparison with Actual Limits

- Current SK limit (2017) [4]: $\tau/B(p \to e^+\pi^0) > 2.4 \times 10^{34}$ year – obtained constraint ∼1.5 times longer than published.

6 References

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[4] K. Abe *et al.*, "Search for proton decay via $p \to e^+\pi^0$ and $p \to \mu^+\pi^0$ in 0.31 megaton · years exposure of the Super-Kamiokande water Cherenkov detector," *Phys. Rev. D*, vol. 95, no. 1, p. 012004, 2017.