$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\Gamma \sim g\partial g$$

$$\eta \partial h$$

$$R \sim \partial\Gamma + \cancel{O(\Gamma^2)} \sim h^2$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$|h_{\mu\nu}| \ll 1$$

$$|\partial_\nu h_{\mu\nu}| \ll 1$$

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2}\eta^{\lambda\rho}\left(\partial_\mu h_{\rho\nu} + \partial_\nu h_{\rho\mu} - \partial_\rho h_{\mu\nu}\right)$$

$$R_{\mu\nu} = \partial_\lambda \Gamma^{\lambda}_{\mu\nu} - \partial_\mu \Gamma^{\lambda}_{\lambda\nu}$$

$$R_{\mu\nu} = \frac{1}{2}\eta^{\lambda\rho}\left(\partial_\lambda \partial_\mu h_{\rho\nu} + \partial_\lambda \partial_\nu h_{\rho\mu} - \partial_\lambda \partial_\rho h_{\mu\nu}\right) + \ldots.$$

$$\frac{1}{2}\left(\partial^\rho \partial_\mu h_{\rho\nu} + \partial^\rho \partial_\nu h_{\rho\mu} - \overbrace{\partial^\rho \partial_\rho}^{\Box} h_{\mu\nu}\right) + \ldots.$$

$$R = \eta^{\mu\nu} R_{\mu\nu}$$

$$\frac{1}{2}\partial_\nu\partial^\lambda h_{\lambda\mu} - \frac{1}{2}\underbrace{\Box h_{\mu\nu}}_{} - \frac{1}{2}\partial_\mu\partial_\nu h + \frac{1}{2}\partial_\mu\partial^\lambda h_{\lambda\nu} -$$

$$= T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$$

$$-\frac{1}{2}\eta_{\mu\nu}\partial^\sigma\partial^\lambda h_{\lambda\sigma} + \frac{1}{2}\eta_{\mu\nu}\Box h = \frac{8\pi G}{c^4}T_{\mu\nu}$$

$$x'^\mu = x^\mu + \xi^\mu$$

$$\partial_\nu \xi^\mu \sim h$$

$$g'_{\mu\nu} = g_{\rho\sigma}\frac{\partial x^\rho}{\partial x'^\mu}\frac{\partial x^\sigma}{\partial x'^\nu} = g_{\rho\sigma}\left(\delta^\rho_\mu - \partial_\mu\xi^\rho\right)\left(\delta^\sigma_\nu - \partial_\nu\xi^\sigma\right)$$

$$\boxed{h'_{\mu\nu} = h_{\mu\nu} - \partial_\mu\xi_\nu - \partial_\nu\xi_\mu}$$

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h$$

$$\bar{h} = -h$$

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\bar{h} \qquad h' = \eta^{\mu\nu} h'_{\mu\nu} = h - 2\partial_\mu \xi^\mu$$

$$\boxed{\bar{h}'_{\mu\nu} = \bar{h}_{\mu\nu} - \partial_\mu \xi_\nu - \partial_\nu \xi_\mu + \eta_{\mu\nu}\partial_\alpha \xi^\alpha}$$

$$\partial^\mu \bar{h}_{\mu\nu} = 0 \qquad \partial_\mu A^\mu = 0 \qquad A^{\mu\prime} = A^\mu + \partial_\mu \chi$$

$$\underbrace{\partial^\mu (\bar{h}_{\mu\nu})'}_{0} = \partial^\mu \bar{h}_{\mu\nu} - \Box \xi_\nu$$

$$\Box \xi_\nu = \partial^\mu \bar{h}_{\mu\nu}$$

$$\boxed{\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}}$$

$$T_{\mu\nu} = 0$$

$$k^\rho = \left(\frac{\omega}{c}, \vec{k}\right)$$

$$\Box \bar{h}_{\mu\nu} = 0$$

$$k^\mu A_{\mu\nu} = 0$$

$$\bar{h}_{\mu\nu} = \mathrm{Re}\left[A_{\mu\nu} e^{i k_\rho x^\rho}\right]$$

$$k_\rho k^\rho = 0 \Rightarrow \frac{\omega^2}{c^2} - |\vec{k}|^2 = 0$$

$$\bar{h}''_{\mu\nu} = \bar{h}'_{\mu\nu} - \partial_\mu \tilde{\xi}_\nu - \partial_\nu \tilde{\xi}_\mu + \eta_{\mu\nu} \partial_\rho \tilde{\xi}^\rho$$

$$(\partial^\mu \bar{h}_{\mu\nu})'' = (\partial^\mu h_{\mu\nu})' - \Box \tilde{\xi}_\nu$$

$$\Box \tilde{\xi}_\nu = 0$$

$$\tilde{\xi}_\nu = \mathrm{Re}[\xi_\nu e^{ik_\rho x^\rho}]$$

$$\bar{h}_{\mu\nu} = \mathrm{Re}[A_{\mu\nu} e^{ik_\rho x^\rho}]$$

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \cancel{\frac{1}{2}\eta_{\mu\nu} h}$$

$$\bar{h} = -h \qquad h = 0$$

$$A'_{\mu\nu} = A_{\mu\nu} - k_\nu \xi_\mu - \xi_\nu k_\mu + \eta_{\mu\nu} k_\alpha \xi^\alpha$$

$$\eta^{\mu\nu} A'_{\mu\nu} = 0 \qquad \bar{h}_{\mu\nu} = h_{\mu\nu} \qquad A_{0i} = 0$$

$\partial^0 A_{00} + \dots \to 0$

$K^0 A_{00} - K^i \cancel{A_{0i}} = 0$

$A_{00} = \text{const}, \quad A_{00} = 0$

$$\boxed{\partial^0 h_{00} - \partial^i h_{0i} = 0 \quad h_{00} = \text{const}}$$

$A^i_i = 0 \quad A_{0i} = 0$

$A_{0\mu} = 0$

$R_{\mu\nu}$

$x'^\mu = x^\mu + \xi^\mu$

$$A_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$k^\mu = \left\{ \frac{\omega}{c}, 0, 0, k_z \right\}$

$k^\mu A_{\mu\nu} = 0$

$\frac{\omega}{c} A_{0\nu} + 0 + 0 + k^3 A_{3\nu} = 0$

$$h_{ij}^{TT}(t, z) = \begin{pmatrix} h_+ & h_\times \\ h_\times & -h_+ \end{pmatrix} \cos\left(\omega\left(t - \frac{z}{c}\right)\right)$$

$$\frac{d^2 x^{i+\xi}}{d\tau^2} + \Gamma^i_{\nu\beta} \frac{dx^\nu}{d\tau} \frac{dx^\beta}{d\tau} = 0$$

$\ddot{\xi} \neq 0 \sim \ddot{h}$

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

$$\partial^\mu \bar{h}_{\mu\nu} = 0$$

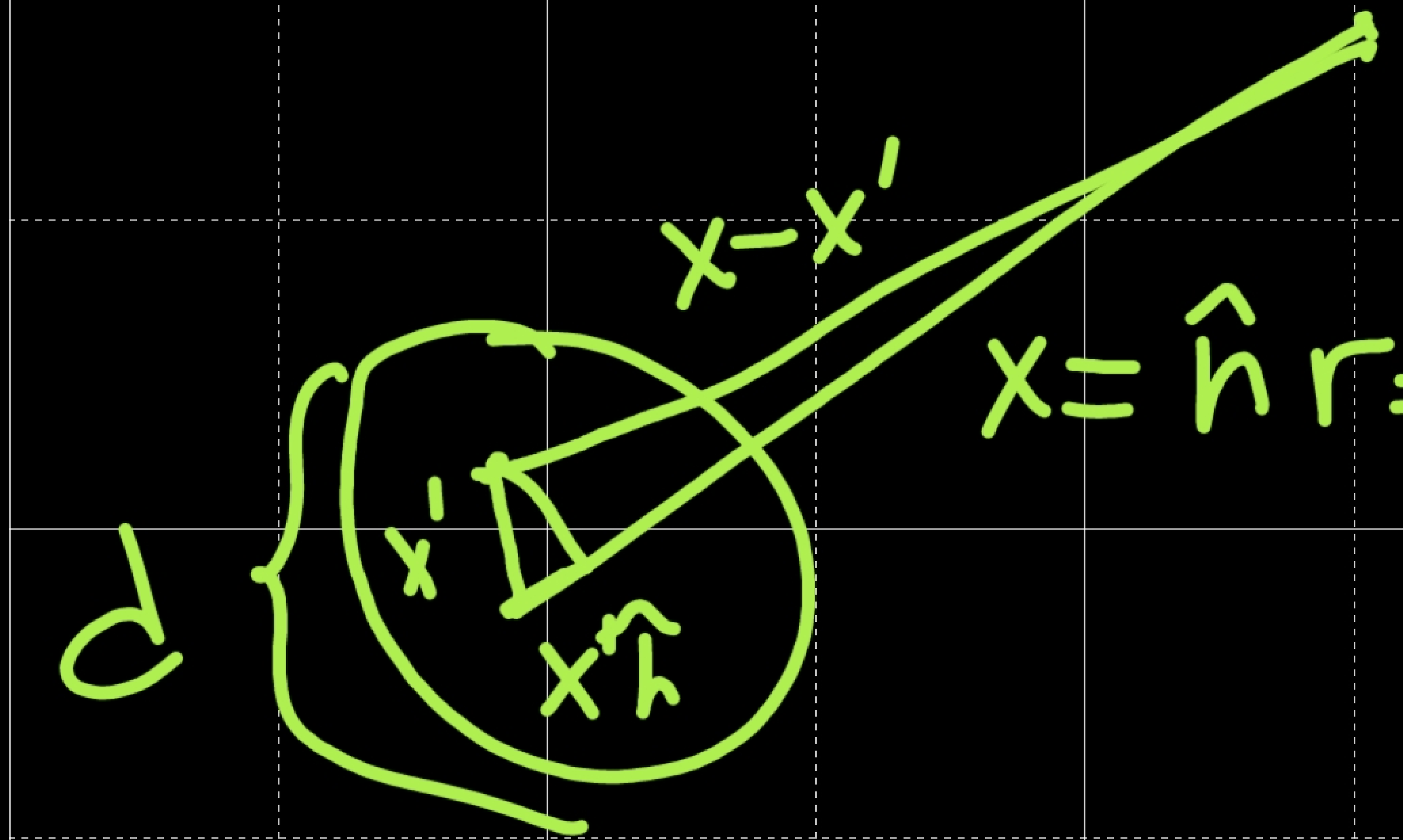
$$p\, k_\mu k_\nu$$

$$\bar{h}_{\mu\nu} = -\int d^4x' \, G(x-x') \, T_{\mu\nu}(x') \cdot \frac{16\pi G}{c^4}$$

$$\Box G(x-x') = \delta^4(x-x')$$

$$G(x-x') = \frac{1}{4\pi |x-x'|} \delta(ct_{ret} - ct') \qquad t_{ret} = t - \frac{|x-x'|}{c}$$

$$\bar{h}_{\mu\nu} = \frac{4G}{c^4} \int d^3x' \, \overbrace{\frac{1}{|x-x'|}}^{r} \, T_{\mu\nu}(t_{ret}, x')$$



$$x = \hat{n} r = \hat{n}\,\underline{|x|}$$

$$\frac{(x' n)\,\omega}{c} \sim \frac{v}{c}$$

$$r \gg d$$

$$|x - x'| = r - (x'\hat{n}) + O\left(\frac{d^2}{r}\right)$$

$$T^{\mu\nu}\left(t - \frac{|x-x'|}{c}, x'\right) = T^{\mu\nu}\left(t + \frac{(x'n)}{c} - \frac{r}{c}, x'\right) = \underbrace{T^{\mu\nu}\left(t - \frac{r}{c}, x'\right)} +$$

$$+ \dot{T}^{\mu\nu}\left(t - \frac{r}{c}, x'\right) \cdot \frac{(x'n)}{c} + \dots$$

$$\bar{h}_{\mu\nu} = \frac{4G}{c^4 r}\int d^3x'\, T_{\mu\nu}\left(t-\frac{r}{c}, x'\right)$$

$$h^{TT}_{ij} = \underbrace{\Lambda_{ij,kl}}\, \bar{h}_{kl} \qquad h^{TT}_{ij} = \Lambda_{ij,kl} \cdot \frac{4G}{c^4 r}\int d^3x'\, T_{kl}\left(t-\frac{r}{c}, x'\right)$$

$$M = \frac{1}{c^2}\int d^3x\, T^{00}(t,x) \qquad S_{kl} = \int d^3x'\, T_{kl}\left(t-\frac{r}{c}, x'\right)$$

$$M^i = \frac{1}{c^2}\int d^3x\, T^{00}(t,x)\, x^i \qquad M(t) = \frac{1}{c^2}\int d^3x\, T^{00}\left(t-\frac{r}{c}, x\right)$$

$$M^{ij} = \frac{1}{c^2}\int d^3x\, T^{00}(t,x)\, x^i x^j$$

$$P^i = \frac{1}{c}\int d^3x\, T^{0i}(t_{ret}, x)$$

$$P^{i,j} = \frac{1}{c}\int d^3x\, T^{0i}(t_{ret}, x)\, x^j$$

$$P^{i,jk} = \frac{1}{c}\int d^3x\, T^{0i}(t_{ret}, x)\, x^j x^k$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_0 T^{00} - \partial_i T^{i0} = 0$$

$$c\dot{M} = \int d^3x\, \dot{T}^{00}(t_{ret}, x) = -\int d^3x\, \partial_i T^{i0} = \int dS^i\, T^{0i} = 0$$

$$c\dot{M}^i = \int d^3x\, \dot{T}^{00}(t_{ret}, x) \cdot x^i = -\int d^3x\, \partial_j T^{j0} \cdot x^i = \int d^3x\, T^{i0} = cP^i$$

$$S^{kl} = \frac{1}{2}\ddot{M}^{kl}$$

$$h_{ij}^{TT} = \frac{2G}{c^4 r}\Lambda_{ij,kl}(\hat{n})\ddot{M}^{kl}\left(t-\frac{r}{c}\right) = \frac{2G}{c^4 r}\Lambda_{ij,kl}(\hat{n})\ddot{Q}^{kl}\left(t-\frac{r}{c}\right)$$

$$Q^{ij} = M^{ij} - \frac{1}{3}\delta^{ij} M = \int d^3x\, \rho(t,x)\left(x^i x^j - \frac{1}{3}\delta_{ij} r^2\right)$$

$$T_{00} = \rho c^2$$